Evidential data mining for Length Of Stay (LOS) prediction problem

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Abstract—Hospitals need to optimize their healthcare planning and organization to minimize costs. The indicator that is often used to measure the efficiency in hospital is the average length of stay. Many studies show a strong and obvious correlation between the costs of patients and the inpatient Length Of Stay (LOS). In this paper, we propose to apply data mining techniques to predict the LOS. An evidential variant of data mining, called evidential data mining, has been used to reduce the impact of uncertainty and missing data. New measures of itemset support and association rule confidence are applied. We introduce the Evidential Length Of Stay prediction Algorithm (ELOSA) that allow the prediction of the length of stay of a new patient. Therefore, the inpatient length of stay (LOS) can be predicted efficiently, the planning and management of hospital resources can be greatly enhanced. The proposal is evaluated on a real hospital dataset using 270 patient traces.

Index Terms—LOS, Association rule; Evidential database, Evidential data mining.

I. INTRODUCTION

The healthcare system integrates many service units according to the needs of the patients. It consists of a number of interacting departments and healthcare units such as outpatient department, emergency department, operating theatre, intensive care unit, and inpatient wards. The operating theatre includes Operating Rooms (OR) and Recovery Rooms (RR). The objectives of a healthcare system are the establishment of an appropriate healthcare planning and organization while minimizing the cost of healthcare, maximizing the utilization of resources (material and human) and improving the quality of healthcare. The OR represents a bottleneck in hospitals [1] and is considered among the most costly resources, it represents more than 9% of the annual budget of hospital [2]. Among the performance indicators in hospital, the average length of stay (ALOS) is considered as the indicator often used to measure the efficiency [3]. The Length of stay (LOS) is the duration of hospitalization patients spend in hospital. This duration is measured in number of days. A patient entering and leaving a hospital on the same day have a length of stay of one. The clinical treatment processes for clinical and financial purposes show a strong correlation between the patients’ cost and LOS [4]. To minimize the cost of healthcare, hospital has to minimize the length of stay of patients.

The uncertainties involved in the healthcare system are significant [5], [6], [7]. They arise mainly due to the number of received patients in emergency department, randomness in service times at various stages of care such as processing time of surgical care in operating rooms and the transitions of an emergency degree of patient during the hospitalization [8]. In this context, the prediction of the LOS is essential to optimize healthcare planning and resources while reducing the impact of uncertainties. A model to predict the LOS hospitalized patients can be an effective tool for healthcare providers. Such a model will enable early interventions to prevent complications and prolonged LOS and guarantee a more efficient use of human resources and facilities in hospitals.

To predict the LOS of patients, several works propose statistical approaches or Artificial Neuronal Networks (ANN) [4]. Tu et al. [9] develop an ANN to predict the LOS in the intensive care unit. Ng et al. [10] present an increment Expectation Maximization (EM)-based learning approach in ANN, it is supported to provide an early prediction of patients requiring a long hospital care. Wrenn et al. [11] develop and valid an ANN to predict LOS for an emergency department. Hachesu et al. [12] use the techniques of classification with three algorithms (decision tree, Support Vector Machines (SVM), and ANN) to predict LOS with various degrees of accuracy. Huang et al. [4] propose a LOS prediction approach based on Case-based Reasoning methodology to predict LOS across various stages of clinical treatment processes. Azari et al. [13] propose an approach for predicting hospital length of stay using a multi-tiered data mining approach. They employ clustering to create the training sets to train different classification algorithms. Jiang et al. [14] use four data mining techniques (logistic regression, neural network, decision tree, and ensemble model) to analyse the inpatient discharge data for average LOS based on input variables. Some other approaches has been used, like Markov process. For instance, it has been used by [15] to estimate the LOS in different severity of illness states in the intensive care unit.

In this work, we tackle the problem of LOS prediction. Starting from a numerical and categorical database, our aim is providing a first length of stay prediction of new coming patients. This prediction is made possible with the use of data mining tools. To ensure a robust method and reduce the impact of uncertainty, the evidential variant of data mining (called also evidential data mining) is used. Those kinds of imperfect data mining support different types of imperfection (imprecision and uncertainty) and even missing data [16]. New measures of itemset (pattern) support and association rule confidence are introduced and applied on the LOS problem. In this paper, we propose to add uncertainty to the length of stay in order to suit the medical staff requirement. In addition, we introduce the Evidential Length Of Stay prediction Algorithm (ELOSA)
that allows the prediction of a new patient the length of stay. The content of this paper is organized as follows: in section II basic concepts of the evidence theory and evidential data mining are recalled. In Section III, the support and the confidence measures are introduced and applied on some examples. The contribution of clustering attributes and specially the LOS is presented in Section IV. In Section V, we introduce our ELOSA algorithm for the length of stay estimation. The proposed prediction algorithm is experimented on a healthcare datasets. Finally, we conclude and we sketch issues of future work.

II. PRELIMINARIES

A. Evidence theory

The evidence theory [17] becomes more and more popular. It is a simple and flexible framework for dealing with imperfect information. It generalizes the probabilistic framework by its capacity to model total and partial ignorance. It is a powerful tool for combining data. Several interpretations have been introduced from evidence theory such as [17], [18], [19]. One of the most used is the Transferable Belief Model (TBM) proposed by Smets [19] to represent quantified beliefs. The TBM model is a non probabilistic interpretation of the evidence theory based on two distinct levels: (i) a credal level where beliefs are entertained and quantified by belief functions; (ii) a pignistic level where beliefs can be used to make decisions and are quantified by probability functions. The evidence theory is based on several fundamentals such as the Basic Belief Assignment (BBA). A BBA $m$ is the mapping from elements of the power set $2^\Theta$ on to $[0, 1]$:

$$m : 2^\Theta \rightarrow [0, 1]$$

where $\Theta$ is the frame of discernment. It is the set of possible answers for a treated problem and is composed of $N$ exhaustive and exclusive hypotheses:

$$\Theta = \{H_1, H_2, ..., H_N\}.$$  

A BBA $m$ do have some constraints such that:

$$\begin{cases} 
\sum_{A \subseteq \Theta} m(A) = 1 \\
 m(\emptyset) \geq 0 
\end{cases}$$

Each subset $X$ of $2^\Theta$ fulfilling $m(X) > 0$ is called focal element. Constraining $m(\emptyset) = 0$ is the normalized form of a BBA and this corresponds to a closed-world assumption [20], while allowing $m(\emptyset) \geq 0$ corresponds to an open world assumption [19].

In the literature, we often come a cross the notion of pignistic probability. The pignistic probability, denoted $BetP$, was proposed by [21] within its Transferable Belief Model (TBM). In the decision phase, the pignistic transformation consists in distributing equiprobably the mass of a proposition $A$ on its included hypotheses. Formally, the pignistic probability is defined by:

$$BetP(H_n) = \sum_{A \subseteq \Theta} \frac{|H_n \cap A|}{|A|} \times m(A) \quad \forall H_n \in \Theta$$

where $|$ is the cardinality operator. In the following section, we present the concept of evidential database based on the evidence theory.

B. Evidential Database

An evidential database stores data that could be perfect or imperfect. Data imperfection in such database is expressed via the evidence theory. An evidential database, denoted by $EDB$, with $n$ columns and $d$ lines where each column $i$ $1 \leq i \leq n$ has a domain $\Theta_i$ of discrete values. Cell of line $j$ and column $i$ contains a normalized BBA as follows:

$$m_{ij} : 2^{\Theta_i} \rightarrow [0, 1] \text{ with }$$

$$\begin{cases} 
m_{ij}(\emptyset) = 0 \\
\sum_{A \subseteq \Theta_i} m_{ij}(A) = 1
\end{cases}$$

Such kind of modelization makes from the evidential database one of the largest representation of any database [22].

<table>
<thead>
<tr>
<th>Transaction</th>
<th>Attribute A</th>
<th>Attribute B</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>$m_{11}(A_1) = 0.7$</td>
<td>$m_{21}(B_1) = 0.4$</td>
</tr>
<tr>
<td></td>
<td>$m_{11}(\Theta_A) = 0.3$</td>
<td>$m_{21}(B_2) = 0.2$</td>
</tr>
<tr>
<td>T2</td>
<td>$m_{12}(A_2) = 0.3$</td>
<td>$m_{22}(\Theta_B) = 0.4$</td>
</tr>
<tr>
<td></td>
<td>$m_{12}(\Theta_A) = 0.7$</td>
<td>$m_{22}(B_1)_{22} = 1$</td>
</tr>
</tbody>
</table>

**TABLE I**

EVIDENTIAL TRANSACTION DATABASE $EDB$

In an evidential database, as shown in Table I, an evidential item corresponds to a focal element. An evidential itemset corresponds to a conjunction of focal elements having different domains. Two different evidential itemsets can be related via the inclusion or intersection operator. Indeed, the inclusion operator [23] for evidential itemsets is defined as follows, let $X$ and $Y$ be two evidential itemsets:

$$X \subseteq Y \iff \forall x_i \in X, x_i \subseteq y_i$$

where $x_i$ and $y_i$ are the $i^{th}$ elements of $X$ and $Y$, respectively. For the same evidential itemsets $X$ and $Y$, the intersection operator [23] is defined as follows:

$$X \cap Y = Z \iff \forall z_i \in Z, z_i \subseteq x_i$$

Finally, an Evidential associative rule $R$ is a causal relationship between two itemsets that can be written in the following form $R : X \rightarrow Y$ fulfilling $X \cap Y = \emptyset$.

**Example 1:** From Table I, $A_1$ is an item and $\Theta_A \times B_1$ is an itemset such that $A_1 \subset \Theta_A \times B_1$ and $A_1 \cap \Theta_A \times B_1 = A_1$. $A_1 \rightarrow B_1$ is considered as an association rule. $A_1$ is the premise and $B_1$ is the conclusion part.
III. EVIDENTIAL SUPPORT AND CONFIDENCE FOR LOS PROBLEM

In this section, we study the contribution of imperfect data mining and specially evidential data mining in LOS problem. Indeed, in real life, gathered information are suffering from imperfection due to many factors such as acquisition reliability, human errors, information absence, etc. In [16], Lee detailed the two sides of imperfection that could manifest in a database. As example, let us take the example of doctor having to pronounce symptoms and several other information regarding the treated patient. Some times doctors can not give a straight and precise answer regarding some measurement. In addition, doctors may omit data in case of lacking further information.

For this reason, the use of imperfect data mining is hugely recommended due to the fact this it adapt to those kind of data. The evidential database allows to model uncertainty and imprecision. In reality, it may be used to model answers for asked questions. Data mining relies on two major definition: the support of itemsets and the confidence measure. The first measure estimates the degree of presence of an itemset within the whole database whereas the second measures the pertinence of a rule. In this following, we present the support and the confidence measure introduced by Samet et al. [23], [24] within evidential databases.

Let us consider an evidential database $E\mathcal{DB}$ and the itemset $X = x_1 \times \cdots \times x_n$ constituted by the product of items (focal elements) $x_i$ ($1 \leq i \leq n$) of the exclusive frame of discernment $\Theta_i$. The degree of presence of an item $x_i$ in a transaction $T_j$ (BBA) can be measured as follow:

$$ Pr : 2^\Theta \rightarrow [0, 1] $$

$$ Pr(x_i) = \sum_{x \subseteq \Theta_i} \frac{|x_i \cap x|}{|x|} \times m(x) \quad \forall x_i \in 2^{\Theta_i}. \quad (5) $$

As illustrated above, the $Pr(.)$ measure allows to compute $x_i$ presence in a single BBA. The $Pr$ measure is equal to the pignistic probability if $x_i \in \Theta_i$. The evidential support of an itemset $X = \prod_{i \in [1...n]} x_i$ is then computed as follows:

$$ Sup_{T_j}^{Pr}(X) = \prod_{x_i \in \Theta_i, i \in [1...n]} Pr(x_i) \quad (6) $$

$$ Sup_{E\mathcal{DB}}(X) = \frac{1}{d} \sum_{j=1}^{d} Sup_{T_j}^{Pr}(X). \quad (7) $$

The confidence of an evidential association rule $R : R_a \rightarrow R_c$ is computed based on the precise measure as follows:

$$ Conf(R) = \frac{\sum_{j=1}^{d} Pr_{T_j}(R_a) \times Pr_{T_j}(R_c)}{\sum_{j=1}^{d} Pr_{T_j}(R_a)} \quad (8) $$

where $d$ is the number of transactions in the evidential database. Thanks to its probabilistic writing, the proposed metric sustains previous confidence measure such that introduced in [25].

Example 2: Let us consider the Table II that represents two patients with their personal data and length of stay. The emergency degree information is represented with a BBA having the following frame of discernment $\Theta_1 = \{High(H), Medium(M), Low(L)\}$. $H$ refers to a high emergency degree. For the age attribute, its frame of discernment is $\Theta_2 = \{Old(O), Adult(A), Young(Y)\}$. For a new patient having a low emergency degree and old we have the following association rules:

$$(L) + (O) \rightarrow 2 \iff Conf = \frac{0.51}{1.51} = 0.33$$

$$(L) + (O) \rightarrow 4 \iff Conf = \frac{1}{1.51} = 0.66$$

Treating the numerical attributes such that the LOS (see example 2) would be a difficult task. A discretization process should be conducted before using data mining tools. In the following section, we provide a method to construct an evidential database from a raw dataset.

IV. EVIDENTIAL CLUSTERING OF THE ATTRIBUTES

In this section, we discuss how to construct an evidential database from numerical and categorical data. It is obvious that in case of categorical data where there is no room for uncertainty, the BBA construction is the most simple task. Indeed, in case of modelling a categoric data such as the sex of the patient a certain BBA is constructed as the following example:

$$ \begin{align*}
    m(Male) &= 1 \\
    M(Female) &= 0 
\end{align*} \quad (9) $$

However for numerical data such as blood pressure measure, temperature, etc, further transformation is required. In the following, we propose a method that allows to construct an evidential database from a numerical dataset. We based our evidential database construction on the ECM [26] clustering approach. It is an C-Means like algorithm based on the concept of credal partition, extending those of hard, fuzzy, and possibilistic ones. From a set of numerical data, it is possible to construct an evidential database with ECM. ECM starts by creating the user requested number of cluster. Then, ECM estimates the distance that separate the considered data from each cluster’ center. A BBA is created depending on the computed distance. Afterwards, ECM tries to minimize the objective function defined in Equation 10. ECM compute

<table>
<thead>
<tr>
<th>Patient</th>
<th>Emergency degree</th>
<th>Age</th>
<th>Length of stay</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>$m_{11}(L) = 0.7$</td>
<td>$m_{21}(O) = 0.4$</td>
<td>2</td>
</tr>
<tr>
<td>P2</td>
<td>$m_{12}(L) = 1$</td>
<td>$m_{22}(O) = 1$</td>
<td>4</td>
</tr>
</tbody>
</table>
recursively the cluster’s center until the objective function is minimized. From evidential data mining point of view, ECM allows to construct for each tuple a BBA that represents its membership to each cluster. The clusters represent the different categories we may recover from an attribute.

Formally, to derive such a structure, we minimized the proposed objective function:

$$J_{ECM}(M, V) \triangleq \sum_{i=1}^{d} \sum_{(j/A_j \neq \emptyset, A_j \subseteq \Theta)} c^2_{ij} m^\beta_{ij} \text{dist}^2_{ij} + \sum_{i=1}^{n} \delta^2 m^\beta_{i\emptyset}$$

subject to:

$$\sum_{(j/A_j \neq \emptyset, A_j \subseteq \Theta)} m_{ij} + m_{i\emptyset} = 1 \quad \forall i = 1, d$$

where $m_{ij}$ and $m_{i\emptyset}$ denote respectively $m_i(\emptyset)$ and $m_i(A_j)$. $M$ is the credal partition $M = (m_1, \ldots, m_d)$ and $V$ is a cluster centers matrix. $c^2_{ij}$ is a weighting coefficient and $\text{dist}_{ij}$ is the Euclidean distance. In our case, the parameters $\alpha$, $\beta$ and $\delta$ were fixed to 1, 2 and 10.

Generally, in prediction problems with association rules, the conclusion part is constituted by a precise element. For example, in an LOS problem, the conclusion part of the association rule have to be the exact number of the days. However, such construction may induce to errors. As example, let us consider a doctor having to pronounce on the length of stay of a patient based on some symptoms. The pronounced length of stay can only be a prediction. Indeed, unexpected events or patient health aggravation could alter the doctor’s prediction. In addition, if we consider the Example 2 in which we prcised the length of stay, we may notice that the confidence is not high for both rules. Indeed, if we have changed the length of stay with an imperfect value, we could have more confidence in the rule. With a use of ECM on the length of stay, the obtained association rule may look as “if the patient have a low emergency degree and he is old, he might stay for short period (around 3.5 days)”. Another contribution, represented in Figure 1 of such use of ECM is imperfection of the conclusion. In fact, we may encounter rules with a disjunction of hypothesis in the conclusion part in case of confusion on the prediction. So, the resulting rule may look like “if the patient have a low emergency degree and he is old, he might stay for short or medium period (around 3.5 days and 10 days)”. In the next section, we introduce the ELOSA algorithm for the inpatient LOS prediction.

V. PREDICTIVE EVIDENTIAL DATA MINING APPROACH FOR LOS PROBLEM

In this section, we introduce our algorithm for patient’s length of stay prediction. Algorithm 1 details the different step in order to predict the length of stay. Starting from non treated database $DB$ representing the records of several patients, we construct the evidential database $EDB$ with use of the $ECM(\cdot)$ function. The same goes for the instance to classify $X$ which is evidentialized. As a result, the constructed $\tilde{X} = \{m_{iX}, i \in [1, n]\}$ is a set of BBA where each $m_{iX}$ represents the membership of $X$ to all clusters within the $i^{th}$ attribute. From $X$, the premise part of the association rule is constructed. Indeed, those focal elements that maximize the pignistic probability (i.e., Equation (2)) are retained and constitute the premise part. Afterwards, the provided algorithm, called ELOSA, construct all association rules having an element from the superset $2^{\Theta}$ in conclusion part. The depth of the focal element must not exceed a fixed value $\text{depth}_{\text{max}}$. The depth can be written as follows:

$$\text{depth}(A)_{A \in \Theta} = |A|$$

The rule that maximize the confidence is retained for the prediction and denoted as $R_{\text{max}}$.

VI. EXPERIMENTAL RESULTS

The experiments were conducted on a real hospital dataset. The dataset contains 270 patients. A sample of the treated dataset is summarized in Table III. The database illustrates some patient personnel informations such as their age and sex. The emergency degree column is a categoric attribute that take value from the following set $\{A, R, D\}$. $A$ indicates the absolute emergency, $R$ stands for relative emergency whereas $D$ for delayed one. Finally, the operation length is measured in hours and the length of stay is the class $\{(\text{Short}(S),\text{Medium}(M),\text{Long}(L))\}$.

Figure 2 shows 70 patients’ length of stay set between the LOS clusters. Three clusters are shown and could be seen as the length of stay classes. Indeed, the red line is the center of the short stay, the blue one refers to the medium stay and the green designates the long stay. So, having the short class in the conclusion part of the association rule means that the length of stay is neighbouring the short stay (nearby 2.4 days). However a rule having $(M) \cup (L)$ means that there is uncertainty about the patient LOS and the numerically it could between 6.8
Algorithm 1 Evidential LOS Prediction Algorithm (ELOSA)

Require: $DB, minconf$, $\Theta_C$, $X$, $depth_{max}$

Ensure: $R_{max}$

1: $\mathcal{EDB} \leftarrow ECM(DB)$
2: $X \leftarrow ECM(X)$
3: $Premise_X \leftarrow \prod_{i<|\mathcal{EDB}|} \arg\max_{H_k \in \Theta_C} BetP(H_k)$
4: Initialize($R_{max}$)
5: for all $l \in 2^{\Theta_C}$ & $depth(l) \leq depth_{max}$ do
6: \hspace{1em} $R \leftarrow \{Premise_X \rightarrow l\}$
7: \hspace{1em} if $Conf(R) > Conf(R_{max})$ & $Conf(R) > minconf$ then
8: \hspace{2em} $R_{max} \leftarrow R$
9: \hspace{1em} end if
10: end for

and 17.7 days. We proceeded to cross validation experiments where the entire database was used for the machine leaning.

Figure 3 and 4 show respectively the number of frequent pattern and association rule depending on the fixed value of $minsup$. The frequent patterns designate the usual scenarios treated in the hospital. They are a valuable information to draw the nature of the admission and maybe could be in help to adapt the hospital in the future for such patients. On the other hand, Figure 4 illustrates the association rules that can be used for the prediction. The number of the pattern and association rule is important due to the clustering of the attributes and the class (LOS).

The ELOSA algorithm was tested on the provided dataset.
Two versions have been proposed: ELOSA$_1$ (Depth = 1) and ELOSA$_2$ (Depth = 2). ELOSA$_1$ uses precise association rule (conclusion constituted only with a singleton class) for prediction as described in Algorithm 1. ELOSA$_2$ generates further rules reaching a Depth = 2 in the conclusion part and retains only the rule with the highest confidence. ELOSA$_1$ provides 70% of good prediction. Since ELOSA$_2$ provides a larger conclusion part, we tested if the truth is member of conclusion. Indeed, 90% of ELOSA$_2$ prediction contains the truth. We also compared our results to an ELOSA$_{LB}$ approach based on [27] evidential support and confidence measure. Our approaches provide better result comparatively to ELOSA$_{LB}$ and this due to the effect of the precise support and confidence.

<table>
<thead>
<tr>
<th>Prediction approach</th>
<th>ELOSA$_1$</th>
<th>ELOSA$_2$</th>
<th>ELOSA$_{LB}$</th>
<th>ELOSA$_{LB}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>% good prediction</td>
<td>70%</td>
<td>90%</td>
<td>63%</td>
<td>82.7%</td>
</tr>
</tbody>
</table>

TABLE IV

PERCENTAGE OF GOOD PREDICTION

VII. CONCLUSION

This paper introduces an inpatient LOS prediction approach based on Evidential data mining. This approach handles the uncertainty, imprecision and missing data within a database. We introduced the Evidential Length Of Stay prediction Algorithm (ELOSA) to predict the length of stay of a new patient in a hospital with different level of uncertainty. The introduced algorithm is based on the precise support and association rule confidence measures. The suitability of this approach was confirmed by testing and experimenting the proposed algorithm on a healthcare datasets containing 270 patients. To predict the inpatient LOS, we consider a priori information, such age, sex and physiological conditions (emergency degree) of patients. In future studies, the proposed approach will be extended to predict other features of healthcare system such as the processing time of operations in the operating theatre. Another important area for future research is the integration and the combination of operational research techniques to optimize healthcare planning and organization such as scheduling of surgical cares in the operating rooms.

REFERENCES