Generic discounting evaluation approach for urban image classification

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Abstract. Belief function theory provides a robust framework for uncertain information modeling. It also offers several fusion tools in order to profit from multi-source context. Nevertheless, fusion is a sensible task where conflictual information may appear especially when sources are unreliable. Therefore, measuring source’s reliability has been the center of many research and development. Existing solutions for source’s reliability estimation are based on the assumption that distance is the only factor for conflictual situations. Indeed, integrating only distance measures to estimate source’s reliability is not sufficient where source’s confusion may be also considered as conflict origin. In this paper, we tackle reliability estimation and we introduce a new discounting operator that considers those two possible conflict origins. We propose an automatic method for discounting factor calculation. Those factors are integrated on belief classifier and tested on high-resolution image classification problem.

Keywords: Belief function theory, Discounting, Classification, Conflict management, Source confusion

1 Introduction

The improvement of image acquisition techniques have led to the treatment of more complex images in terms of details. This detail complexity comes generally from the multi-band nature of the image. The multi-source information is too valuable for decision process but needs an adequate formalism able to manipulate it. In order to synthesize more useful information related to the observed scene, many formalisms were proposed manipulating those multi-source information and allowing to formalize mathematically uncertain and imprecise data as the Bayesian theory, fuzzy set theory,…

The belief function theory, introduced by Dempster [1] and formalized by Shafer [2], presents a powerful mathematical background in information fusion domain. It not only allows modeling mathematically uncertainty and imprecision information but it also integrates many combination tools allowing source fusion. The fusion ability of this formalism is granted by several combination rules; the
oldest is the Dempster’s rule of combination. However, Zadeh in [3] highlighted its counter-intuitive behavior. Additionally, a lot of studies have been interested on fusing information in the context of image classification [4].

Despite, the fact that belief function theory excels in extracting the most truthful proposition from a multi-source context, it nevertheless presents a major inconvenient that is conflict. The conflict is weight accorded to the empty set proposition and generally appears after source combination. Many works have been done in this domain allowing conflict elimination. In literature, we distinguish two main conflict management family approaches. The first family consists of managing the conflict meaning the combination operator allowing fit conflict redistribution. From those works we can cite [5, 6]. Some authors tried to unify these combination rules [7, 8].

Another method to reduce conflict is the use of discounting factors [2] before combining sources. Indeed, those kinds of approaches rely on the fact that conflict is inducted and generated by the unreliability of at least one source. Many works have been done finding those discounting factors such that [9–11]. Shafer in [2] has proven that the resulting conflict may not only come from source’s contradiction in combination phase. Indeed, the confusion rate of a source may generate conflict. This assumption means that the more the source is less informative, the higher the conflict is. To the best of our knowledge, rare are the discounting based approaches that addressed conflict taking those two conflict origins into consideration.

In this paper, we consider two possible factors for conflict and should be taken into consideration. The intrinsic conflict caused by the unreliability of a source to determine certain classes. The second considered conflict origin is the extrinsic conflict which indicates to what extent the obtained sources are in contradiction. In this work, we propose a new conflict management approach denoted Generic Discounting Approach (GDA) based on discounting factors determination. Those discounting factors are found by studying not only the confusion of the source but also its contradiction with the other ones. The GDA discounting approach was experimented on a high-resolution urban image classification problem. We take advantage of the belief function theory in order to modelize imperfect data extracted from the image and benefit from combination rules. Our classification approach is based on two main stages where, in the first phase, we combine multi-source information in order to obtain a reference classification. In the second stage, we try to improve the classification result by adding the GDA discounting approach.

This paper is organized as follows: in the second section we briefly introduce the basics of the belief function theory. In section 3, we detail several extrinsic and intrinsic reliability measures developed in the framework of belief function theory. In the following, we introduce our Generic Discounting Approach (GDA) capable of estimating the reliability of a source regarding its two sided conflict measures. In section 5, we experiment our approach on a high resolution urban image providing comparative results with other notable works. Finally, we conclude and we sketch issues of future work.
2 Belief function theory

The belief function theory or the evidence theory was introduced by Dempster [1] in order to represent some imprecise probabilities with upper and lower probabilities. Then, it was mathematically formalized by Shafer [2]. The belief function theory is used for representing imperfect (uncertain, imprecise and/or incomplete) information. In this section, we present the main concepts of this theory.

2.1 Frame of discernment

The frame of discernment is the set of possible answers for a treated problem and generally noted $\theta$. It is composed of exhaustive and exclusive hypotheses:

$$\theta = \{H_1, H_2, \ldots, H_N\}.$$

These elements are assumed to be mutually exclusive and exhaustive. From the frame of discernment $\theta$, we deduce the set $2^{\theta}$ containing all the $2^N$ subsets $A$ of $\theta$:

$$2^{\theta} = \{A, A \subseteq \theta\} = \{H_1, H_2, \ldots, H_N, H_1 \cup H_2, \ldots, \theta\}.$$

This set constitutes a reference to assess the veracity of any proposal.

2.2 Basic Belief Assignment

A Basic Belief Assignment (BBA) $m$ is the mapping from elements of the power set $2^\theta$ onto $[0, 1]$ such that:

$$m : 2^\theta \rightarrow [0, 1]$$

having as constraints:

$$\begin{cases} \sum_{A \subseteq \theta} m(A) = 1 \\ m(\emptyset) = 0. \end{cases} \quad (1)$$

Each subsets $X$ of $2^\theta$ verifying $m(X) > 0$ is called focal elements. Constraining $m(\emptyset) = 0$ is the normalized form of a BBA and this corresponds to a closed-world assumption [12], while allowing $m(\emptyset) > 0$ corresponds to an open world assumption [5].

2.3 Combination rules

The belief function offers many advantages. One of its proposed asset is the information fusion allowing extracting the more veracious proposition from a multi-source context. This benefit is granted by the combination rules. Several
operators were defined such the conjunctive rule allowing fusion without any normalization (conflict management). For two sources $S_1$ and $S_2$ having respectively $m_1$ and $m_2$ as BBA, the conjunctive rule is defined by:

$$m_{\otimes}(A) = \sum_{B \cap C = A} m_1(B) \times m_2(C) \quad \forall A \subseteq \emptyset.$$  

(2)

A normalized version of conjunctive rule proposed by Dempster [1] integrates a conflict management approach that redistributes the generated conflictual mass. The Dempster’s rule is defined as follows:

$$m_{\oplus}(A) = \frac{1}{1 - K} \sum_{B \cap C = A} m_1(B) \times m_2(C) = \frac{1}{1 - K} m_{\otimes}(A) \quad \forall A \subseteq \emptyset, A \neq \emptyset$$  

(3)

where $K$ is defined as:

$$K = \sum_{B \cap C = \emptyset} m_1(B) \times m_2(C) = m_{\otimes}(\emptyset).$$  

(4)

$K$ represents the conflict mass between $m_1$ and $m_2$.

2.4 Decision operators

In literature, among several functions that were proposed, we distinguish the pignistic probability. The pignistic probability noted $BetP$ was proposed by Smets [5] within his Transferable Belief Model (TBM) approach. TBM is based on the differentiation between the knowledge representation and decision-making level. In the decision phase, the pignistic transformation consists in distributing equiprobably the mass of a proposition $A$ on its contained hypotheses, formally:

$$BetP(H_n) = \sum_{A \subseteq \emptyset} \frac{|H_n \cap A|}{|A|} \times m(A) \quad \forall H_n \in \emptyset.$$  

(5)

2.5 Discounting

Assuming that an information source has a reliability rate equal to $(1 - \alpha)$ where $(0 \leq \alpha \leq 1)$, such meta-knowledge can be taken into account using the discounting operation introduced by Shafer [2], and defined by:

$$\begin{align*}
m^\alpha(B) &= (1 - \alpha) \times m(B) \quad \forall B \subseteq \emptyset \\
m^\alpha(\emptyset) &= (1 - \alpha) \times m(\emptyset) + \alpha.
\end{align*}$$  

(6)

A discount rate $\alpha$ equal to 1 means that the source is not reliable and the piece of information it provides cannot be taken into account. On the contrary, a null discount rate indicates that the source is fully reliable and the piece of information it provides is entirely accepted. Thanks to discounting, an unreliable source’s BBA is transformed into a function assigning a larger mass to $\emptyset$. 
3 Reliability measure and discounting

Empty set mass resulting from the conjunctive rule can be considered as the most obvious conflict measure in the belief function theory. The conflictual mass could be seen as a measure of contradiction between BBA. Shafer in [2], has defined the confusion of a source as a possible cause for conflict appearance after combination. The conflictual mass resulting from fusion could be seen as the result of the source confusion and the contradiction between fused BBA. In this part, we distinguish between two possible causes for the conflict. At first, the intrinsic conflict which is the conflict generated by the confusion rate of a source. The extrinsic conflict is the conflict resulting from source contradiction. In this section, we present notable state of art of metrics used to measure those two types of conflict.

3.1 Intrinsic conflict

The intrinsic conflict measures the consistency between the different focal elements inside the BBA. Several measures have been proposed in literature. These measures take into account the inclusion relations between the focal elements present in the BBA. Several measures were proposed such that the auto-conflict [13]. Nevertheless, auto-conflict is a kind of contradiction measure that depends on order, it was therefore necessarily to define an independent measure that get rid of this constraint. Smarandache et al. [14] proposed the contradiction measure, defined by:

$$\text{contr}(m) = c \sum_{X \subseteq 2^\Theta} m(X) \cdot D(m, m_X)$$

(7)

where $m_X(X) = 1$, $X \in 2^\Theta$ is the categorical BBA, $c$ is normalization constant and $D$ is the Jousselme distance [15] defined for two mass function $m_1$ and $m_2$ by:

$$D(m_1, m_2) = \sqrt{\frac{1}{2} \cdot (m_1 - m_2)^t \cdot D \cdot (m_1 - m_2)}$$

(8)

$$D(A, B) = \begin{cases} 1 & \text{if } A = B = \emptyset \\ 
\frac{|A \cap B|}{|A \cup B|} & \text{if } A, B \subseteq 2^\Theta.
\end{cases}$$

(9)

From the other intrinsic distance, we can cite the confusion measure [16], the auto-conflict [13], ...

3.2 Extrinsic conflict

Several measures of extrinsic conflict have been studied in order to model the disagreement between sources. Indeed, if one source opinion disagree the other, their fusion will lead to an important conflictual mass. Some authors have defined distance between the mass functions directly, such as Jousselme’s distance [15]...
focal elements. From other distances, we can cite Euclidean distance [8], Tessem’s distance [17], Milan’s distance [18]. Martin et al. [9] proposes using a function that quantifies the conflict between BBA. This function, called $Conf(., .)$, is defined as:

$$Conf(i, E) = \frac{1}{M-1} \sum_{k=1; k \neq i}^{M} Conf(i, k)$$  \hspace{1cm} (10)

with $M$ is the number of belief functions produced respectively by $M$ sources called $S_1, \ldots, S_M$ and $E$ is the set of BBA such that $\{m_k|k = 1, \ldots, M \text{ and } k \neq i\}$. The function $Conf(i, k)$ is obtained using a BBA distance introduced by Jousselme et al. [15]:

$$Conf(i, k) = D(m_i, m_k).$$  \hspace{1cm} (11)

The value $Conf(i, E)$ quantifies the average conflict between the BBA $m_i$ and the BBAs of the set $E$. Once the conflict measure is obtained, the authors have proposed to compute discounting rates as follows:

$$\alpha_i = f(Conf(i, M))$$  \hspace{1cm} (12)

where $f$ is a decreasing function. The authors propose to choose the function $f$ as follows:

$$\alpha_i = (1 - Conf(i, M)^\lambda)^{1/\lambda}$$  \hspace{1cm} (13)

with $\lambda > 0$. The authors recommend setting $\lambda$ to 1.5. Extensions of this work use the idea of sequential discount to manage the conflict when combining belief functions [11].

## 4 Generic Discounting Approach (GDA) for conflict management

Many works have been proposed for finding discounting factors in order to eliminate conflict. Most of proposed discounting approaches rely on extrinsic measures rather than intrinsic measures. Rare were works that tried to associate those two conflict origins to estimate source reliability. In this section, we propose an automatic method to find those discounting factors depending on the two conflict measures. The GDA is a discounting approach that estimates source’s reliability based on the two conflict origins. It is a function $f$ satisfying several constraints:

- $f$ is an increasing function from $[0, 1]^2 \rightarrow [0, 1]$
- $f(1, 1) = 1$ and $f(0, 0) = 0$.

$$\begin{cases} f := [0, 1]^2 \rightarrow [0, 1] \\ (\delta, \beta) \rightarrow \delta^{(1 - \beta)} \end{cases}$$  \hspace{1cm} (14)
where $\delta$ is an extrinsic measure and $\beta$ is an intrinsic measure that can be chosen as follows.

$$\delta = \text{Dist}(m)$$  \hspace{1cm} (15)  \\
$$\beta = \text{contr}(m).$$  \hspace{1cm} (16)

Thus, the GDA is a function $f$ that can be written as follows:

$$f := [0, 1]^2 \to [0, 1] \quad \text{such that} \quad \langle \delta, \beta \rangle \to (\text{Dist}(m))^{(1-\text{contr}(m))}.$$  \hspace{1cm} (17)

where $\text{Dist}(m)$ designates the distance between $m$ and other BBA and can be written:

$$\text{Dist}(m) = \frac{\sum_{m_i \in [1..I]} D(m, m_i)}{I - 1}.$$  \hspace{1cm} (18)

$D$ is the Jousselme’s distance and $\text{contr}(m)$ is the contradiction value indicating the confusion rate of the source itself. The discounting can be written as follows:

$$m^{GDA}(B) = (1 - f(\delta, \beta)) \times m(B) \quad \forall B \subseteq \theta$$  \\
$$m^{GDA}(\varnothing) = (1 - f(\delta, \beta)) \times m(\varnothing) + f(\delta, \beta)$$  \hspace{1cm} (19)

In [19], we proposed another version for source discounting regrouping both intrinsic and extrinsic measures. The Table 1 shows the discounting value that could be associated to a BBA depending on the confusion and distance rates.

<table>
<thead>
<tr>
<th>Source</th>
<th>With Confusion</th>
<th>Without Confusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distant</td>
<td>$f(\delta, \beta) = 1$</td>
<td>$f(\delta, \beta) = \text{Dist}(m, \overline{m})$</td>
</tr>
<tr>
<td>Near</td>
<td>$f(\delta, \beta) = 1$</td>
<td>$f(\delta, \beta) = 0$</td>
</tr>
</tbody>
</table>

**Example 1.** Let’s consider the frame of discernment $\theta = \{H_1, H_2\}$ and three sources $S_1$, $S_2$ and $S_3$. The belief function values associated to those sources and their discounting values are calculated in Table 2.

We can remark that the distance between a source and the other directly affects the GDA discounting coefficient. The BBA’s distance (extrinsic measure) is powered by the confusion (intrinsic measure) that is why the GDA factor is equal to the extrinsic measure when the source is not confused. However, it increases the more confused the source is. As it is shown in Table 2, GDA decreases drastically conflict where comparatively to the conjunctive sum, it fell from 0.717 to 0.0748. GDA also improved $\{H_1\}$ and $\{H_2\}$ hypothesis credibilities by considering $S_1$ and $S_3$ unconfused nature.
Table 2. Evaluation of discounting approach on an example

<table>
<thead>
<tr>
<th></th>
<th>$S_1$</th>
<th>$S_2$</th>
<th>$S_3$</th>
<th>$m^\cap$</th>
<th>$m^\cup GDA$</th>
</tr>
</thead>
<tbody>
<tr>
<td>${H_1}$</td>
<td>0.7</td>
<td>0.6</td>
<td>0.2</td>
<td>0.187</td>
<td>0.3061</td>
</tr>
<tr>
<td>${H_2}$</td>
<td>0.1</td>
<td>0.3</td>
<td>0.7</td>
<td>0.0094</td>
<td>0.1613</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.2</td>
<td>0.1</td>
<td>0.1</td>
<td>0.002</td>
<td>0.4578</td>
</tr>
<tr>
<td>$\emptyset$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.717</td>
<td>0.0748</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.5845</td>
<td>0.7891</td>
<td>0.6904</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.3452</td>
<td>0.2691</td>
<td>0.4562</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$f(\delta, \beta)$</td>
<td>0.6428</td>
<td>0.7581</td>
<td>0.7842</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

5 Classification optimization by discounting factors determination

In this part, we detail our image classifier and how we managed to integrate the GDA discounting to optimize the result of high-resolution urban image first classification. We experimented our approach on a Quickbird image covering urban areas of Strasbourg, taken in 2008, having four bands, each band with 2.44-2.88m/px. From the variety of object constituting this image, we are interested in finding roads, buildings and vegetation (see figure 1). Those three classes will constitute our frame of discernment $\theta = \{Roads, Building, Vegetation\}$. In order to extract correctly those classes, we used five different sources. Each source corresponds to a band from the image. The five considered sources are: the R/G/B bands, the NDVI band and the PIR band.

5.1 First classification

In belief function estimation and classification, we distinguish two main family approaches. Likehood based approaches [2], rely on density estimation where they assume known the class-conditional probability densities for each class. The second family, is the distance based approaches introduced by Zouhal and Denœux [20]. Both methods are applicable in our image classification problem, but we have chosen to work with the distance based model for its simplicity of its generated BBA. The distance classifier relies on training base constituted by $I$ vectors $x_i$. Each training vector, belonging to $H_n^i$, sufficiently close to the vector to classify $x$ constitutes a piece of evidence and should be taken into consideration. Indeed, this piece of evidence influences our belief concerning the class membership of the entity under consideration. A fraction of the unit mass is assigned by $m$ to the singleton $\{H_n^i\}$, and the rest is assigned to the whole frame of discernment. Denœux in [21], proposed a new alternative to the training base by characterizing each studied class by a prototype (a value that represents the class) rather than using training vectors $x_i$. Two strategies for distance modeling can be differentiated which are the mono-dimensional and multi-dimensional variants. The difference consists in fusion’s level where the multi-dimensional strategy considers the vector $x$ as a single information leading
to a unique fusion level. In the other hand, the mono-dimensional strategy apply
the described distance estimation for each vector \( x \) component that lead to a
two fusion level. In our case, we studied the mono-dimensional rather than the
multi-dimensional variant which is also applicable. As training set we associated
the distance estimation to a prototype base. For every \( x \) component \( x_j \) (with
\( j \in [1, .., J] \)), we estimate our BBA following this expression:

\[
\begin{align*}
m_{sj}(\{H_i\}) &= \alpha^s_j \phi^s_j(d^s_j) \\
m_{sj}(\emptyset) &= 1 - \alpha^s_j \phi^s_j(d^s_j)
\end{align*}
\]

where \( 0 < \alpha^s_j < 1 \) is a constant. \( \phi^s_j \) is a decreasing function verifying \( \phi^s_j(0) = 1 \)
and \( \lim_{d \to \infty} \phi^s_j(d) = 0 \), \( d^s_j \) represent the distance between the constituent \( x_j \) of
the vector \( x \) and the component \( j \)-th prototype of source \( s \). The \( \phi^s_j \) function
might be an exponential function following this form:

\[
\phi^s(d^s_j) = \exp(-\gamma^s_j (d^s_j)^2).
\]

The use of Dempster combination operator allows merging those \( J \) belief func-
tions. \( m_s \) is the resulting belief function:

\[
m_s = \bigoplus_{j \in [1,J]} m_{sj}.
\]

A unique belief function \( m \) is obtained by the application of the same fusion
principle on those resulting \( S \) BBA:

\[
m = \bigoplus_{s \in [1,S]} m_s
\]

with \( S \) the number of source. The described method constitutes the Distance
Classifier (DC). However, for our first classification, we replace the Dempster’s
combination rule (equation 23) by the conjunctive sum (equation 2) in ord er
to generate conflict and analyze source’s discordance. Those two methods are
applied on each pixel of the image following to the studied sources.

5.2 Classification optimization

After a first classification, a high amount of conflict is generated making the results more or less acceptable. As it is shown in Table 3, more than 70 percent of our BBA present a conflict rate exceeding 0.6. This result means that 70 percent of the obtained BBA can be potentially attributed to another suited class. A conflict management approach is needed in order to improve results. In this section, we present how we managed to optimize and improve our classification results by adding an extra process which is conflict management.

The pixels (BBA) that present a conflict rate superior to a threshold are reanalyzed by a new combination phase. In this new fusion step, we aim to discard unreliable sources following GDA discounting factors (equation 19).
Table 3. Conflict rate after the first classification

<table>
<thead>
<tr>
<th>Conflict rate</th>
<th>&lt; 0.4</th>
<th>&lt; 0.6</th>
<th>&gt; 0.6</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5.006%</td>
<td>24.195%</td>
<td>70.779%</td>
</tr>
</tbody>
</table>

5.3 Application and results

In order to test our approach we compared it to the DC approach (described in section 5.1) based on distance estimation where conflict management is operated thanks to the Dempster’s combination rule. The comparison were also conducted to Martin discounting approach [9]. The test has been done 8458 images pixel where 1825 represent building points, 1666 road points and 4967 vegetation pixels.

Table 4. Comparative classification results

<table>
<thead>
<tr>
<th>Building</th>
<th>DC</th>
<th>Mart</th>
<th>GDA</th>
<th>DC</th>
<th>Mart</th>
<th>GDA</th>
<th>DC</th>
<th>Mart</th>
<th>GDA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Building</td>
<td>68.25%</td>
<td>72.79%</td>
<td>71.52%</td>
<td>28.19%</td>
<td>25.90%</td>
<td>27.46%</td>
<td>3.56%</td>
<td>1.31%</td>
<td>1.02%</td>
</tr>
<tr>
<td>Road</td>
<td>18.03%</td>
<td>15.21%</td>
<td>13.63%</td>
<td>81.69%</td>
<td>84.51%</td>
<td>86.12%</td>
<td>0.28%</td>
<td>0.28%</td>
<td>0.25%</td>
</tr>
<tr>
<td>Vegetation</td>
<td>0.21%</td>
<td>0%</td>
<td>0%</td>
<td>1.53%</td>
<td>0.36%</td>
<td>0.19%</td>
<td>98.26%</td>
<td>99.64%</td>
<td>99.81%</td>
</tr>
</tbody>
</table>

As it is shown in the Table 4, the proposed discounting factor approach (GDA) presents quite satisfying results. The versatility of the proposed discounting factors associated to the confused nature of the BBA have improved results. By comparing our approach to the DC approach, we can notice that we did improve all class detection. We can conclude that we did optimize the first classification.

The Figure 4 represents the classification of the original image (Figure 1) with DC approach. A first classification is applied as announced in section and instead of using the conjunctive combination rule, we apply Dempster’s rule that integrates a conflict manager. The Figure 3 represents the initial image classification with Martin discounting approach.

The Figure 2 represents the same urban image classified using the proposed conflict manager. For each pixel in this image, we apply the first classification. For each high conflict pixel (BBA with high conflict rate), we calculate the discounting factor using the BBA’s extrinsic and intrinsic rate.

6 Conclusion

In this paper, we presented a classification approach for urban high-resolution image. This method is based on two stages, a first classification based on the
belief function framework. This classification is improved and optimized with a new conflict management approach. The conflict management is based on an automatic discounting factor calculation. The discounting factors are found using not only the BBA distance measure but also the confusion rate of belief function. The first classification result has improved thanks to the discounting factors determination. In future work, we will try to propose a complete classification approach based on fusion discounted pieces of evidence. This discounting will also associate an intrinsic and extrinsic measures for more adequate combination. Even if the results are satisfying on image classification, tests can be extended to UCI benchmarks to verify GDA contribution.

References


