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APPLICATION OF A SUBSPACE-BASED FAULT DETECTION METHOD TO INDUSTRIAL STRUCTURES†

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Early detection and localization of damage allow increased expectations of reliability, safety and reduction of the maintenance cost. This paper deals with the industrial validation of a technique to monitor the health of a structure in operating conditions (e.g. rotating machinery, civil constructions subject to ambient excitations, etc.) and to detect slight deviations in a modal model derived from in-operation measured data. In this paper, a statistical local approach based on covariance-driven stochastic subspace identification is proposed. The capabilities and limitations of the method with respect to health monitoring and damage detection are discussed and it is explained how the method can be practically used in industrial environments. After the successful validation of the proposed method on a few laboratory structures, its application to a sports car is discussed. The example illustrates that the method allows the early detection of a vibration-induced fatigue problem of a sports car.

1. INTRODUCTION

Over the last years, industry is showing an increasing interest in techniques aiming at the detection, localisation and quantification of damage in structures such as rotating machinery, aircraft, bridges, off-shore platforms, etc. Early identification of damage is of crucial importance, not only for safety reasons, but also for economical reasons as it allows to efficiently program maintenance and repair actions and, consequently, to reduce the associated costs. Nowadays, visual, systematic inspections of the system are typically performed, often requiring that the system is shutdown and disassembled. The key idea is to replace these inspections by health monitoring systems which regularly acquire and analyse response data and indicate a malfunction or damage in an early stage. A relevant approach is to monitor the vibrations of the structure in operating conditions. Several techniques to identify damage from vibration data have been proposed. They include advanced signal processing procedures such as wavelet analysis and neural networks, as well as model-based

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methods such as experimentally derived modal models or FE models. An overview of techniques on health monitoring of mechanical systems is given in [1].

Generally speaking, four levels can be distinguished in the damage identification problem, as proposed in [2]:

- *Level* 1 Determination that damage is present in the structure.
- *Level* 2 Determination of the geometric location of the damage.
- *Level* 3 Quantification of the severity of the damage.
- *Level* 4 Prediction of the remaining service life of the structure.

This paper is concerned with the first level using experimentally derived modal models. The objective is to answer the simple question whether there is damage or not, only using vibration data measured on the structure in operating condition. From economical and practical points of view, such an output-only damage identification technique offers the great advantage that the system does not need to be shutdown or artificially excited (e.g. shaker or hammer).

A straightforward strategy to tackle the damage detection problem would be the comparison of the reference modal model with the modal model derived from the fresh data. This requires the extraction of modal parameters from output-only data. Over the last years, several techniques have been proposed and successfully validated for output-only system identification. They include auto-regressive moving average models (ARMA) [3], stochastic subspace methods [4], and the natural excitation technique (NeXT) [5]. In [6, 7], the capabilities and limitations of the NeXT technique and the stochastic subspace method have been evaluated for their applicability to industrial cases. In many cases (e.g. civil engineering structures), the change in modal parameters due to damage is very small and it is extremely difficult to statistically decide whether the difference is due to damage or due to measurement and modelling precision, changing excitation, environmental influences, etc. In addition, the modal parameters need to be extracted from each acquired data set. As the modal parameter extraction requires user interaction, this approach is not suitable to health monitoring applications, unless the modal analysis can be automated.

A relevant approach to the vibration monitoring problem has been proposed based on the modelling of modes through state-space representations [8], the use of *output-only* and *covariance-driven* identification methods (such as instrumental variables or balanced realization algorithms) [9], and the computation of specific χ^2-type tests based on the so-called instrumental statistics [10], and more generally [11] on subspace-based linear systems identification methods [4, 12, 13]. In practice, these tests turn out to be robust w.r.t. non-stationary excitation.

In this paper, these algorithms are briefly described, and their capabilities are investigated for a few test cases. The paper is structured in four parts. Section 2 describes the algorithm developed in [11]. Section 3 discusses some practical considerations. Section 4 presents the experimental results obtained on simple laboratory set-ups, and finally Section 5 discusses the application of the algorithm to operating data measured on a sports car.

2. THEORY

In Section 2.1, we recall the mechanical model and its modal representation. Then, we describe the subspace-based identification methods in Section 2.2. Finally, in Section 2.3 we present the proposed damage detection method.

2.1. MECHANICAL MODEL

We start with the following finite elements model of the mechanical structure:

$$\begin{aligned} M\ddot{Z}(t) + C\dot{Z}(t) + KZ(t) &= v(t) \\ Y(t) &= LZ(t) \end{aligned} \tag{1}$$

where t denotes continuous time, the vector Z represents the displacement of the degrees of freedom (dof) of the FE model, M is the mass matrix, C the damping matrix and K the stiffness matrix. The external (non-measured) force v is modelled as a non-stationary white noise with time-varying covariance $Q_v(s)$. Measurements are collected in the (low-dimensional) vector Y, and matrix L indicates where the the sensors are located.

The mechanical characteristics (M, C, K) of the system cannot be recovered from output-only data. Hence identifiable modal characteristics of the system are introduced: the vibration eigenfrequencies denoted generally by μ, the eigenvectors denoted by ψ_μ and the mode shapes or observed eigenvectors denoted by ϕ_μ. These quantities are solutions of the following equation:

$$(M\mu^2 + C\mu + K)\psi_\mu = 0, \qquad \phi_\mu = L\psi_\mu. \tag{2}$$

Sampling model (1) at rate $1/\tau$ yields the discrete time model in state-space form:

$$\begin{aligned} X_{k+1} &= FX_k + \varepsilon_{k+1} \\ Y_k &= HX_k + v_k \end{aligned} \tag{3}$$

where the state and the output are

$$X_k = \begin{bmatrix} Z(k\tau) \\ \dot{Z}(k\tau) \end{bmatrix}, \quad Y_k = Y(k\tau) \tag{4}$$

the state transition and observation matrices are

$$F = \mathrm{e}^{\mathscr{L}\tau}, \quad \mathscr{L} = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix}, \quad H = [L \quad 0] \tag{5}$$

and where state noise ε_{k+1} is zero mean, white noise, with covariance matrix

$$Q_{k+1} = E(\varepsilon_{k+1}\varepsilon_{k+1}^T) = \int_{k\tau}^{(k+1)\tau} \mathrm{e}^{\mathscr{L}s}\tilde{Q}(s)\,\mathrm{e}^{\mathscr{L}^T s}\,\mathrm{d}s \tag{6}$$

with

$$\tilde{Q}(s) = \begin{bmatrix} 0 & 0 \\ 0 & M^{-1}Q_v(s)M^{-T} \end{bmatrix}$$

and where $E(\cdot)$ denotes the expectation operator.

The measurement noise process (v_k) is added to take noise over sensor measurements into account. Generally, it is assumed to be an *unmeasured* MA(ι) Gaussian sequence with zero mean. In this paper, we will assume that $\iota = 0$, corresponding to white (i.i.d.) measurement noise. Note that, with this MA assumption for its structure, measurement noise *not* affect the eigenstructure of system (3).

The modal characteristics defined in equation (2) are equivalently found in the eigenstructure (λ, ψ_λ) of F:

$$\mathrm{e}^{\tau\mu} = \lambda, \quad \text{and} \quad \phi_\mu = \phi_\lambda = H\psi_\lambda. \tag{7}$$

The problem we consider is to monitor the observed system eigenstructure, i.e. the collection of m pairs (λ, ϕ_λ), where λ ranges over the set of eigenvalues of state transition matrix F. In the sequel, such a pair (λ, ϕ_λ), is called a mode. The set of the m modes is considered as the system parameter θ:

$$\theta \overset{\text{def}}{=} \begin{pmatrix} \Lambda \\ \operatorname{vec} \Phi \end{pmatrix}. \tag{8}$$

In equation (8), Λ is the vector whose elements are the eigenvalues λ, Φ is the matrix whose columns are the ϕ_λ's, and vec is the column stacking operator. Parameter θ has size $(r+1)m$. The problem is to detect changes in the parameter vector θ.

At first glance, it seems reasonable to consider that diagnosis of failures can be performed by comparing a new modal signature to a reference one. However, the comparison of two different modal signatures is an extremely difficult task. In order to decide how eigenfrequencies and corresponding mode shapes have been modified, eigenfrequencies of different signatures must be correlated and compared. Moreover, how do you decide whether a given change in eigenfrequency or mode shape is really significant? This is really a difficult point if numerical approaches are used to estimate such signatures. In the same vein, heuristic approaches lead to false alarms, since it is difficult for the human operator to evaluate how sensor noise affects the uncertainty in the eigenfrequency or mode shape estimates.

For this reason, the statistical approach has been developed, to overcome both drawbacks. Given a reference modal signature and new data, the basic idea is to evaluate whether the new data agree with the signature, without determining the signature from the new data. This makes the proposed procedure much simpler than a modal comparison approach since it requires much less involvement of the human operator.

2.2. SUBSPACE-BASED IDENTIFICATION

We are given a sequence of covariances:

$$R_j \overset{\text{def}}{=} \mathbf{E}(Y_{k+j} Y_k^T) \tag{9}$$

of output Y_k of a state model (3). For $q \geqslant p+1$, let $\mathcal{H}_{p+1,q}$ be the block-Hankel matrix:

$$\mathcal{H}_{p+1,q} = \begin{pmatrix} R_1 & R_2 & \cdots & R_q \\ R_2 & R_3 & \cdots & R_{q+1} \\ \vdots & \vdots & \ddots & \vdots \\ R_{p+1} & \cdots & \cdots & R_{p+q} \end{pmatrix}. \tag{10}$$

Choosing the eigenvectors of matrix F as a basis for the state space of model (3) yields the following particular representation of the observability matrix introduced in [10]:

$$\mathcal{O}_{p+1}(\theta) = \begin{pmatrix} \Phi \\ \Phi\Delta \\ \vdots \\ \Phi\Delta^p \end{pmatrix} \tag{11}$$

where diagonal matrix Δ is defined as $\Delta = \operatorname{diag}(\Lambda)$, and Λ and Φ are as in equation (8).

From [10], we get that the following property characterizes whether a nominal parameter θ_0 agrees with a given output covariance sequence (R_j) [13]:

$$\mathcal{O}_{p+1}(\theta_0) \text{ and } \mathcal{H}_{p+1,q} \text{ have the same left kernel space.} \tag{12}$$

Property (12) can be checked as follows:

1. From θ_0 as defined in equation (8), form $\mathcal{O}_{p+1}(\theta_0)$, and pre-multiply it by some invertible weighting matrix W_1.
2. Pick an orthonormal basis of the left kernel space of matrix $W_1\mathcal{O}_{p+1}(\theta_0)$, in terms of the columns of some matrix S of co-rank m such that

$$S^T S = I_s \tag{13}$$

$$S^T W_1 \mathcal{O}_{p+1}(\theta_0) = 0 \tag{14}$$

3. The parameters θ_0 which actually corresponds to the output covariance sequence $(R_j)_j$ is characterized by

$$S^T(\theta_0) W_1 \mathcal{H}_{p+1,\,q} W_2^T = 0 \tag{15}$$

where W_2 is another invertible weighting matrix.

It has been shown in [11] that under exact known model order assumption any choice of such weightings should give the same test result. So any weightings, including those corresponding to the Balance Realization (BR) and Canonical Variate Analysis (CVA) methods are acceptable (see [4] for a description of these methods). For BR, the weighting matrices equal the identity matrix. We stress that the above only holds in the case of known system order. In practice, both BR and CVA methods were tested and gave similar results. Small differences might occur from numerical issues and from the effect of model reduction. In the following we retain the BR method for the test computation, as this method does not require the computation of the weightings.

2.3. THE DAMAGE DETECTION METHOD

Assume we have at hand a nominal model θ_0, and some newly collected data $Y_1, \ldots, Y_n$. Form the empirical covariance sequence, perform steps 1 and 2 of Section 2.2, and replace step 3 by:

3. Define the residual vector

$$\zeta_n(\theta_0) \stackrel{\text{def}}{=} \sqrt{n}\,\mathrm{vec}\,(S^T(\theta_0)\, W_1\, \hat{\mathcal{H}}_{p+1,\,q}\, W_2^T) \tag{16}$$

where $\hat{\mathcal{H}}_{p+1,\,q}$ is the empirical block-Hankel matrix obtained by substituting $\hat{R}_j$ for R_j in equation (10).

From equation (15) we already know that the expectation of the residual vector should be zero if and only if this residual vector is computed using samples generated under the reference θ_0 parameter value. Testing if the hypothesis is valid requires knowledge of the distribution of $\zeta_n(\theta_0)$ when the actual parameter for the new data sample is θ. Unfortunately, this distribution is unknown, in general. One manner to circumvent this difficulty is to use a local approach, that is to assume close hypotheses:

$$(\mathit{Safe})\ \mathrm{H}_0\colon \theta = \theta_0 \quad \text{and} \quad (\mathit{Faulty})\ \mathrm{H}_1\colon \theta = \theta_0 + \frac{\Upsilon}{\sqrt{n}} \tag{17}$$

where vector Υ is unknown, but fixed.

More precisely, let $\mathbf{E}_\theta$ and cov_θ be the expectation and the covariance, respectively, when the actual system parameter is θ. We define the mean deviation

$$M(\theta_0) \stackrel{\text{def}}{=} -\frac{1}{\sqrt{n}} \frac{\partial}{\partial\theta} \mathbf{E}_{\theta_0}\, \zeta_n(\theta) \bigg|_{\theta=\theta_0} \tag{18}$$

and the residual covariance matrix

$$\Sigma(\theta_0) \stackrel{\text{def}}{=} \lim_{n\to\infty} \mathbf{E}_{\theta_0}(\zeta_n \zeta_n^T). \tag{19}$$

According to Basseville *et al.* [11] and Delyon [14], a deviation in the system parameter θ is reflected into a change in the mean value of residual ζ_n, which is asymptotically Gaussian distributed with constant covariance matrix. The sensitivity of residual ζ_n w.r.t. parameter θ is captured by the Jacobian matrix $M(\theta_0)$. The uncertainty in residual ζ_n is captured by the covariance matrix $\Sigma(\theta_0)$.

Let $\hat{M}$, $\hat{\Sigma}$ be consistent estimates of $M(\theta_0)$, $\Sigma(\theta_0)$. The detection problem, namely deciding that residual ζ_n is *significantly* different from zero, can be achieved as follows.

Theorem 2.1. (χ^2-test [11]). *Assume additionally that*

$$\textit{Jacobian matrix } M(\theta_0) \textit{ is full column rank (f.c.r).} \tag{20}$$

Then the test between the hypotheses H_0 *and* H_1 *defined in equation* (17) *is achieved through*

$$\chi_n^2 \stackrel{\text{def}}{=} \zeta_n^T \hat{\Sigma}^{-1} \hat{M} (\hat{M}^T \hat{\Sigma}^{-1} \hat{M})^{-1} \hat{M}^T \hat{\Sigma}^{-1} \zeta_n \tag{21}$$

which should be compared to a threshold. In equation (21), *the dependence on* θ_0 *has been removed for simplicity. Test statistics* χ_n^2 *is asymptotically distributed as a* χ^2*-variable, with* rank(M) *dofs and with non-centrality parameter under* H_1:

$$\Upsilon^T M^T \Sigma^{-1} M \Upsilon. \tag{22}$$

3. PRACTICAL IMPLEMENTATION

Application of the proposed damage identification method is basically a 3 or 4 steps procedure, depending whether training data sets are available or not:

1. Identification of the nominal modal model θ_0.
2. Estimation of the residual covariance and the sensitivity matrix.
3. Use of training data sets for the estimation of the mean and variance of the test for the healthy structure, resulting in an alarm threshold for the χ^2-test.
4. Evaluation of the χ^2-test on newly collected data sets, making use of the data prepared in steps 1–3.

When no training data are available, step 2 is immediately followed by step 4. The four steps are detailed below.

Step 1: The first step is the identification of the nominal model θ_0 with data from the same sensor locations as those used in the monitoring application. In principle, any kind of modal parameter extraction technique can be used. However, as the laboratory conditions may differ from the in-operation conditions, it is preferred to extract the modal parameters from output-only data measured in operating conditions. Therefore, the stochastic subspace identification method can be employed. Our simulations have shown that the quality of the nominal model is of crucial importance for monitoring. In case one is not sure about a particular mode in the identification process, we highly recommend not to retain it. One drawback is that mechanical expertise is required in order to distinguish the physical poles from the spurious ones.

Step 2: In order to perform the χ^2-test in (21), the sensitivity matrix (18) and the covariance matrix (19) need to be estimated. As they do not depend on the fault vector Υ in equation (17), they do not need to be re-estimated when testing the hypotheses. Instead, they can be estimated prior to the testing, using data acquired on the safe system. Typically, the same data as used in step 1 for the identification of the nominal model will be used. A consistent estimate of the sensitivity matrix can be obtained from a data sample using a simple average, while an empirical estimate based on a sample version of the jackknife method is recommended for the estimation of the residual covariance matrix [3]. The latter basically means that the n-size data sample is divided into a number of segments of n_{seg} samples. For each segment, the residual vector is computed according to equation (16) and multiplied by its transpose. The matrices obtained for each segment are then averaged, resulting in an estimate of the residual covariance matrix. Via an eigenvalue decomposition and truncation of the negative eigenvalues, this estimate is forced to be positive definite. Important to note is that the number of averages should exceed the number of parameters in θ, i.e. $m(r+1)$. On the other hand, the number of samples should be larger than $p+q-1$. Thus,

$$p + q - 1 \ll n_{seg} \ll \frac{n}{m(r+1)}. \tag{23}$$

After the choice of the n_{seg} parameter, the user has to specify p and q and the weighting matrices W_1 and W_2. Simulations have shown that good results are obtained with $q = p + 1$ and with a minimal value for p satisfying the inequality $pr > m$. Taking p and q too high loads to huge matrices and high computational load. The weighting matrices are the same as in the stochastic subspace identification methods BR and CVA.

The nominal model θ_0, the choice for p and q as well as the estimates of the sensitivity and the covariance matrices can be validated by performing the χ^2-test on the same data as the data used to derive the nominal model. The value of the χ^2-test, further referred to as the calibration value, should be close to the number of degrees of freedom, i.e. $m(r+1)$. In case this is not true, some further tuning of the parameters p and n_{seg} or a new extraction of the nominal model might be needed. Another reason for a poor calibration value could be that the number of data samples is insufficient to accurately estimate the sensitivity and the covariance matrices. Especially in case the unknown excitation is non-stationary (e.g. civil structure subject to ambient excitation), a huge amount of data is necessary to obtain accurate estimates.

Step 3: As mentioned above, in case of no damage, the theory says that the expected value of the χ^2-test equals the number of dofs. In practice, data measured on the healthy structure will not yield this value. Therefore, the value of the χ^2-test needs to be compared with a threshold. In order to determine this threshold, training data sets can be used, allowing to estimate the mean and the variance of the χ^2-test. Basically, this means that the test is applied to several data sets measured on the healthy structure. The data could e.g. be acquired at the beginning of the monitoring process. The mean and variance of the test value can be easily computed, allowing to determine a realistic threshold. In case no training data are available, user's experience is needed to specify a good threshold (e.g. 100 times higher than the calibration value).

Step 4: The nominal model, the estimates for the sensitivity and covariance matrices, the weighting matrices and the threshold for the χ^2-test, derived in steps 1–3 are fed to the health monitoring system. The monitoring system acquires new data, reduces them to covariance data and performs the χ^2-test on a regular basis. In case the test exceeds the threshold, the system raises an alarm. This alarm generation mechanism is fast, automatic and does not require any human intervention.

4. THE WHOLE PROCEDURE AS A LABORATORY EXPERIMENT

In this section, the χ^2-test is applied to laboratory structures and the results are discussed. Our goal is to detect, as soon as possible, a small change in the modal parameters due to a slight damage in the structure. The performance of the χ^2-test is critically evaluated for progressive damage tests. For each damage scenario, the subspace-based identification method is also employed to extract the modal parameters, in order to be able to address the changes in the modal parameters.

4.1. FIRST EXPERIMENT

We use a horizontal aluminium beam which is clamped at one end and free at the other end. The characteristics of the beam are as follows: the beam is 700 mm long, 20 mm wide, and has a thickness of 2 mm. At the free end, it is excited by a shaker with white noise. We place eight sensors on the beam (distance between sensors is 100 mm), with the first sensor located at the same location as the shaker (see Fig. 1). The signals are sampled at 1600 Hz: 32 000 samples are taken, corresponding to a duration of 26 s.

After measuring the response signals on the safe structure, damage is introduced to the beam by making a cut, 24.4 cm apart from the shaker position. The initial depth of the cut is 3 mm (first damage). The depth of the cut is then increased to 7.05 mm (second damage), 9.15 mm (third damage) and 12.1 mm (fourth damage).

Table 1 contains the results of the damage test for each damage scenario: we show the ratio of the χ^2-tests values obtained on the possibly damaged sets and the reference data set. This is further referred to as the normalised χ^2-test value. This value should be close to 1 if the structure is not damaged and much higher if damage occurred.

Table 2 contains the frequencies of the modes identified by the BR-identification method. It is important to note that the modal parameters listed in Table 2 are not required for the test. They are shown in Table 2 in order to be able to assess the changes in the modal parameters. It can be clearly seen that the differences are quite small, making their interpretation in terms of damage or not difficult. Damping ratios are not shown in Table 2 as they are typically less accurately identified from output-only data. Therefore, we recommend not to monitor them.

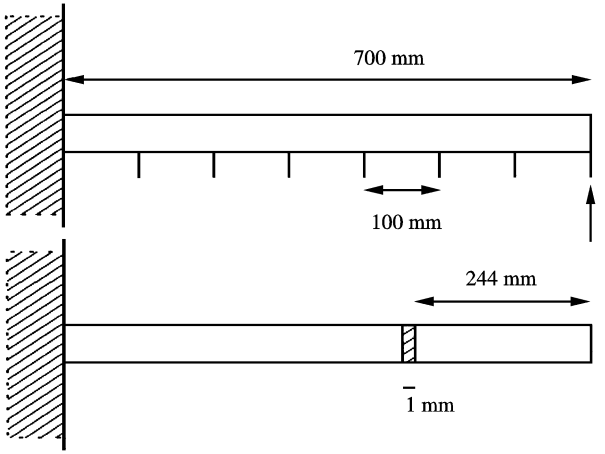

Figure 1. Set-up of the first experiment: (top) the sensor and shaker locations, (bottom) the damage location and size.

TABLE 1
χ^2-test results for the first experiment

Safe structure	Validation structure	1st damage	2nd damage	3rd damage	4th damage
1	4	6600	16 600	94 000	400 000

TABLE 2
For each damage scenario, we give the natural frequencies of all modes (modal analysis of the first experiment) identified by the BR-identification method

Safe structure	35.6	77.1	127.5	186.7	252.9	339.4	486.2	584.8	698.1
1st damage	35.5	77.0	127	186.5	251.6	339	485.6	583.8	698
2nd damage	35.5	76.9	126.4	186.4	251.5	337.9	484.3	580.1	696.8
3rd damage	35.2	76.9	126.2	186.3	251.4	337.5	483	577.7	696.6
4th damage	35.1	76.8	124.9	185.6	251.3	336.3	480.2	572.6	694.5

Now we investigate the robustness of the test against changes in excitation. The test was applied to response data of the healthy structure for a different excitation. The second column of Table 1 illustrates this. In this case, the test has a higher value than expected, but much lower than a test value corresponding to a damage scenario. This result however suggests that the algorithm should be trained with a large set of safe structures (damaged structures) to define upper (lower) bounds for the H_0 (H_1) test using some sort of training method as described in step 3 of Section 3.

To investigate the importance of the choice of the number and location of sensors, we display in Fig. 2, the normalized χ^2-test values on a log-scale, for experiments with four sensors (instead of 8) located at different positions on the beam. The best results are obtained with sensors near the damage or with sensors uniformly spread over the beam. This illustrates the importance of the choice of the location of sensors for monitoring. This question is addressed in [3, 15, 16], where some techniques are developed to find the optimal set of sensors. This data information can be obtained from measurements on the healthy structure.

4.2. SECOND EXPERIMENT

A calibration test is performed on a breadboard model of a steel frame (see Fig. 3). The structure resembles a subframe of a car to be connected to the body at four locations, and on which the engine has to be mounted. The frame is approximately 720 mm long and 170 mm wide and its weight is about 9.8 kg. The subframe is suspended on four flexible threads, the response is measured in the vertical direction at 27 points for vertical excitation at two points. Using dual random shaker excitation, 32 000 samples are measured, for each output, sampled at 1024 Hz.

The modal frequencies of the structure, identified with only three sensors, are shown in Fig. 4, for three different states: safe structure, decrease in mass, increase in stiffness by adding a stiffener between two frames. For both damages, the frequencies increase, but to a lower extent for the stiffness modification. The corresponding χ^2-test values are shown in Table 3. As expected the test is highly sensitive to the mass change, and less, but still significantly sensitive to the stiffness change.

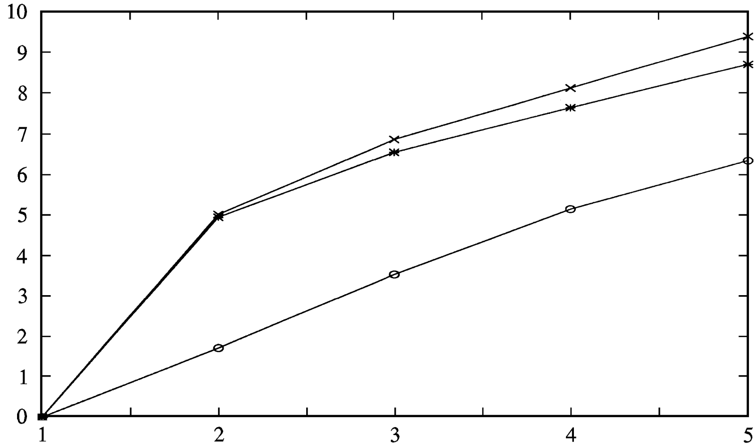

Figure 2. Different locations of sensors for the first experiment: the horizontal axis represents the damage scenarios (1–5, 1 being safe), the vertical axis is the normalized test value on a log-scale: ($\times$) sensors near the cut; ($*$) sensors spread over the beam, ($\bigcirc$) sensors located at the other end.

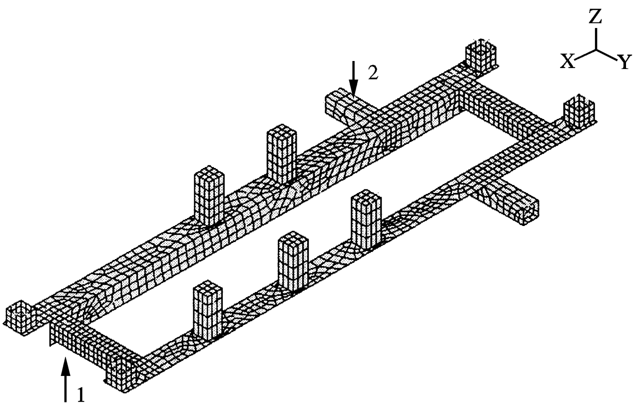

Figure 3. Finite elements representation of the steel frame in the second experiment.

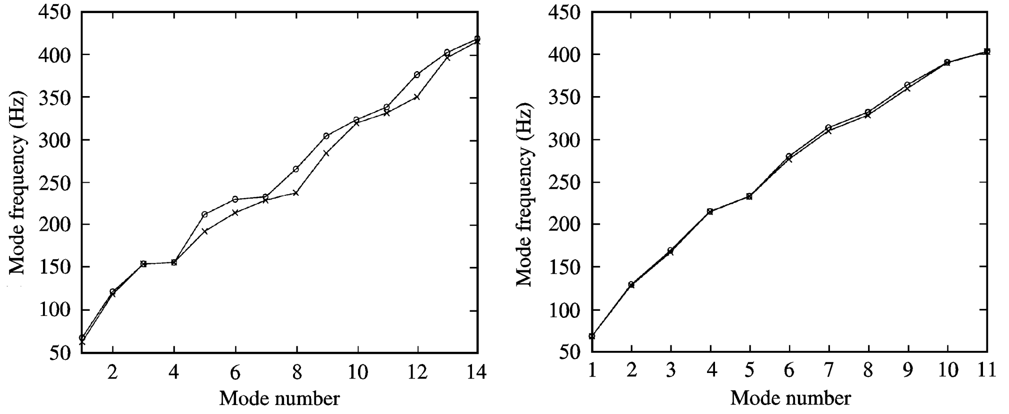

Figure 4. Modal analysis of the second experiment (left) decrease of mass, (right) increase of stiffness. The lower curve ($\times$ symbol) displays the modal frequencies of the safe structure, whereas the upper curve ($\bigcirc$ symbol) represents the modified structure.

TABLE 3

Test result for the second experiment: all values are normalised by the test value obtained on the reference data set

	Safe structure	Validation structure	Damaged structure
Mass change	1	12	340 000
Stiffness change	1	5	250

5. APPLICATION OF THE χ^2-TEST TO A SPORTS CAR

Prior to assessing the performance of the proposed approach to detect a fatigue failure of a sports car related to the gearbox mounting with the car body, a scale model was manufactured which showed some geometrical similarity to the fatigue problem. Tests are performed on the scale model in Section 5.1, then on the sports car in Section 5.2.

5.1. APPLICATION TO A SCALE MODEL

The construction consists of two vertical plates supported by a very stiff bottom plate. Between the two plates, a mass is connected by four rubber elements. The structure is vertically excited such that the mass moves up and down while introducing bending moments in the vertical plates. In order to obtain stress concentration, notches in the plates are introduced at the position of maximal bending strains. As the construction is light, large deformations are possible so the fatigue testing can be done in an acceptable time frame. Figure 5 shows the scale model mounted to the shaker. The structure is mounted under an angle of 15° in order to better excite the modes. The centre of gravity is above the vertical axis of the shaker. The design of the foot allows the measurement of the vertically introduced forces.

The structure is submitted to an endurance test. A flat force spectrum is realised during the whole test by controlling the voltage sent to the shaker. This favours the output-only identification process which assumes that the unknown excitation is white. Measurements are recorded every 3 min. One accelerator is located on the vertical plate, two on the mass and one on the bottom plate. About 8000 samples were recorded for each data set at a sampling rate of 250 Hz.

First, the modal model of the healthy structure is extracted from the acceleration data. The nominal model θ_0 consists of the first three modes, which are well identified:

- *Mode* 1 Bending of plates in phase at 15 Hz.
- *Mode* 2 Bending of plates out of phase at 30.8 Hz.
- *Mode* 3 Roll mode of the mass at 50.8 Hz.

The identification process is then repeated for each record. The frequency of the second mode is shown in Fig. 6 (left): the resonance frequency is decreasing. This process starts quite soon after the beginning of the test. The crack initiation period is very short and the accelerometers pick up the changes very soon during the crack growth. This reduces the stiffness characteristics and, consequently, reduces the resonance frequency.

Figure 6 (right) shows the χ^2-test value as a function of test time. The values are normalised by the calibration value, i.e. the test value obtained on the first data set, which is used to estimate the nominal model and the sensitivity and covariance matrices. So, in case

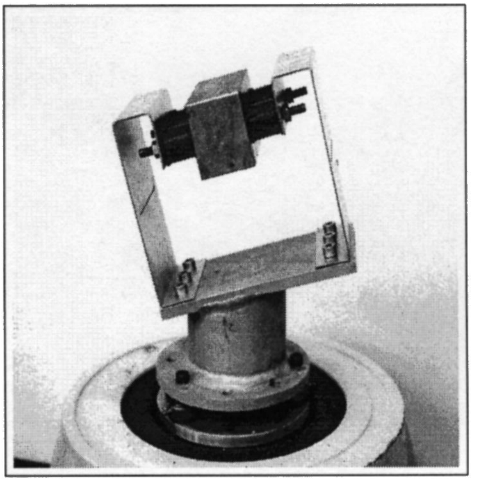

Figure 5. Scale model mounted to the shaker.

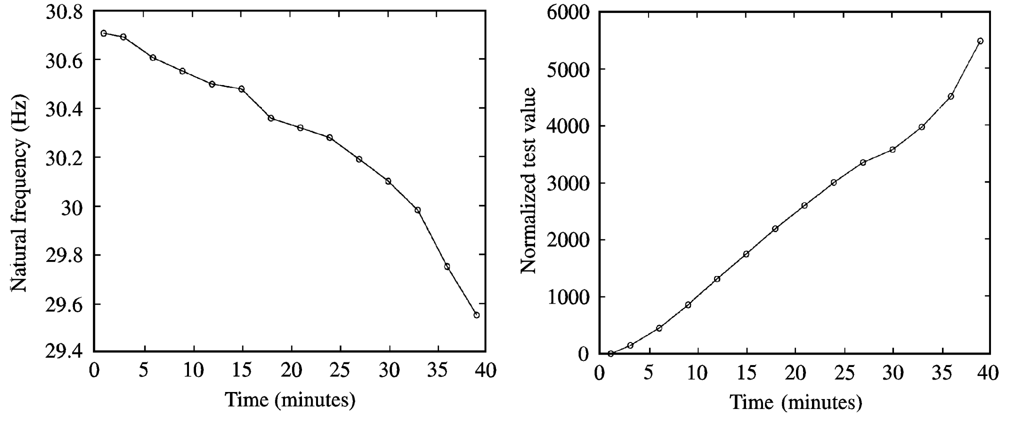

Figure 6. Natural frequency of mode 2 (left) and χ^2-test (right) *vs* time.

of no damage, the test value should be close to 1. For the covariance estimation in equation (23), the size of the segment n_{seg} is set to 50 samples. The χ^2-test is not too sensitive to this parameter: similar results were found for a segment size of e.g. 100 samples. Figure 6 (right) shows that the decrease in natural frequency is well detected. By setting a threshold of e.g. 500, the fatigue problem can already be detected after 6 min without any user interaction.

5.2. APPLICATION TO A SPORTS CAR

5.2.1. Introduction to the fatigue problem

A car is driven on the endurance track until a fatigue problem of the gearbox mounting with the car body occurs. With this knowledge of the failure, a second test car is instrumented to measure the relevant strain and acceleration signals [17].

Figure 7 depicts the test wireframe. There are six measurement points on the car body and four points on the powertrain. All measurements are done in the vertical direction.

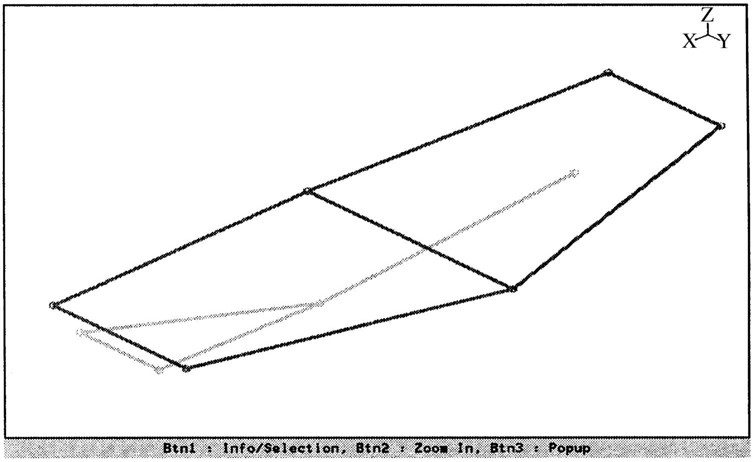

Figure 7. Test wireframe of the car.

A spectral and coherence analysis of the strain and acceleration signals identifies the acceleration at the gearbox mounting and at the front differential in vertical direction around 17 Hz as the most important accelerations for the description of the dynamic behaviour causing the fatigue crack. The modal parameters of the car are derived from the in-operation acceleration data. Correlation of these modes with operational deflection shapes of the first, second and third principal component at 17 Hz revealed that three modes highly contribute to the response:

- Vertical front differential.
- Rolling mode of the powertrain coupled to the car body-torsion.
- Vertical powertrain coupled to car body-bending.

The mode shape of the second mode at 17.2 Hz is shown in Fig. 8.

5.2.2. Health monitoring during the endurance test in the lab

An endurance 4-shaker test on a body-in-white equipped with the powertrain was then undertaken in the lab. To avoid problems in the attachment areas of the shakers to the body, the front and rear suspension including the wheels are also built in and the wheels are directly put on the four hydraulic shakers. Some local masses are added so that the dynamic behaviour is roughly equivalent to the one of the car which has been tested on the test track. For the excitation, a narrowband period random signal with a trapezoidal frequency content (10–14–18–22 Hz) and random phase is used. Two independent excitation signals are sent to the left and right hydraulic front shakers. The same signals, but delayed with a time corresponding to the critical speed (50 km/h), are sent to the rear shakers. Objective of the test is to reproduce the same failure, but in a much shorter time and in well-controlled conditions. The result of the test is that cracks are obtained in exactly the same locations as on the test track. The damage is however less severe in the test lab than on the test track: the critical areas had to be processed with special contrast lack to reveal the crack. During the test, the acceleration and strain signals are recorded every half an hour in order to see whether early detection of the fatigue problem is possible. About 10 000 samples are taken at a sampling rate of 378 Hz. The tests have been performed within the framework of the Esprit project 2486 'DYNAMO'.

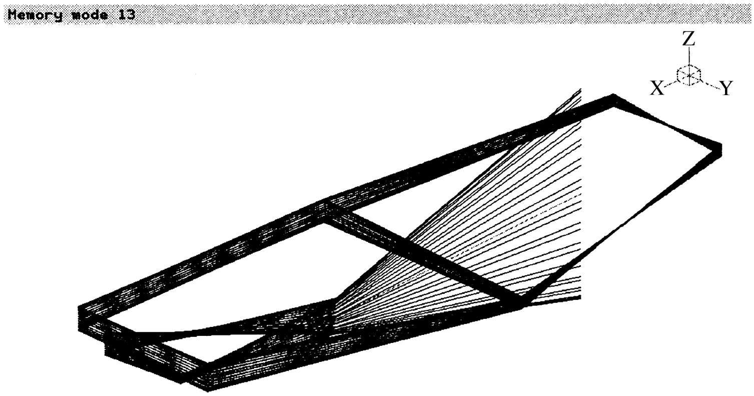

Figure 8. Mode shape of the mode at 17.2 Hz. Rolling mode of the powertrain coupled to the car body-torsion.

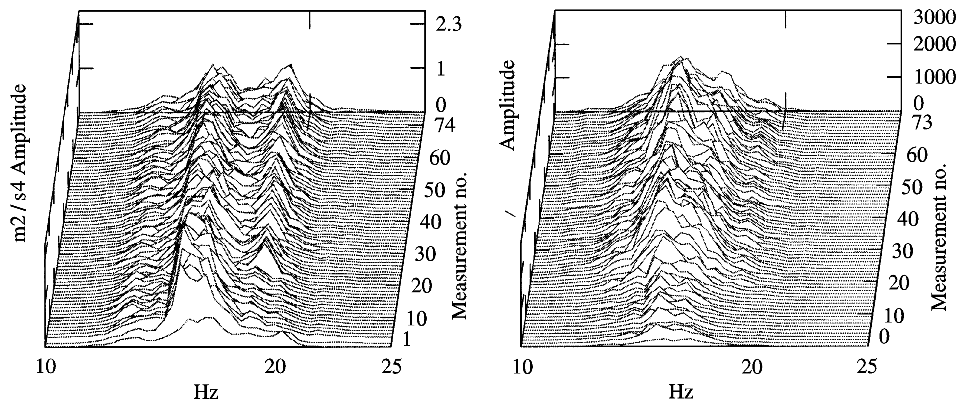

Figure 9. Waterfalls of autopower: (left) strain signal, (right) gearbox acceleration.

In a first analysis, the autopowers of a strain signal close to the crack area and the gearbox acceleration are estimated for each measurement taken every 30 min and plotted in a waterfall diagram, as shown in Fig. 9.

Figure 9 shows that the height of the peaks first remains approximately constant, increases next, but after some time, stays about constant again till the end of the test. The gearbox acceleration shows no change first, a slight decrease next and a quite sudden change when the strain becomes constant again. This can be interpreted as follows. The first period refers to the initiation period of the crack. When the crack starts to be macroscopic (crack growth), the strain gauge starts to register the change. This is also reflected in the acceleration signals. After some time, the crack growth stops and during the remaining time, the component is flapping in response to the input.

The data measured at the beginning of the endurance test are used to derive the modal model using the stochastic subspace identification method. Stabilisation diagrams have to be used to select the physical poles. About five modes below 22 Hz were found. Then, the residual sensitivity and covariance matrix are estimated from the time data and the calibration χ^2-test value is computed.

This calibration value is about 20 times higher than its expected value. The main reason is that the data length was too short for an accurate estimation of the residual covariance

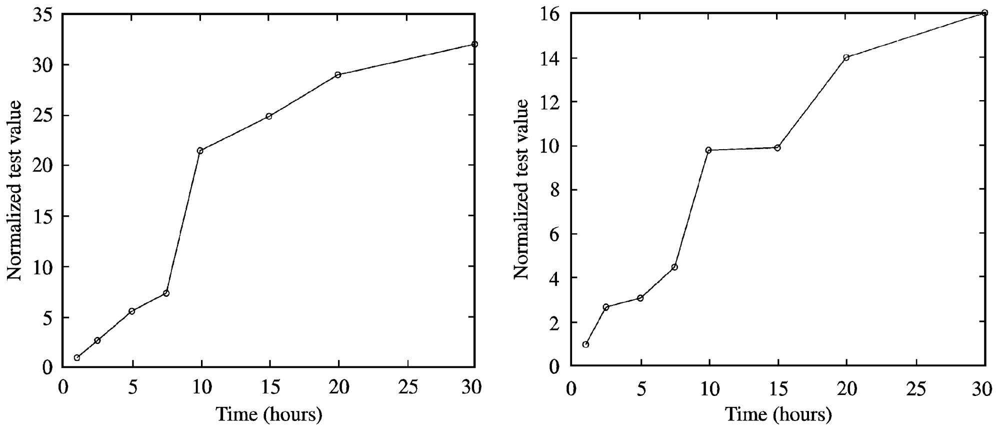

Figure 10. Results of the χ^2-test as a function of test time: (left) six sensors on the body and four sensors on the powertrain; (right) six sensors on the body.

matrix. The calibration value is used to normalise the test values and it is verified whether the χ^2-test could detect the change for short time signals. Two groups of sensor position are evaluated. The first group consisted of the six sensors on the body and four sensors on the powerstrain. The χ^2-test values are shown in Fig. 10 (left) as a function of time. The value slightly increases during the crack growth. When the crack growth ends, the test value significantly increases. Similar conclusions can be drawn when using the six sensors on the body, as shown in Fig. 10 (right).

6. CONCLUSION

This paper illustrates how a subspace-based damage detection method can be successfully used to detect damage in vibrating structures. It shows that the proposed method is capable of detecting damage in an early stage of damage occurring during the experiments. The proposed approach was successfully applied to a few laboratory structures and acceleration data measured on a scale model and a sports car during an endurance test. The paper described how the method can be practically implemented and incorporated in a health monitoring system. Once the user has prepared some data like the nominal modal model and the residual sensitivity and covariance matrix, the χ^2-test can be quickly evaluated. By comparing the test value with a threshold, the simple question whether the structure is healthy or not can be successfully answered.

REFERENCES

1. S. W. Doebling, C. R. Farrar, M. B. Prime and D. W. Shevitz 1996 *Los Alamos National Laboratory Report LA-13070-MS.* Damage identification and health monitoring of structural and mechanical systems from changes in their vibration characteristics: a literature review.
2. A. Rytter 1993 Ph.D. Dissertation. *Department of Building Technology and Structural Engineering, Aolborg University, Denmark.* Vibration based inspection of civil engineering structures.
3. B. Devauchelle-Gach 1991 *Paris IX Dauphine University Thesis*, in French. Diagnostic Mécanique des Fatigues sur les Structures Soumises à des Vibrations en Ambiance de Travail.
4. P. Van Overschee and B. De Moor 1996 *Subspace Identification for Linear Systems: Theory–Implementation–Applications.* Dorderecht: Kluwer Academic Publishers.

5. G. H. III JAMES, T. G. CARNE and J. P. LAUFER 1995 *International Journal of Analytical and Experimental Modal Analysis* **10**, 260–277. The natural excitation technique (NExT) for modal parameter extraction from operating structures.
6. L. HERMANS, H. VAN DER AUWERAER, M. ABDELGHANI and P. GUILLAUME 1998 *Proceedings of the 16th International Modal Analysis Conference*, 300–306. Evaluation of subspace identification techniques for the analysis of flight test data.
7. L. HERMANS, H. VAN DER AUWERAER and M. ABDELGHANI 1998 *Proceedings of the 16th International Modal Analysis Conference*, 1000–1006. Modal testing and analysis of a car under operational conditions.
8. M. PREVOSTO, M. OLAGNON, A. BENVENISTE, M. BASSEVILLE and G. LEVEY 1991 *Journal of Sound and Vibration* **148**, 329–342. State-space formulation, a solution to modal parameter estimation.
9. A. BENVENISTE and J.-J. FUCHS 1985 *IEEE Transactions on Automatic Control* **AC-30**, 66–74. Single sample modal identification of a non-stationary stochastic process.
10. M. BASSEVILLE, A. BENVENISTE, G. MOUSTAKIDES and A. ROUGÉE 1987 *Automatica* **23**, 479–489. Detection and diagnosis of changes in the eigenstructure of non-stationary multi-variable systems.
11. M. BASSEVILLE, M. ABDELGHANI and A. BENVENISTE 1999 *Automatica* **35**, Subspace-based fault detection algorithms for vibration monitoring.
12. M. VIBERG 1995 *Automatica* **31**, 1835–1853. Subspace-based methods for the identification of linear time-invariant systems.
13. M. VIBERG, B. WAHLBERG and B. OTTERSTEN 1997 *Automatica* **33**, 1603–1616. Analysis of state space system identification methods based on instrumental variables and subspace fitting.
14. B. DELYON, A. JUDITSKY and A. BENVENISTE 1997 *Research Report*, *Irisa*, **1104**. On the relationship between identification and local tests.
15. M. BASSEVILLE, A. BENVENISTE, B. GACH-DEVAUCHELLE, M. GOURSAT, D. BONNECASE, P. DOREY, M. PREVOSTO and M. OLAGNON 1993 *Mechanical Systems and Signal Processing* **17**, 401–423. Damage monitoring in vibration mechanics: issues in diagnostics and predictive maintenance.
16. M. BASSEVILLE, A. BENVENISTE, G. MOUSTAKIDES and A. ROUGÉE 1987 *IEEE Transactions on Automatic Control* **AC-32**, 1067–1075. Optimal sensor location for detecting changes in dynamical behavior.
17. L. HERMANS and H. VAN DER AUWERAER 1999 *Mechanical Systems and Signal Processing* (*Special issue on modal testing*) **13**, 193–216. Modal testing and analysis of structures under operational conditions: industrial applications.