Argumentation Framework Based on Evidence Theory

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Abstract. In many fields of automated information processing it becomes crucial to consider imprecise, uncertain or inconsistent pieces of information. Therefore, integrating uncertainty factors in argumentation theory is of paramount importance. Recently, several argumentation based approaches have emerged to model uncertain data with probabilities. In this paper, we propose a new argumentation system called \textit{evidential argumentation framework} that takes into account imprecision and uncertainty modeled by means of evidence theory. Indeed, evidence theory brings new semantics since arguments represent expert opinions with several weighted alternatives. Then, the evidential argumentation framework is studied in the light of both Smets and Dempster-Shafer interpretations of evidence theory. For each interpretation, we generalize Dung’s standard semantics with illustrative examples. We also investigate several preference criteria for pairwise comparison of extensions in order to select the ones that represent potential solutions to a given decision making problem.

Keywords: Argumentation Theory, Evidence Argumentation Framework, Pignistic Scenario Graph, Belief Scenario Graph.

1 Introduction

Argumentation has long been a major topic in Artificial Intelligence (see e.g.,\cite{12} and for more recent accounts e.g.,\cite{3}) that has concerned a large variety of application domains for more than a decade, like e.g., medicine \cite{4}, law \cite{5,6}, negotiation \cite{7}, decision making \cite{8}, multiagent systems \cite{9,10}, semantic web \cite{11}, and databases \cite{12}, etc. Argumentation is basically concerned with the exchange of interacting arguments. This set of arguments may come either from a dialogue between several agents but also from the available (and possibly contradictory) pieces of information at the disposal of one unique agent. Usually, the interaction between arguments takes the form of a conflict, called \textit{attack}. Two main families of computational models for argumentation have been studied in the literature: namely, the abstract and the logic-based argumentation frameworks. Following
the seminal work of [13], the first family is based on graph-oriented representations and focuses mainly on the interaction between arguments without taking the possible internal structure of the involved arguments into account. Different acceptability semantics for abstract argumentation frameworks have also been proposed that highlight different aspects of argumentation. Basically, each of these semantics corresponds to some properties which certify whether a set of arguments can be profitably used to support a point of view in a discussion. On the contrary, the logic-based approaches (e.g., [14,15,16,17]) exploit the logical internal structure of arguments and adopt inconsistency as a pivotal paradigm: any pair of conflicting arguments must be contradictory.

Although Dung’s frameworks are widely approved tools for abstract argumentation, their abstractness make expressing notions such as support or uncertainty very difficult. So far a plethora of works have been introduced in order to model the uncertainty of an argument with a probability [18,19,20]. However, weighting arguments in this manner has several drawbacks, due mainly to uncertainty representation. To illustrate this point, let us consider, for example the following three doctors that prescribe medication to a patient. To cure the patient headache, the first doctor prescribes several paracetamol-family drugs (e.g. \(P_1\) and \(P_2\)) with weights to express his/her preferences. The second doctor favours all biological medical products and therefore (s)he prescribes an andrographis drug. Finally, a third doctor is hesitating whether to give the paracetamol-based product \(P_1\) or the andrographis one without a further complementary analysis on the patient. In this argumentation context, uncertainty is ubiquitous in all opinions. In addition, doctors are confronting arguments about the convenience and the applicability of drug alternatives (drug brands) rather a type of medication. This kind of example is difficult to handle with classical probability-based approaches.

The aim of this paper is to extend uncertainty consideration in argumentation theory. In fact, uncertainty is modelled thanks to evidence theory rather with classical probabilities to solve problems as the one described above. A new framework, called evidential argumentation framework, is introduced to model experts’ opinion over alternatives. The latter is studied and interpreted in the light of two evidence theory interpretations: the non probabilistic model of Smets [21] and the probabilistic one of Dempster-Shafer [22,23]. Therefore, several acceptability semantics are generalized based on scenario graphs derived from the evidential argumentation framework.

The roadmap of this paper is organized as follows. The basic foundations of abstract argumentation theory are detailed in Section 2. In Section 3, the evidential argumentation framework is motivated and introduced. Then, our approach is studied and interpreted in the light of two evidence theory interpretations. Moreover, acceptability semantics are generalized for evidential argumentation framework. We also introduce various preference relations at the semantics level in order to determine what are desirable outcomes of the argumentation framework. Finally, we conclude and sketch potential issues for the future work.
2 Abstract Argumentation Framework: Brief Overview

In this section, we briefly outline the notion of abstract argumentation framework and various semantics studied in the literature.

An abstract argumentation framework (AF), where \( A \) is a finite set, whose elements are called arguments, and \( R \subseteq A \times A \) is a binary relation over \( A \), whose elements are referred to as attacks. An argument is an abstract entity whose role is entirely determined by its relationships with other arguments.

An AF can simply be represented as a directed graph, called attack graph, where nodes are the arguments and edges represent the attack relation. Throughout the paper examples are using this graph representation.

Given two arguments \( A \) and \( B \), we say that \( A \) attacks \( B \) iff there is \((A, B) \in R\). Moreover, a set \( S \subseteq A \) attacks an argument \( B \in A \) iff there is \( A \in S \) s.t. \( A \) attacks \( B \). A set \( S \subseteq A \) of arguments is said to be conflict-free if there are no arguments \( A, B \in S \) s.t. \( A \) attacks \( B \). An argument \( A \) is defended by a set \( S \subseteq A \) iff \( \forall B \in A \) s.t. \( B \) attacks \( A \), there is \( C \in S \) s.t. \( C \) attacks \( B \).

Using the notions of conflict-freeness and defense, we can define a number of argumentation semantics, each embodying a particular rationality criterion, in order to identify reasonable sets of arguments, called extensions.

**Definition 1 (Acceptability semantics).** Given an argumentation framework \( F = \langle A, R \rangle \). A set \( S \subseteq A \) of arguments is said to be:

- admissible iff \( S \) is conflict-free and all its arguments are defended by \( S \)
- a complete extension iff \( S \) is conflict-free and \( S \) attacks each argument in \( A \setminus S \)
- a preferred extension iff \( S \) is a minimal (w.r.t. set inclusion) complete set of arguments
- a preferred extension iff \( S \) is a maximal (w.r.t. set inclusion) admissible set of arguments
- an ideal extension iff \( S \) is admissible and \( S \) is contained in every preferred set of arguments.

**Example 1.** Consider the AF \( F = \langle A, R \rangle \) such that \( A = \{A, B, C, D, E\} \) and \( R = \{(A, B), (C, B), (C, D), (D, C), (D, E)\} \). The graph representation of \( F \) is indicated on Figure 1. This argumentation framework has two preferred extensions: \( \mathcal{E}_1 = \{A, C, E\} \) and \( \mathcal{E}_2 = \{A, D\} \); these are also the unique stable extensions. Moreover, \( F \) possesses a unique ideal extension \( \{A\} \).

3 Evidence Theory Based Argumentation Framework

One of the abstract argumentation frameworks shortcomings is the insufficient handling of the levels of uncertainty, an aspect which typically occurs in domains, where diverging opinions are raised. In this section, we describe a new framework...
of abstract argumentation based on evidence theory in which uncertainty has two dimensions. In other words, we intend to model an argumentation system that handles imprecise opinions as arguments. Indeed, each argument (i.e., opinion) is described over the powerset of alternatives (hypotheses). In the sequel, we denote by \( \Theta_A \) the frame of discernment of the alternatives of the argument \( A \).

**Definition 2 (Evidential Argumentation Framework).** An evidential argumentation framework is a tuple \( V = \langle A, R, m \rangle \) where \( \langle A, R \rangle \) is an AF, \( m = \{ m_A, A \in A \} \), and \( m_A \) (basic belief assignment (bba)) is a mapping from elements of the powerset \( 2^{\Theta_A} \) onto \([0, 1]\) such that:

\[
\begin{align*}
    m_A(\emptyset) &= 0 \\
    \sum_{a \in \Theta_A} m_A(a) &= 1
\end{align*}
\]

In Definition 2, \( a \) corresponds to a hypothesis of the argument \( A \) and is called an alternative of \( A \). Clearly, an evidential argumentation framework differs from classical argumentation models since it deals with arguments’ alternatives rather than arguments. In this study, the arguments are supposed to be independent.

**Example 2.** Let us consider the evidential argumentation graph shown in Figure 2. Five arguments are considered \( A = \{ A, B, C, D, E \} \). Each argument highlights the diagnostic of a doctor and the intended prescription. Each prescription contains either one of two alternatives from a drug family. In other words, an argument represents a drug type family having several brands. A drug brand of a single family has its own properties, effects and prescriptions. Therefore, a mass is given to each drug brand depending on the treated patient. For example, \( A = \) “Patient has hypertension so prescribe drug family \( A \) with a higher preference to \( A_1 \) of all other same family products”. In addition, a doctor can hesitate between same drug family alternatives. For example argument \( E = \) “Patient has hypertension so prescribe one of the medication of the drug family \( E \) without any preference to any of them.”. Here, we assume that \( C \) and \( D \) attack each other because we should only give one treatment and so giving one precludes the other, and we assume that \( A \) and \( C \) attack \( B \) because they provide a counterargument to all \( A \) family medications.

Several interpretations exist for evidence theory such as \([21,22,25]\). In the remainder, we build our contributions following the Transferable Belief Model.
Evidential Argumentation Framework

(TBM), that was introduced by Smets [21] and the probabilistic interpretation of Dempster-Shafer [22,23]. The TBM model is a non-probabilistic interpretation of the theory, that aims at representing quantified beliefs based on two levels: (i) a credal level where beliefs are entertained and quantified by belief functions; (ii) a pignistic level where beliefs can be used to make decisions and are quantified by probability functions. In the following subsection, we provide the argumentation framework based on the TBM interpretation.

3.1 Pignistic Argumentation Framework

In the following, we analyse the evidential argumentation framework using Smets TBM interpretation. Even if TBM is not limited to normalized bbas (i.e., bba with a null mass over the empty-set), we restrict our study to bbas without conflict.

**Definition 3.** Let $V = \langle A, R, m \rangle$ be an evidential argumentation framework. A pignistic scenario graph is a tuple $G_p = \langle A, R, P \rangle$ such that $P = \{ P_A, A \in A \}$ where $P_A$ is a pignistic probability defined as:

$$P_A(a) = \sum_{x \subseteq \Theta_A} \frac{|a \cap x|}{|x|} \times m_A(x) \quad \forall a \in \Theta_A$$

where $m_A$ is the bba of the argument $A \in A$ and $\cdot$ is the cardinality operator.

Obviously, the value assigned by $P_A(a)$ represents the probability that the alternative $a$ actually occurs. This probability also considers the absolute ignorance (i.e., $\Theta_A$) when probabilities are built. Comparatively to the state-of-the-art works, the pignistic scenario graph analyses the attacks of the hypotheses of arguments rather than arguments. This could be seen as an extension of previous works [24]. Indeed, to recover the method of [24], a certain bba\footnote{A certain bba expresses the total certainty. It is defined as follows: $m(A) = 1$ and $m(B) = 0$ for all $B \neq A$ and $B \subseteq \Theta$, where $A$ is a singleton event of $\Theta$.} must be constructed over single hypothesis. In addition, it is important to note that the number of pignistic scenario graphs $N$ that could be retrieved from an evidential argumentation framework is computed as $N = \prod_{A \in A} |\Theta_A|$. Indeed, the number of scenarios depends on the arguments’ frame of discernment size.

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Fig. 2: Example of an evidential argumentation graph
However, a simple heuristic could be applied to drop alternatives with a low pignistic probability. This approach consists of dropping alternatives with a pignistic probability lower than 0.5. Thus, the number of retrieved scenario graphs is drastically reduced.

Example 3. Figure 3 shows an example of pignistic scenario graph built from the evidential argumentation framework presented in Example 2.

![Fig. 3: A pignistic scenario graph from Example 2](image)

Note that some of the provided constraints of [26] in probabilistic argumentation framework could be recovered in our pignistic argumentation framework. The pignistic probability function may take different aspects of the structure of the argument graph into account. More formally,

- $P$ is **coherent** w.r.t. $G_p$ if for every $a \in \Theta_A$, $\sum_{a \subseteq \Theta_A} P_A(a) = 1$.
- $P$ is **semi-founded** w.r.t. $G_p$ if $P_A(a) \geq 0.5$ for every unattacked $a \in \Theta_A$.
- $P$ is **founded** w.r.t. $G_p$ if $P_A(a) = 1$ for every unattacked $a \in \Theta_A$.
- $P$ is **semi-optimistic** w.r.t. $G_p$ if $P_A(a) \geq 1 - \sum_{(a,b) \in R} P_B(b)$ for every $a \in \Theta_A$ that has at least one attacker.
- $P$ is **optimistic** w.r.t. $G_p$ if $P_A(a) \geq 1 - \sum_{(a,b) \in R} P_B(b)$ for every $a \in \Theta_A$.
- $P$ is **justifiable** w.r.t. $G_p$ if $P$ is coherent and optimistic.
- $P$ is **rational** w.r.t. $G_p$ if $P_A(a) \in \{0, 0.5, 1\}$ for every $a \in \Theta_A$.
- $P$ is **neutral** w.r.t. $G_p$ if $P_A(a) = 0.5$ for every $a \in \Theta_A$.
- $P$ is **involuntary** w.r.t. $G_p$ if for every $a \in \Theta_A$, $b \in \Theta_B$, if $(A, B) \in R$ then $P_A(a) > 0.5$ implies $P_B(b) \leq 0.5$.
- $P$ is **maximal** w.r.t. $G_p$ if $P_A(a) = 1$ for every $a \in \Theta_A$.
- $P$ is **minimal** w.r.t. $G_p$ if $P_A(a) = 0$ for every $a \in \Theta_A$.

**Definition 4.** Let $G_p = (A, R, P)$ be a pignistic scenario graph such that $a \in \Theta_A$ and $b \in \Theta_B$. Then, $a$ is **stronger than** $b$, denoted by $a \succ_s b$, if and only if $P_A(a) > P_B(b)$.

In order to characterize a particular argumentation semantics for a pignistic argumentation framework, we define a version of the semantics introduced in Definition 1.
Definition 5. Let $G_p = \langle A, R, P, \Theta_A \rangle$ be a pignistic scenario graph s.t. $a \in \Theta_A$, $b \in \Theta_B$ and $S \subseteq \Theta_A$. We say that:

- $a$ defeats $b$ if and only if $a$ attacks $b$, $b$ does not attack $a$, and $a \succ_s b$.
- $a$ is p-acceptable with respect to $S$, if \forall b which defeats $a$ there exists $c \in S$ such that $c$ defeats $b$.
- a set $S$ of arguments is p-conflict-free if there are no arguments $a, b \in S$ such that $a$ attacks $b$.
- $S$ is p-admissible iff $S$ is conflict-free and all its arguments are acceptable w.r.t. $S$.
- $S$ is a p-stable extension iff $S$ is conflict-free and $S$ attacks each argument in $\Theta_A \setminus S$.
- $S$ is a p-complete extension iff $S$ is admissible and $S$ contains all and only the arguments it defends.
- $S$ is a p-grounded extension iff $S$ is a minimal (w.r.t. set inclusion) complete set of arguments.
- $S$ is a p-preferred extension iff $S$ is a maximal (w.r.t. set inclusion) admissible set of arguments.
- $S$ is a p-ideal extension iff $S$ is admissible and $S$ is contained in every preferred set of arguments.

Let $\text{Ext}_x(G_p)$ denote the set of extensions of the pignistic scenario graph $G_p$ under semantics $x$ where $x \in \{a, s, c, g, p, i\}$ and $a$ (resp. $s, c, g, p, i$) stands for p-admissible (resp. p-stable, p-complete, p-grounded, p-preferred and p-ideal).

When the semantics are not important, or when it is clear from the context to which semantics we refer to, we use the notation $\text{Ext}(F)$ for short.

Example 4. Let us consider the pignistic scenario graph depicted in Figure 3. We have two p-preferred extensions $E_1 = \{A_1, C_2, E_1\}$ and $E_2 = \{A_1, D_1\}$.

By applying a basic argumentation semantics to a pignistic scenario graph, one can infer different extensions which represent potential solutions to a given decision making problem. Since an extension has different arguments that argue for a particular decision, some criteria for selecting a suitable decision are worth defining. In other words, it is desirable to compare extensions based on the arguments which support a decision with level of certainty. There exist several different approaches to induce a preference relation over extensions. The first comparison criterion is based on the cardinality of the set of arguments.

Definition 6. Let $G_p = \langle A, R, P \rangle$ be a pignistic scenario graph and $\mathcal{E}_1, \mathcal{E}_2 \in \text{Ext}(G_p)$. Then, $\mathcal{E}_1$ is cardinality-preferred to $\mathcal{E}_2$, denoted by $\mathcal{E}_1 \succeq_c \mathcal{E}_2$, iff $|\mathcal{E}_1| > |\mathcal{E}_2|$.

In certain applications, counting is not the best method of defining an order between extensions. Therefore, a more cautious preference relation can be defined based on probability of the arguments induced in a given extension. For the next preference relation, we need to provide a new metric. To do so, let
Let \( \mathcal{E} = \{a_1, \ldots, a_n\} \subset \text{Ext}(G_p) \) be a given extension, the weight of \( \mathcal{E} \) is \( W(\mathcal{E}) = \sum_{i=1}^{n} P_A(a_i) \). Now, let us define an ordering for pairwise comparison of extensions as follows.

**Definition 7.** Let \( G_p = \langle A, \mathcal{R}, \mathcal{P} \rangle \) be a pignistic scenario graph. Let \( \mathcal{E}_1, \mathcal{E}_2 \in \text{Ext}(G_p) \). Then, \( \mathcal{E}_1 \) is probability-preferred to \( \mathcal{E}_2 \), denoted by \( \mathcal{E}_1 \succ_p \mathcal{E}_2 \), iff \( W(\mathcal{E}_1) > W(\mathcal{E}_2) \).

**Example 5.** Let us consider again the pignistic scenario graph depicted in Figure 3. We have \( W(\mathcal{E}_1) = 1.7 \) and \( W(\mathcal{E}_2) = 1.8 \), then \( \mathcal{E}_2 \succ_p \mathcal{E}_1 \).

Notice that several extensions can be obtained through each pignistic scenario graph. Now, in order to select the outcome of the original argumentation framework, it will be of interest to compare different extensions from the different pignistic scenario graphs. To do this, let us consider the following definition.

**Definition 8.** Let \( \mathcal{V} = \langle A, \mathcal{R}, m \rangle \) be an evidential argumentation framework and \( G^1_p, \ldots, G^n_p \) the set of pignistic scenario graphs obtained from \( \mathcal{V} \). Let \( W(G^i_p) = \sum_{\mathcal{E}} W(\mathcal{E}_i) \) such that \( \{\mathcal{E}_1, \ldots, \mathcal{E}_m\} \) are the extensions over the pignistic scenario graph \( G^i_p \). Then, the outcome of \( \mathcal{V} \), denoted by \( \text{Ext}(\mathcal{V}) \), is defined as:

\[
\text{Ext}(\mathcal{V}) = \{\mathcal{E} \mid \mathcal{E} \in \text{Ext}(G_p), \ G_p = \text{argmax}_{i=1}^{n} W(G^i_p)\}
\]

The pignistic scenario graph represents the TBM-based approach to handle the evidential argumentation framework. Another interpretation can be obtained based on Dempster-Shafer works. In fact, a lower and an upper bound on the degree of belief of a single argument’s alternative could be assigned.

### 3.2 Belief Argumentation Framework

In the following, we intend to analyse the evidential argumentation framework in the light of Dempster-Shafer interpretation. In this context, each alternative’s pertinence is bounded by an upper and a lower bound.

**Definition 9.** Let \( \mathcal{V} = \langle A, \mathcal{R}, m \rangle \) be an evidential argumentation framework. A belief scenario graph is a tuple \( G_B = \langle A, \mathcal{R}, \text{Bel}, \text{Pl} \rangle \) such that for all \( A \in \mathcal{A} \), \( \text{Bel}_A : \Theta_A \rightarrow [0, 1] \) and \( \text{Pl}_A : \Theta_A \rightarrow [0, 1] \) are, respectively, the belief and the plausibility functions over the hypotheses of \( A \) where:

\[
\text{Bel}_A(a) = \sum_{\emptyset \neq x \subseteq a} m_A(x) \quad (1)
\]

\[
\text{Pl}_A(a) = \sum_{a \cap x \neq \emptyset} m_A(x) \quad (2)
\]

**Example 6.** Figure 4 shows an example of a belief scenario graph built from the evidential argumentation framework of Example 2. Each argument is labelled by a belief and a plausibility functions of a single alternative.
Definition 10. Let \( \Theta_A \) be a belief scenario graph and \( a \in \Theta_A \). The strength of \( a \) is defined as \( \text{Bel}(a) \).

The use of a dual value for dealing with the strength of an argument, as it is done in Definition 10, was explored in the context of possibilistic theory in [27]. The strength of an argument allows us to compare pairs of arguments. Informally, an argument is all the better as it uses more certain knowledge and refers to an important goal. This can be formally captured by a Pareto-based comparison criterion [4].

Definition 11. Let \( \Theta_B \) be a belief scenario graph and \( b \in \Theta_B \). Then, \( a \) is stronger than \( b \), denoted \( a \succ_b b \), if and only if:

\[
(\text{Bel}(a), \text{Pl}(a)) \succeq_{\text{pareto}} (\text{Bel}(b), \text{Pl}(b))
\]

4 Let \( x_1, x_2, x'_1, x'_2 \) be four alternatives. Then \( (x_1, x_2) \succeq_{\text{pareto}} (x'_1, x'_2) \) iff \( \forall i \in [1, 2] \), \( x_i \geq x'_i \) and \( \exists j \), such that \( x_j > x'_j \).
Definition 12. Let $G_B = \langle A, R, Bel, Pl \rangle$ be a belief scenario graph s.t. $a \in \Theta_A$, $b \in \Theta_B$ and $S \subseteq \Theta_A$. We say that:

- $a$ defeats $b$ if and only if $a$ attacks $b$, $b$ does not attack $a$, and $a \succ b$.
- $a$ is $b$-acceptable with respect to $S$, if $\forall b$ which defeats $a$ there exists $c \in S$ such that $c$ defeats $b$.
- A set $S$ of arguments is $b$-conflict-free if there are no arguments $a, b \in S$ such that $a$ attacks $b$.
- $S$ is $b$-admissible iff $S$ is conflict-free and all its arguments are acceptable w.r.t. $S$.
- $S$ is a $b$-stable extension iff $S$ is conflict-free and $S$ attacks each argument in $\Theta_A \setminus S$.
- $S$ is a $b$-complete extension iff $S$ is admissible and $S$ contains all and only the arguments it defends.
- $S$ is a $b$-grounded extension iff $S$ is a minimal (w.r.t. set inclusion) complete set of arguments.
- $S$ is a $b$-preferred extension iff $S$ is a maximal (w.r.t. set inclusion) admissible set of arguments.
- $S$ is a $b$-ideal extension iff $S$ is admissible and $S$ is contained in every preferred set of arguments.

Proposition 2. Let $G_B = \langle A, R, Bel, Pl \rangle$ such that $F = \langle A, R \rangle$ be a belief scenario graph and $E \in \text{Ext}(G_B)$. Then, $\succ_B$ is a partial order relation over arguments in $E$.

Notice that the condition in Definition 11 follows the principle of Pareto optimality according to which an argument is preferred if it is better or equal to another in all attributes and strictly better in at least one attribute. The set of best arguments is represented by the Pareto frontier which contains arguments which are not dominated by any other arguments. A way for computing the Pareto frontier is by means of the skyline operator [28]. It is important to observe that the Pareto relation can be used for defining the acceptability of arguments. This means that a belief argumentation framework can be instantiated as $G_B = \langle F, Bel, Pl, \succ_B \rangle$. In this case, any basic argumentation semantics applying to $G_B$ could use $\succ_{\text{pareto}}$ for defining the acceptability of arguments from $A$.

Now in order to compare extensions, of a given belief scenario graph, we can consider the following ordering criteria.

Definition 13. Let $G_B = \langle A, R, Bel, Pl \rangle$ and $E_1, E_2 \in \text{Ext}(G_B)$. Then:

1. $E_1 \succ_{B1} E_2$ if $|E_1| > |E_2|$.
2. $E_1 \succ_{B2} E_2$ if $\forall a \in E_1, \forall b \in E_2, a \succ_B b$.
3. $E_1 \succ_{B3} E_2$ if the number of arguments in $E_1$ non attacked by $E_2$ is greater than the number of arguments in $E_2$ non attacked by $E_1$.

Note that the first relation is a basic ordering based on the size of extensions. The last two ones give a more fine-grained ordering since they are based on the degree of certainty of arguments ($\succ_{B2}$) and the number of non-attacked arguments ($\succ_{B3}$).
Example 7. Consider the belief scenario graph from Example 6. Then, $\text{Ext}_a(G_B) = \{\mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3\}$ where $\mathcal{E}_1 = \{A_1\}$ and $\mathcal{E}_2 = \{D_1\}$, $\mathcal{E}_3 = \{A_1, D_1\}$. Therefore, we have $\mathcal{E}_3 \succ_B \mathcal{E}_1$, $\mathcal{E}_3 \succ_B \mathcal{E}_2$, and $\mathcal{E}_3 \succ_B \mathcal{E}_2$.

Last, Definition 8 can be naturally extended in order to compute the outcome of the original argumentation framework. This can be done by considering all the belief scenario graphs obtained from the original evidential argumentation framework.

Conclusion

In this paper, we have presented a new argumentation framework that handles uncertainty based on evidence theory. The evidential argumentation framework allows to model arguments expressed as opinions and preferences over several alternatives. From this argumentation framework, two families of scenario graphs are distinguished. Each one relies on a specific interpretation of the evidence theory. Moreover, new acceptability semantics are provided on the pignistic and the belief scenario graphs to select acceptable arguments. We have also introduced several criteria for pairwise comparison of extensions and a method for selecting only the best extensions given the winners of pairwise duels.

There is still work needed on the topic. First to propose other criteria to compare and rationalize extensions and to explore the notion of skyline in argumentation theory. Second, we plan to study aggregation methods for evidential abstract argumentation by taking as input a profile of evidential argumentation frameworks, and give as result an argumentation framework that represents the beliefs of the group. Finally, an ambitious research agenda would be to study the computational complexity of our framework and practical algorithms to compute extensions.

References