Assessment of Subspace Fault Detection Algorithms on a Realistic Simulator-Based Example *

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Abstract

We present in this paper a simulation-based assessment of our recently-presented subspace-based monitoring and diagnostics algorithms [2]. The study is based on a simple but realistic simulation of a rocked during flight with known dynamics. The faults are represented by changes in the mass matrix, and stationary as well as non-stationary excitations are considered. A new idea for a symmetrical use of the test is introduced that could have applications in FE-model updating.

1 INTRODUCTION

Modal identification/monitoring and diagnostics of structures and machines under their operating conditions is an important issue especially for systems where the excitation can not be measured and may be of complex nature. Examples are Offshore structures subject to swell or rotating machinery suject to turbulent excitation.

Recently some of the present authors proposed a new set of fault detection and isolation algorithms based on subspace algorithms that use output-only measurements [1][2][3]. These algorithms are based on the so-called asymptotic local approach used in statistics and work under the hypothesis of small changes in the model parameters.

The purpose of this paper is the thorough exploration of these new algorithms on a simple but realistic simulated example.

Experimental investigations involve 1/ damage detection under both stationary and non-stationary excitation, for small and non-small changes: confronting a nominal model with fresh simulated damaged data, and 2/ model-data correlation (possible use for FE-model updating). This occurs to be a symmetrical use of the above damage detection algorithm: confronting different damaged models with a single nominal data set.

The study is based on a realistic simulator of a rocket during flight with known dymanics. The simulated faults consist of changes with different magnitudes in the model mass matrix.

2 MODELLING ISSUES

We assume that the behavior of the mechanical system can be described by a stationary linear dynamical system, and that, in the frequency range of interest, the input forces can be modeled as a non-stationary white noise. Consequently the relevant model is the following matrix differential equation :

$$\left\{ \begin{array}{rcl} M\ddot{\mathcal{Z}}(t) + C\dot{\mathcal{Z}}(t) + K\mathcal{Z}(t) & = & \nu(t) \\ Y(t) & = & L\mathcal{Z}(t) \end{array} \right. \quad (1)$$

where t denotes continuous time, M, C, K are the mass, damping and stiffness matrices respectively, (high dimensional) vector $\mathcal{Z}$ collects the displacements of the degrees of freedom of the structure; the external (non measured) force ν is modeled as a non-stationary white noise with time-varying covariance matrix $Q_\nu(t)$, measurements are collected in the (low dimensional) vector Y, and matrix L states where the sensors are located. The mechanical characteristics (M, C, K) of the system cannot be recovered from the output measurements. Hence, identifiable modal characteristics of the system are introduced : the vibration modes or eigen-frequencies denoted generically by μ, and the modal shapes or eigenvectors denoted generically by ψ_μ. These quantities are solutions of the following equation :

$$(M\mu^2 + C\mu + K)\,\Psi_\mu = 0 \ \ , \ \psi_\mu = L\,\Psi_\mu \quad (2)$$

Sampling model (1) at rate $1/\tau$ yields the discrete time model in state space form :

$$\left\{ \begin{array}{rcl} X_{k+1} & = & FX_k + \varepsilon_{k+1} \\ Y_k & = & HX_k \end{array} \right. \quad (3)$$

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where the state and the output are :

$$X_k = \begin{bmatrix} \mathcal{Z}(k\tau) \\ \dot{\mathcal{Z}}(k\tau) \end{bmatrix}, \; Y_k = Y(k\tau),$$

the state transition and observation matrices are :

$$F = e^{\mathcal{L}\tau},\; \mathcal{L} = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix},\; H = \begin{bmatrix} L & 0 \end{bmatrix},$$

and where state noise ε_{k+1} is zero-mean, white, with covariance matrix :

$$Q_{k+1} \stackrel{\text{def}}{=} \mathbf{E}\left(\varepsilon_{k+1}\, \varepsilon_{k+1}^T\right)$$

where $\mathbf{E}(.)$ denotes the expectation operator. The modal characteristics defined in (2) are equivalently found in the eigenstructure $(\lambda, \varphi_\lambda)$ of F : $e^{\tau\mu} = \lambda, \quad \psi_\mu = \phi_\lambda \stackrel{\text{def}}{=} H\varphi_\lambda$. We stress that sinusoidal or colored noise excitation can be encompassed as well [5].

From now on, we consider linear multi-variable systems described by a state space model (5), where state X and observed output Y have dimensions m and r respectively, with r (much) smaller than m in practice[1]. The state noise process $(\varepsilon_k)_k$ is an *unmeasured* Gaussian white noise sequence with zero mean and constant covariance matrix Q.

The problem we consider is to monitor the observed system eigenstructure. The observed system eigenstructure is the collection of m pairs (λ, ϕ_λ), where λ ranges over the set of eigenvalues of state transition matrix F, $\phi_\lambda = H\varphi_\lambda$ and φ_λ is the corresponding eigenvector. In all what follows, we assume that the system has no multiple eigenvalues, and thus that the λ's and φ_λ's are pairwise complex conjugate. In particular, 0 is *not* an eigenvalue of state transition matrix F. We stress that the collection of (λ, ϕ_λ) provides us with a canonical parameterization of the pole part of system (5). In particular, it does not depend on the state space basis [6]. In the sequel, referring to vibration monitoring, such a pair (λ, ϕ_λ) is called a mode. The set of the m modes is considered as the system parameter θ :

$$\theta \stackrel{\text{def}}{=} \begin{pmatrix} \Lambda \\ \operatorname{vec}\Phi \end{pmatrix} \tag{4}$$

In (4), Λ is the vector whose elements are the eigenvalues λ, Φ is the matrix whose columns are the ϕ_λ's, and vec is the column stacking operator. Parameter θ has size $(r+1)m$. The FDI problem is to detect and isolate changes in parameter vector θ. By isolation, we mean deciding which mode(s) has(ve) changed.

[1] As explained in [5], when few sensors are available, we identify and monitor a number of modes which is much smaller than the number of solutions of (2), but much larger than the number of sensors.

3 Subspace identification methods

We consider linear multi-variable systems described by a discrete-time state space model :

$$\begin{cases} X_{k+1} &= FX_k + \varepsilon_k \\ Y_k &= HX_k + \nu_k \end{cases} \tag{5}$$

where state X and observed output Y have dimensions m and r respectively.

The state noise process $(\varepsilon_k)_k$ is an *unmeasured* Gaussian white noise sequence with zero mean. We assume noise ε_k to be stationary, that is of constant covariance matrix; the issue of robustness with respect to non-stationary excitation is addressed in detail in [6] [4].

The measurement noise process $(\nu_k)_k$ is assumed to be an *unmeasured* MA(ι) Gaussian sequence with zero mean. In the sequel, we use the notational convention that $\iota = -1$ for no measurement noise, and $\iota = 0$ for white (i.i.d.) measurement noise. Note that, with this MA assumption for its structure, measurement noise does *not* affect[2] the eigenstructure of system (5).

Let $G \stackrel{\text{def}}{=} \mathbf{E}\left(X_k Y_k^T\right)$ be the cross-correlation between state X_k and observation Y_k, and let :

$$\mathcal{O}_p = \begin{pmatrix} H \\ HF \\ \vdots \\ HF^{p-1} \end{pmatrix} \text{ and } \mathcal{C}_p = \begin{pmatrix} G & FG & \dots & F^{p-1}G \end{pmatrix} \tag{6}$$

be the pth-order observability matrix of system (5) and controllability matrix of pair (F, G), respectively. We assume that, for p large enough, both observability and controllability matrices have full rank m.

Consider now a sequence of covariances :

$$R_j \stackrel{\text{def}}{=} \mathbf{E}\left(Y_{k+j}\; Y_k^T\right) \tag{7}$$

of output Y_k of a state space model (5). For $q \geq p+1$, let $\mathcal{H}_{p+1,q}$ be the block-Hankel matrix :

$$\mathcal{H}_{p+1,q} = \begin{pmatrix} R_{\iota+1} & R_{\iota+2} & \dots & R_{\iota+q} \\ R_{\iota+2} & R_{\iota+3} & \dots & R_{\iota+q+1} \\ \vdots & \vdots & \ddots & \vdots \\ R_{\iota+p+1} & \dots & \dots & R_{\iota+p+q} \end{pmatrix} \tag{8}$$

As mentioned above, integer ι reflects the assumed dependency in the measurement noise sequence $(\nu_k)_k$. It should be considered as a design parameter for the algorithms described in the paper.

Choosing the eigenvectors of F as a basis[3] for the state

[2] The same would not hold true with an AR assumption for the measurement noise structure.

[3] This is called the modal basis in the vibration monitoring application [5].

space of model (5) yields the following particular representation of the observability matrix introduced in (6) [5] :

$$\mathcal{O}_{p+1}(\theta) = \begin{pmatrix} \Phi \\ \Phi\Delta \\ \vdots \\ \Phi\Delta^p \end{pmatrix} \tag{9}$$

where diagonal matrix Δ is defined as $\Delta = \mathrm{diag}(\Lambda)$, and Λ and Φ are as in (4). For any other state basis, the observability matrix $\mathcal{O}_{p+1}$ can be written as :

$$\mathcal{O}_{p+1} = \mathcal{O}_{p+1}(\theta)\, U \tag{10}$$

for a suitable $m \times m$ invertible matrix U. Because of the definition of $\mathcal{H}_{p+1,q}$, $\mathcal{O}_p$ and $\mathcal{C}_q$ in (8) and (6) respectively, a direct computation of the R_j's from the model equations :

$$R_{\iota+j+1} = H\ F^{\iota+j+1}\ G \qquad (j \geq 0) \tag{11}$$

leads to the following well known factorization property :

$$\mathcal{H}_{p+1,q} = \mathcal{O}_{p+1}\ \left(F^{\iota+1}\ \mathcal{C}_q\right) \tag{12}$$

Let W_1, W_2 be two user-defined invertible weighting matrices of size $(p+1)r$ and qr, respectively. Let :

$$W_1\ \mathcal{H}_{p+1,q}\ W_2^T = \begin{pmatrix} P & P^\perp \end{pmatrix} \begin{pmatrix} D & 0 \\ 0 & 0 \end{pmatrix} V^T \tag{13}$$

be the singular value decomposition (SVD) of this matrix, where $D = \mathrm{diag}(\sigma_1, \ldots, \sigma_m)$ contains the m non-zero singular values in decreasing order, the m columns of matrix P are the corresponding left singular vectors, and the $(qr-m)$ columns of matrix $P^\perp$ are the left singular vectors associated with singular value 0. Note that P is full column rank (f.c.r.). The factorization property of the Hankel matrix (eq. 12) results in :

$$W_1\ \mathcal{H}_{p+1,q}\ W_2^T = W_1\ \mathcal{O}_{p+1}\ \left(F^{\iota+1}\ \mathcal{C}_q\right) W_2^T \tag{14}$$

with, for example : $W_1\ \mathcal{O}_{p+1} = PD^{1/2}$, where T is a non-singular transformation. The observability matrix can then be recovered, up to a change of basis or equivalently for some pair (H, F), as :

$$\mathcal{O}_{p+1}(H, F) = W_1^{-1} P D^{1/2} T \tag{15}$$

The particular choice of factor $D^{1/2}$, together with T identity, corresponds to the balanced realization (BR) case when weighting matrices W_1 and W_2 are identity. Any other non-singular factor may be used.

The pair (H, F) is recovered from $\mathcal{O}_{p+1}(H, F)$ in a standard way [6]. The eigenstructure of the state transition matrix F is then recovered from : $F = \Phi\ \Delta\ \Phi^{-1}$, obtained, e.g., by eigenvalue solving.

It has been shown in [6] that this algorithm is robust against non-stationary excitation and exhibit good numerical conditionning.

From (13), (9) and (10), we get that the following property characterizes whether a nominal parameter θ_0 agrees with a given output covariance sequence $(R_j)_j$:

$$S^T(\theta_0) W_1\ \mathcal{H}_{p+1,q}\ W_2^T = 0 \tag{16}$$

where matrix $S(\theta_0)$ has dimensions $(p+1)r \times s$ where $s = (p+1)r - m$, and satisfies the following:

$$\begin{aligned} S^T\ S &= I_s & (17) \\ S^T\ W_1\ \mathcal{O}_{p+1}(\theta_0) &= 0 & (18) \end{aligned}$$

Property (**??**) plays an important role for the design of the subspace-based FDI tests, as will be seen later.

4 Subspace-based tests

4.1 The Central Limit Theorem

Assume we have at hand :(i) a nominal model θ_0, and (ii) newly collected data $Y_1, \ldots, Y_n$.
Form the empirical covariance sequence :

$$\widehat{R}_j \stackrel{\text{def}}{=} \frac{1}{n-j} \sum_{k=1}^{n-j} Y_{k+j}\ Y_k^T \tag{19}$$

Define the residual vector :

$$\zeta_n(\theta_0) \stackrel{\text{def}}{=} \sqrt{n}\ \mathrm{vec}\left(S^T(\theta_0)\ W_1\ \widehat{\mathcal{H}}_{p+1,q}\ W_2^T\right) \tag{20}$$

where $\widehat{\mathcal{H}}_{p+1,q}$ is the empirical block-Hankel matrix obtained by substituting $\widehat{R}_j$ for R_j in (8).

From (16) we already know the following. Let θ denote the actual value of the parameter, for the system which generated the new data sample. Then :

$$\mathbf{E}_\theta\ \left(\zeta_n(\theta_0)\right) = 0 \quad \text{iff} \quad \theta = \theta_0 \tag{21}$$

Thus it is natural to replace criterion (16) by the requirement that statistics $\zeta_n(\theta_0)$ should have zero mean. Based on a data sample, testing if this hypothesis is valid requires the knowledge of the statistical properties of the residual $\zeta_n(\theta_0)$. Unfortunately, the distribution of this residual is unknown, in general. One manner to circumvent this difficulty is to use a local approach, that is to assume close hypotheses :

$$(\textit{Safe system})\ \mathbf{H}_0 :\ \theta = \theta_0 \quad \text{and} \tag{22}$$

$$(\textit{Faulty system})\ \mathbf{H}_1 :\ \theta = \theta_0 + \frac{\Upsilon}{\sqrt{n}}$$

where vector Υ is unknown, but fixed. Note that for large n, hypothesis $\mathbf{H}_1$ in (22) corresponds to small deviations in θ.

As formally stated and described in [4] [3], it turns out that residual ζ_n in (20) is asymptotically Gaussian distributed, under both $\mathbf{H}_0$ and $\mathbf{H}_1$, making easy its evaluation. More precisely, it has been proven in [8] that the residuals are distributed as follows:

$$\zeta_n(\theta_0) \rightsquigarrow \begin{cases} \mathcal{N}(\ 0,\ \Sigma(\theta_0)\ ) & \text{under} \quad \mathbf{H}_0 \\ \mathcal{N}(\ M(\theta_0)\,\Upsilon,\ \Sigma(\theta_0)\ ) & \text{under} \quad \mathbf{H}_1 \end{cases} \tag{23}$$

$M(\theta_0)$ and $\Sigma(\theta_0)$ are the Jacobian and covariance matrices, respectively. Detailed derivations of these two matrices are given in [4] [3].

This theorem means that a deviation in the system parameter θ is reflected into a change in the mean value of residual ζ_n, which is asymptotically Gaussian distributed with constant covariance matrix. The sensitivity of residual ζ_n w.r.t. parameter θ is captured by the Jacobian matrix $M(\theta)$. The uncertainty in residual ζ_n is captured by the covariance matrix $\Sigma(\theta)$. Note that matrices $M(\theta_0)$ and $\Sigma(\theta_0)$ depend on neither the size n of the data sample nor the particular fault vector Υ in hypothesis $\mathbf{H}_1$. Thus we do not need to re-estimate them when testing the hypotheses. They can be estimated prior to testing, using data on the safe system (exactly as the reference parameter θ_0). Jacobian matrix M can be estimated from a data sample using a simple sample average.The estimation of covariance matrix Σ is more tricky [9]. A closed form formula for its estimation is given in [4], but we recommend to use an empirical estimate instead, based on a simple version of the jackknife method [5, 9].

4.2 Use for damage detection

We have shown in the previous subsection that using close hypotheses, the change detection problem in the parameter vector θ is transformed into the universal problem of detecting the change in the mean of a Gaussian distributed variable with known covariance matrix. This problem is easily solved using the following χ^2 test.
Let $\widehat{M}, \widehat{\Sigma}$ be consistent estimates of M, Σ. The detection problem, namely deciding that residual ζ_n is *significantly* different from zero can be achieved as follows.

Theorem 4.1 (χ^2-test.) *Assume additionally that :*

$$\textit{Jacobian matrix } M(\theta_0) \textit{ is full column rank (f.c.r.)} \tag{24}$$

Then the test between the hypotheses $\mathbf{H}_0$ and $\mathbf{H}_1$ defined in (22) is achieved through :

$$\chi^2_n \stackrel{\text{def}}{=} \zeta_n^T\ \widehat{\Sigma}^{-1}\ \widehat{M}\left(\widehat{M}^T\ \widehat{\Sigma}^{-1}\ \widehat{M}\right)^{-1} \widehat{M}^T\ \widehat{\Sigma}^{-1}\ \zeta_n \tag{25}$$

which should be compared to a threshold. In (25), the dependence on θ_0 has been removed for simplicity. Test statistics χ^2_n is asymptotically distributed as a χ^2-variable, with rank(M) *degrees of freedom*[4]*, and with non-centrality parameter under $\mathbf{H}_1$:*

$$\Upsilon^T\ M^T\ \Sigma^{-1}\ M\ \Upsilon \tag{26}$$

In formula 25, the Jacobian and covariance matrices, $\widehat{M}, \widehat{\Sigma}$, are computed using the nominal model θ_0 and the nominal data set. The residual ζ_n, on the other hand, is computed using the nominal model and the freshly (damaged) collected data.
The isolation problem is to decide which (subsets of) components of the parameter vector are affected by the change. In the case of the present paper, this allows us to formulate the diagnostics of changes in terms of eigenfrequencies, damping coefficients, and eigenvectors. The isolation can be performed based on the above asymptotic Gaussianity of residual ζ_n and the tools described in detail in [4, 9]. Note that condition (24) is instrumental in performing the isolation of changes.

4.3 Use for model-data correlation

While for the damage detection use of the test one confronts a nominal model to freshly collected (damaged) data, in many applications it is desired to confront one nominal data set to different (damaged) models. This corresponds to the case of Finite-Element model updating, for instance. This happens to be a symmetric use of the proposed test:

- Use the (damaged) Finite Element model and the nominal data set to compute the Jacobian and covariance matrices.
- Compute the residuals ζ_n using the (damaged) FE-model and the nominal data set.
- Compute the χ^2 test (eq. 25) using the obove estimated quantities.

The test hence computes the mismatch between the data and the model, and using the sensitivity tests it is possible to decide which parameters of the model that are mostly reponsible for the mismatch.

5 Simulation Study

5.1 The simulator

This simple model is intended to be representative of the variations of modal properties of a rocket during flight.

[4]If Jacobian matrix M is not f.c.r., it should be reduced to a f.c.r. matrix. The number of degrees of freedom of the χ^2-test is then the column rank of the reduced matrix.

The representativity is based on the fact that the structure is almost axisymmetric and that the mass inside the tube is varying during flight.
The model used is composed of 168 quad4 elements for the external shell and 10 hexa8 solid elements for the representation of the mass loading effects of the fluid. This effect is simply represented by varying the density of the internal mass which is a very simplified representation of an emptying tank. The initial 1020 DOF model is re-

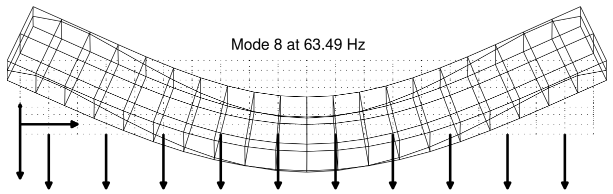

Figure 1: An example of the deflection shape of the structure

duced on a basis of 40 nominal modes of the structure with maximum mass loading complemented with 4 shapes associated to the configuration without mass loading. The equation of motion is represented by the following:

$$\begin{array}{c}(M_0 + p\Delta M)\ddot{q}(t) + C_0\dot{q}(t) + K_0 q(t) = bu(t) \\ y(t) = cq(t)\end{array} \tag{27}$$

The mass parameter `p` varies in the range 10 to 0.1 using a logarithmic spacing. The frequency range of interest is [0–110 Hz]. 13 collocated inputs and outputs are defined. The first 3 inputs correspond to the engine force which has a variable direction defined by two angles `ang` as follows (in matlab notation):

$$F_t = b(:, 1:3)*$$

$$[sin(ang(1)) * [cos(ang(2)); sin(ang(2))]; cos(ang(1))] * u_t$$

where u_t is a scalar variable representing the force magnitude. The next 10 inputs are transverse forces distributed over the height and are assumed to be representative of wind loading.
The angles, the spectrum of the engine force, and the distribution of transverse forces are unknown and would typically not be measured. The following table presents the first 10 modes of the structure for the mass parameter $p = 10$, as computed from the FE-model.

5.2 Results and discussion

In this study, stationary as well as non-stationary excitation were simulated. For the stationary case,the angles of the engine force were kept constant and equal to zero.The amplitude of the engine force was simulated using a white

Frequency (hz)	Damping(%)
14.61	0.21
14.76	0.22
17.23	0.23
29.60	0.23
36.00	0.17
36.36	0.17
39.77	0.22
51.89	0.17
56.37	0.24
62.26	0.20

Table 1: Frequencies and damping factor of the simulated structure for $p = 10$

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noise sequence with standard deviation of 10000. For simulating a non-stationary excitation, the engine force angles were allowed to change during time. The first angle (`ang1`) varies between 0-0.5 deg, while the second angle (`ang2`) varies between 0-360 deg. Each angle was computed as the output of a linear filter (Butterworth type filter of order 4 and cutt-off frequency about 10 Hz) driven by white noise.The amplitude of the driving white noise was automatically adjusted so that the amplitudes of the angle signals correspond to the variations of the angles. Using this procedure, 16000 data points were generated. Different signals corresponding to different mass changes were also generated for use in the fault detection experiments. The Balanced Realisation (BR) algorithm was used for computing the modal signature using different sensor pools.
Concerning the damage detection use of the test, the information available consists of the identified modal signature and the Jacobian computed using the nominal data set, on one hand, and the fresh data corresponding to the damaged structure, on the other.

Figures (2-3) show the values of the global test for different changes in the structure's mass, for the case of stationary and non-stationary excitation and for different sensor pools. Inspection of the figure shows that in general the test detects very well the damage and that for very small (less than 1%) as well as higher changes (up to 10 %). It is interesting to notice that for very small changes (less than 1%), the test seems to be proportional to the changes in the mass. This property however is no longer valid for higher changes and the test tends to stay relatively constant. This illustrates the effeciency of the test in detecting small changes. The modal sensitivity tests are given in Table 2 for the particular case of 0.1%

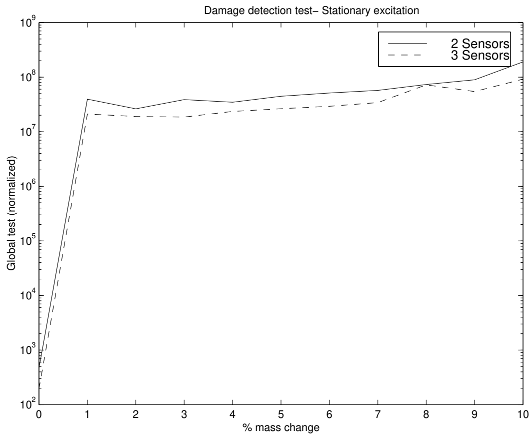

Figure 2: Global test vs. changes in the mass for different sensors pools.

Mode No.	No Change	Mass Change
1	271.6	16828550.1
2	258.9	16477968.3
3	254.8	14551770.8
4	249.7	14318555.2
5	241.3	13359136.0
6	236.0	12852576.0
7	234.3	12397815.1
8	232.4	12140711.7
9	220.1	11617259.4
10	206.7	11579474.6
11	146.6	10381200.3

Table 2: Modal sensitivity tests (usual test) under no change condition and under the condition of mass change(0.1%). 2 sensors example.

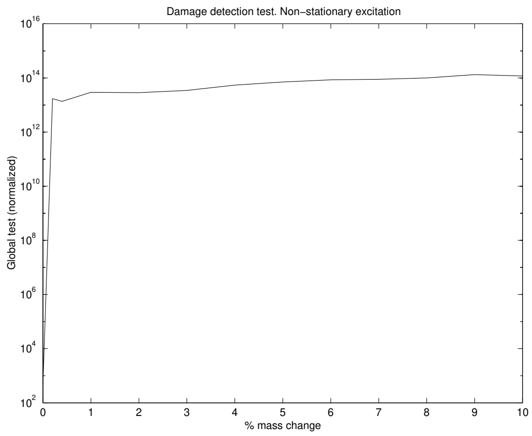

Figure 3: Global test vs. changes in the mass for the case of non-stationary excitation (sensors 4-5).

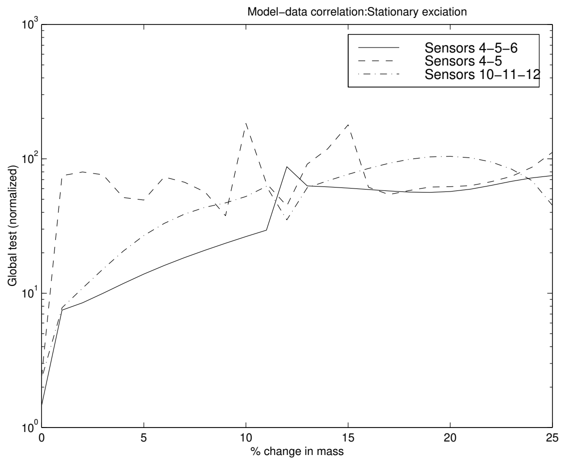

Figure 4: Global test vs. % change in mass. Model-data correlation test and stationary excitation.

change in the mass. The modal sensitivity tests indicate that all modes have been changed, something that is expected from a mass change on the structure.
For the model-data correlation use of the test on the other hand, the only information one has at hand is the modal signatures computed from (damaged) FE-models, and a nominal data set corresponding to the nominal structure. Figures (4-5) show the values of the global test for different changes in the mass and for stationary as well as non-stationary excitation. It is obvious to notice that the test is less sensitive to the changes than its usual use. Higher changes in the mass, compared to the usual use of the test, are needed in order to have significant changes in the global test compared to its no-change value. Nonetheless, the values of the global test clearly indicate the mismatch. The modal sensitivity tests in Table 3 also indicate clearly the change in all modes due to the mass change.

6 Conclusions

We presented in this paper a simulation-based study for the assessment of the recently proposed subspace damage detection algorithms. The study was based on a realistic simulation of a rocket during flight with known dymanics. The simulated damage consists of varying the mass parameter in the model. A symmetric use of the test is introduced and investigated with possible applications to FE-model updating.
The main conclusions are that the subspace tests are quite sensitive to changes in the model parameters and that for small as well as higher changes. The use of the model-data correlation version of the test on the other hand is less sensitive and higher changes are needed to well detect the damage.

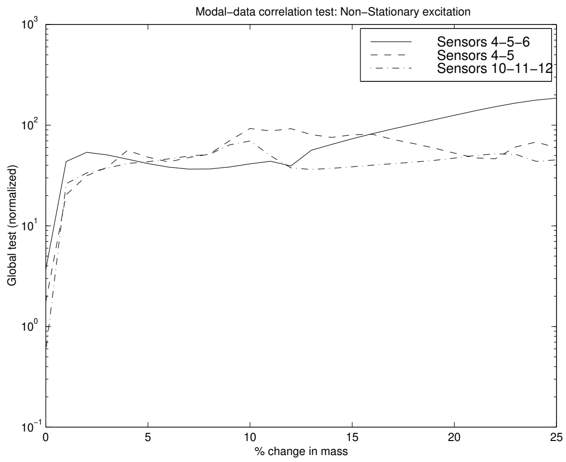

Figure 5: Global test vs. % change in mass. Symmetric test and non-stationary excitation.

Mode No.	No Change	Mass Change
1	227.0	3317.8
2	225.3	2955.2
3	218.8	2910.4
4	215.3	2873.6
5	213.8	2855.5
6	209.2	2790.9
7	208.2	2782.2
8	201.9	2764.5
9	199.5	2716.3
10	190.5	2704.3

Table 3: First 10 modal sensitivity tests (symmetric test) under no change condition and under the condition of mass change(10%). 3 sensors example.

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