NON-UNIFORM SOURCE MODELING FOR DISTRIBUTED VIDEO CODING

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ABSTRACT

We introduce a novel correlation model, called predictive Binary Symmetric Channel (BSC), for Distributed Source Coding (DSC). We then consider non-uniform binary sources and show that, for the predictive BSC model, the non-uniformity does not reduce the compression rate in asymmetric DSC. We assess the minimum achievable rate loss induced by a mismatch between the assumed and the true correlation models. We also propose an LDPC-based decoder adapted to both the classical (additive) BSC and the new (predictive) BSC correlation models. Finally, for Distributed Video Coding (DVC) application, we propose a criterion that allows us to switch between the two correlation models.

1. INTRODUCTION

Distributed Source Coding (DSC) refers to the problem of separate encoding and joint decoding of correlated sources. Slepian and Wolf (SW) have established in 1973 [1] that, for two dependent binary sources \( X \) and \( Y \), the lossless compression rate bound \( H(X, Y) \) could be approached when encoding the two sources separately provided that they are decoded jointly. The lossy equivalent of the SW theorem for two correlated continuous-valued sources has been formulated by Wyner and Ziv (WZ) [2]. Most Slepian-Wolf and Wyner-Ziv coding systems are based on channel codes (e.g., convolutional codes [3], turbo codes [4] or Low Density Parity Check (LDPC) codes [5]). The statistical dependence between two correlated sources \( X \) and \( Y \) is modeled as a virtual correlation channel analogous to Binary Symmetric Channels (BSC) or additive white Gaussian noise (AWGN) channels. In the sequel we consider asymmetric DSC in which the source \( Y \) (called the side information) is usually regarded as a noisy version of \( X \). Using error correcting codes, the compression of \( X \) is achieved by transmitting only parity or syndrome bits which are decoded given the side information \( Y \).

Video compression has been recast into a distributed source coding framework leading to Distributed Video Coding (DVC) systems. The video sequence is structured into Groups Of Pictures (GOP), in which key frames (KF) are intra-coded and intermediate frames are WZ-coded. Each WZ frame is encoded independently of the other frames. In this paper, we consider the DVC codec described in [6] developed by the European IST-DISCOVER project and which will be referred to as the DISCOVER codec in the sequel. The WZ data is transformed and then quantized, and the quantized coefficients are binarized. Each resulting bit plane is then encoded with a channel coder. We will use an LDPC coder in the experiments reported in this paper. The decoder constructs the side information via motion-compensated interpolation of previously decoded key frames. The encoder first sends per bit plane a subset of the syndrome bits coded by the LDPC encoder. If the decoder cannot properly decode the current bit plane, more syndrome bits are requested from the encoder via a feedback channel.

All DVC codec implementations often assume the binary distribution of the bit planes to be uniform, which is actually far from being the case (as we will see in the experiments section). The correlation model between the WZ \( (X) \) and the SI \( (Y) \) bit planes is also assumed to be additive symmetric \( (Y = X + Z) \). In this additive channel model, the correlation noise \( Z \) is assumed to be independent of \( X \) but not of \( Y \). In this paper, we show that, in DVC systems, the correlation noise \( Z \) is on contrary, most of the time, independent of the SI \( Y \). This has led us to introduce another correlation model which we called the predictive channel model \( (X = Y + Z) \).

This paper also describes a modified LDPC decoder which accounts for the non-uniformity of the sources and adapts to the correlation model. The source distribution is iteratively estimated along with the symbol bits, and the reliability of this estimation is assessed. Experimental results show that the channel modeling the correlation between the WZ and SI bit planes in a DVC codec is actually a mixture of the additive and predictive channel models. The dynamic decoder adaptation to the correlation model and to the non-uniformity of the sources leads to a rate gain of about 5.7%.

This paper is structured as follows. Section 2 reviews the theory of DSC for non-uniform sources, and presents the rate bounds that can be achieved with the additive and the predictive BSC. Section 3 describes the proposed LDPC decoder in order to take the source non-uniformity, as well as the type of the BSC, into consideration; the performance for the coding of synthetic sources is shown. Section 4 presents the exploitation of the source non-uniformity for DVC. We prove the accuracy of the non-uniform source modeling besides the existence of both additive and predictive channels, then our experimental results show that the PSNR versus rate performance of the codec is improved.

2. DISTRIBUTED SOURCE CODING OF NON-UNIFORM SOURCES

This section first revisits the principle of asymmetric DSC with the classical binary symmetric correlation model, which is referred to as additive BSC. A new model called predictive BSC is then introduced. The rate bounds are derived for both models in the case the sources are non-uniform.

2.1 Review of the asymmetric DSC

Let \( X \sim B(p_X) \) denote a binary variable. Bernoulli distributed with parameter \( p_X = \mathbb{P}(X = 1) \). In DSC, a second
source $Y$, with realization $y$, is correlated to $X$ and available at the decoder. The correlation is modeled as a virtual $(X,Y,p)$ BSC as defined below.

**Definition 1.** An $(X,Y,p)$ additive BSC is a channel with binary input $X$, binary output $Y$. The noise $Z \sim \mathcal{B}(p)$ is independent of the channel input, and the channel output is obtained by $Y = X \oplus Z$.

Note that our additive BSC is the classical BSC. In the remainder of this paper, "$X$" denotes the source that has to be decoded from its syndrome $S_X$ and the side-information is "$Y$". This setup is commonly called "asymmetric DSC". In [7], Wyner showed that binary linear codes can reach the SW bounds with the syndrome approach. More precisely, consider an $(N,K)$ channel code $\mathcal{C}$ defined by its $(N - K) \times N$ parity check matrix $H = (h_{nm})$. This code defines a partition of the $N$-long sequences into cosets, where all the sequences in a coset $\mathcal{C}_n$ share the same syndrome $s$: $\mathcal{C}_n = \{ x : Hx = s \}$. To encode $x$, the encoder transmits its syndrome $s_n = Hx$, achieving a compression ratio of $N : (N-K)$; $y$ is sent at its entropy $H(Y)$ and can therefore be retrieved. At the decoder, the sequence $X$ is found as the closest sequence to $y$, having syndrome $s_n$. In the sequel, we use LDPC codes which have been shown to perform closely to the SW bound [5], while keeping their encoding and decoding complexity linear with the code length.

### 2.2 Properties of the sources and correlation models

The achievable lower bounds depend not only on the correlation between $X$ and $Y$, but also on their respective distributions. Let $X \sim \mathcal{B}(p_X)$ and $Y \sim \mathcal{B}(p_Y)$. Let $Z$ be the noise, modeled as a BSC. First, when the source $X$ is uniform, i.e. $p_X = 0.5$, the source $Y$ is also uniform, and $H(X) = H(Y) = 1$, and $H(X|Y) = H(Y|X) = H(Z) = H(p)$, where $H(p) = -\log(p) - (1-p)\log(1-p)$.

Now, consider that $X$ is non-uniform ($p_X \neq 0.5$). Then $Y$ is also non-uniform. We claim the following results to characterize the rate gain that is expected depending on the type of BSC.

**Claim 1.** Let $X \sim \mathcal{B}(p_X)$ and $Y \sim \mathcal{B}(p_Y)$ be two correlated sources, where the correlation is modeled as a virtual $(X,Y,p)$ additive BSC. We consider the asymmetric distributed problem, where $Y$ is available at the decoder and compressed at its entropy $H(Y)$ and $X$ is compressed at its conditional entropy $H(X|Y)$. If the source $X$ is non-uniform, the compression rate for $X$ is reduced by $H(Y) - H(X) \geq 0$ compared to the compression rate achieved for uniform sources.

**Proof.** Since the BSC is additive, $\exists Z$ independent of $X$ s.t. $X \sim \mathcal{B}(p_X)$ and $Y = X \oplus Z$. Then $p_Y = p_X(1-p) + (1-p_X)p$. The concavity of $H(\cdot)$ implies $H(Y) \geq H(X)$. Moreover $H(X|Y) = H(Z) - H(Y) - H(X)$. Since $H(Y) - H(X) \geq 0$ (with equality iff the source $X$ is uniform), the non-uniformity of $X$ reduces the lower bound $H(X|Y)$, and the rate gain is $H(Y) - H(X)$.

**Definition 2.** An $(X,Y,p)$ predictive BSC is a channel with binary input $X$, binary output $Y$. The noise $Z \sim \mathcal{B}(p)$ is independent of the channel output s.t. $X = Y \oplus Z$.

This model corresponds to the case where $Y$ represents a prediction of $X$. $Z$ is therefore an innovation noise independent of $Y$. When the correlation between the sources $X$ and $Y$ is a predictive channel, we get the following result:

**Claim 2.** Let $X \sim \mathcal{B}(p_X)$ and $Y \sim \mathcal{B}(p_Y)$ be two correlated sources, where the correlation is modeled as a virtual $(X,Y,p)$ predictive BSC. We consider the asymmetric distributed problem, where $Y$ is available at the decoder and compressed at its entropy $H(Y)$ and $X$ is compressed at its conditional entropy $H(X|Y)$. The non-uniformity of $X$ does not reduce the compression rate for $X$.

**Proof.** Here $H(X|Y) = H(Z)$. Therefore, the compression rate for $X$ only depends on the noise statistics, and the non-uniformity of $X$ does not reduce its compression rate.

The introduction of the predictive channel is motivated by the DVC application. In this context, $X$ represents the current image to be compressed and $Y$ represents the prediction of $X$ based on previous and future images obtained at the decoder side. Therefore, the noise $Z$ is an innovation noise and is more likely to be independent of $Y$ than of $X$. Unfortunately, for a predictive channel the compression rate for $X$ does not reduce as the non-uniformity of $X$ increases. In the following, we show that a mismatch between the true and the assumed correlation models always degrades the performance of the decoder.

**Claim 3.** Let $X \sim \mathcal{B}(p_X)$ and $Y \sim \mathcal{B}(p_Y)$ be two correlated sources, where the correlation is modeled as a virtual $(X,Y,p)$ (predictive or additive) BSC. We consider the asymmetric distributed problem, where $Y$ is available at the decoder and compressed at its entropy $H(Y)$, and $X$ is compressed at its conditional entropy $H(X|Y)$. A mismatch between the true correlation model and the one assumed by the codec implies a rate loss if the sources are non-uniform.

**Proof.** If the decoding is performed with a predictive channel model, then the lower achievable rate will be $H(Z)$. A rate loss of $H(Z) - H(X|Y)$ will then result. Let us now assume that the correlation channel model is predictive. The lower rate bound is in this case $H(X|Y) = H(Z)$. If the decoding is performed by considering the additive BSC model, then the minimal achievable that can be achieved will be $H(Z) - [H(Y) - H(X)]$ with $H(Y) - H(X) \leq 0$.

Motivated by the expected rate gain when the source distribution is non-uniform and the correlation channel is additive, we describe how to modify the existing syndrome-based LDPC decoder to assess and exploit the source non-uniformity as well as the nature of the BSC.

### 3. DISTRIBUTED CODING OF NON-UNIFORM SOURCES USING LDPC CODES

LDPC codes can be represented either by a sparse parity-check matrix (binary matrix of low proportion of 1) or by a bipartite graph. For an LDPC code yielding a compression rate $N : (N-K)$, there are $N$ variable nodes and $(N-K)$ check nodes in the graph. Here, we modify the standard decoding proposed by Liveris et al. [5] to exploit the non-uniformity of the sources and to deal with the type of BSC. The encoder is not modified.

#### 3.1 The proposed syndrome-based LDPC decoder

Consider the following notation and definition of the messages that are passed in the graph. Here, the correlation channel is assumed to be a BSC; if some other channel is used, the intrinsic information only needs to be adapted correspondingly. In particular, in section 4, the correlation between the sources is modeled as a Laplacian channel.
• $x_n, n \in [1, N]$ are the source symbols, represented by the variable nodes; its estimate is $\hat{x}_n$
• $y_n, n \in [1, N]$ are the side-information symbols, represented by the side-information nodes;
• $s_{m}, m \in [1, (N - K)]$ are the syndrome symbols, representing the check nodes. $x_n$ is connected to $s_m$ if $h_{mn} = 1$;
• $d_m$ is the degree of $x_n$;
• $d_m$ is the degree of $s_m$;
• $I_n, n \in [1, N]$ are the intrinsic, passed from $y_n$ to $x_n$;
• $E_{n,e}, e \in [1, d_{un}]$ are the extrinsic information, passed from $x_n$ on their $e$-th edge to the check nodes;
• $Q_{m,e}, m \in [1, (N - K)], e \in [1, d_{sm}]$ are the messages passed from $s_m$ on their $e$-th edge to the variable nodes;
• $p_X$ denotes the estimate of $p_X$. It is updated throughout the iterations, after each update of $\hat{x}$.

All the messages are Log-Likelihood Ratio (LLR), they are labeled (in) or (out) if they come to or from the nodes.

3.1.1 Intrinsic information computation

The intrinsic information depends on the type of BSC. It is defined by $I_n(p_X) = \log \left( \frac{p_{X=x_n=1|y_n}}{p_{X=x_n=0|y_n}} \right) = \begin{cases} (1 - 2y_n) \log \left( \frac{1 - p}{p} \right), & \text{if the BSC is predictive} \\ (1 - 2y_n) \log \left( \frac{1 - p}{p} \right) + \log \left( \frac{1 - p_X}{p_X} \right), & \text{if additive} \end{cases}$ (1)

Since $p_X$ is not known, each $I_n$ is initialized to $I_n(\hat{p}_Y)$ where $\hat{p}_Y$ is the probability of 1’s in $Y$, that is the best guess on $p_X$ so far. Each $E^{(in)}_{n,e}$ is initialized to 0.

3.1.2 Messages from the variable nodes to the check nodes

$$E^{(out)}_{n,e} = I_n(\hat{p}_X) + \sum_{k=1, k \neq e}^{d_n} E^{(in)}_{n,k}$$

where $I_n(\cdot)$ is defined in (1) and the estimate $\hat{p}_X$ is explained in 3.1.4. Each $E^{(out)}_{n,e}$ is mapped to the corresponding $Q^{(in)}_{m,e}$ according to the connections in the graph.

3.1.3 Messages from the check nodes to the variable nodes

$$Q^{(out)}_{m,e} = 2\tanh^{-1} \left( (1 - 2s_m) \prod_{k=1, k \neq e}^{d_m} \tanh \left( \frac{Q^{(in)}_{m,k}}{2} \right) \right)$$

Each $Q^{(out)}_{m,e}$ is mapped to the corresponding $E^{(in)}_{n,e}$.

3.1.4 Decision, and update of $\hat{p}_X$

We denote $E_n = I_n(\hat{p}_X) + \sum_{k=1}^{d_n} E^{(in)}_{n,k} = \log \left( \frac{1 - p}{p} \right)$ where $p_n$ is the best guess on $p(X_n = 1|y, s_X)$ so far. Then $p_n = \frac{e_n}{1 + e_n}$, and $\hat{x}_n = \begin{cases} 1, & \text{if } p_n \geq 0.5 \\ 0, & \text{otherwise} \end{cases}$

$\hat{p}_X$ is estimated as the probability of 1’s in $\hat{x}$, expressed with the soft values $p_n$. The updated value of $\hat{p}_X$ is thus given by

$$\hat{p}_X = \frac{\sum_{n=1}^{N} p_n}{N}$$

3.1.5 Stopping criteria: syndromes check, convergence test, and maximum number of iterations

The decoding algorithm stops either if the estimated $\hat{x}$ satisfies the parity check equation ($H\hat{x} = s_x$) or if the maximal number of iterations has been reached (100 iterations is a good compromise between performance and complexity). Moreover if the syndrome test has failed, while no symbols of $\hat{x}$ have been updated during the current iteration, even if we have not reached the maximal number of iterations yet, we decide that the decoder has converged to a wrong word.

3.2 Simulation results

Non-uniformity and additive BSC

We test the proposed decoding algorithm using an LDPC code of rate $\frac{1}{2}$ created using the Progressive Edge Growth (PEG) principle [8]. The non-uniform sources are drawn for two values of $p_X = \{0.15, 0.2275\}$; and, for each $p_X$, a range of values of $p$ is considered. The source sequences are of length $N = 1584$ - the same length as the bit planes extracted from the video frames (section 4). The syndrome $s_x$, as well as the side-information $y$, are transmitted to three different decoders: (1) the standard decoder that views $X$ as a uniform source, (2) the proposed decoder that knows that $X$ is non-uniform and has to estimate $p_X$, (3) a genie-aided decoder that knows $p_X$ (in order to quantify the sub-optimality introduced by the parallel estimation of $\hat{p}_X$). When the BSC is additive, Fig. 1 shows the performance of the decoding in terms of its Bit Error Rate (BER) versus $H(p)$.

![Figure 1: Performance of the standard and the modified LDPC decoders, for non-uniform sources with $p_X = \{0.15, 0.2275\}$, over an additive BSC. When considering that $X$ is uniform, (1), $H(X|Y) = 0.5$ is achieved for $H(p) = 0.5$ ($p = 0.11$) regardless of the source distribution. When exploiting the non-uniformity, (2, 3), $H(X|Y) = 0.5$ occurs for $H(p) = 0.622$ ($p = 0.155$) when $p_X = 0.2275$, and for $H(p) = 0.79$ ($p = 0.239$) when $p_X = 0.15$.

The standard decoder (1) has the same performance regardless of the source distribution. Meanwhile, the decoders (2) and (3) exploiting the non-uniformity are able to retrieve $X$ from considerably greater $H(p)$; the rate gain increases with the non-uniformity. The estimation of the source Bernoulli parameter, only induces a loss lower than 0.02bit when the BER is under $10^{-5}$, which is acceptable regarding the rate gain with respect to the standard decoder.

Effect of a mismatch between the true correlation model and the one assumed by the decoder

Now, we assess the impact of a wrong guess of the type of BSC between the correlated sources. Let us first consider the case where the true correlation model is additive while the decoder assumes a predictive one. The curve (1) in Fig. 1 shows the performance of such a mismatched decoder while curves (2) and (3) show the performance of the matched de-
obtained transform coefficients are quantized (Block 3 based Discrete Cosine Transform (DCT, Block 3). Frames are encoded in Block 3: they first undergo a block-based H264/A VC encoder and transmitted to the decoder. The WZ frames are conventionally encoded in Block 2 using an encoder. As expected in Claim 3, the mismatch induces a significant rate loss. For the other case, we consider a predictive model while the decoder assumes an additive one. To that end, we generate \( Y \sim \mathcal{B}(p_Y) \) and \( X = Y \oplus Z \), making sure that \( p_X \) is constant, and we plot the BER of that system on Fig. 2.

![BER vs H(p) for different decoders](image)

Figure 2: Influence of a wrong guess on the channel (the BSC is assumed to be additive while it is predictive in reality), for non-uniform sources with \( p_X = \{0.15, 0.2275\} \).

As expected (see Claim 3) the mismatched decoder performs worse than the correct decoder.

**Parallel parameter estimation**

Our decoder presented in section 3.1 estimates the distribution parameter \( p_X \). Fig. 3 shows the estimated parameter averaged over \( 5 \times 10^4 \) realizations of the source.

![Estimated p_X vs H(p) for different realizations](image)

Figure 3: Performance of the parameter estimation that is performed in parallel, for non-uniform sources with \( p_X = \{0.15, 0.2275\} \).

When the correlation level is such that the operating point is far from the SW bound (\( H(p) < 0.52 \) for \( p_X = 0.2275 \) and \( H(p) < 0.62 \) for \( p_X = 0.15 \), the decoding of \( X \) is successful (BER < \( 10^{-2} \)) and the parameter is well estimated (the gap to the actual \( p_X \) is lower than \( 10^{-3} \)); but when the correlation is lower, the decoding of \( X \) fails and the parameter estimation fails as well.

4. DISTRIBUTED VIDEO CODING EXPLOITING THE NON-UNIFORMITY OF THE BIT PLANES

4.1 Review of the DISCOVER codec

Fig. 4 shows the DISCOVER codec block diagram. The encoder consists into the Blocks 1, 2, and 3. Block 1 splits the video frames into key frames (KF) and WZ frames. The KF are conventionally encoded in Block 2 using an HD264/AVC encoder and transmitted to the decoder. The WZ frames are encoded in Block 3: they first undergo a block-based Discrete Cosine Transform (DCT, Block 3a), and the obtained transform coefficients are quantized (Block 3b). The quantized coefficients are organized into bands, where every band contains the coefficients associated to the same frequency in different blocks. Then, organized bit plane by bit plane, the quantized coefficients bands are fed to a SW encoder, which computes their syndromes. In this work, we model those bit planes as non-uniform sources.

At the decoder, the KF are first decoded in Block 4 using a conventional video decoder. Then a motion compensated interpolation between every two closest KF is performed in Block 5, in order to produce the SI for a given WZ frame. The correlation between the WZ bit planes and SI is approximated by a Laplacian channel, in Block 6. In Block 7, the same transform used at the encoder is applied to the SI to obtain an estimation of the WZ bit planes. Based on that information, and thanks to the syndromes, Block 8 performs the proper WZ decoding. More particularly, Block 8a performs the rate-adaptive SW decoding [9], using feedback channels between Blocks 3f and 8b; that is to avoid re-encoding the data. Once the bit planes are correctly decoded, Block 8d makes an inversion of the transform applied by Block 3a, and the decoded video frames are obtained by conveniently multiplexing the decoded KF and the WZ frames.

4.2 Accuracy of the model proposed for the bit planes

We investigate on the distributions of the coded bit planes by assessing off line their Bernoulli distributions. We show in Fig. 5 the probability of 1’s in some WZ bit planes from the three 15Hz QCIF video sequences Hall monitor, Foreman, Soccer.

![Probability of 1's in some WZ bit planes](image)

Figure 5: Probability of 1’s in the bit planes taken individually. The WZ bit planes of the three video sequences are non-uniformly distributed, which justifies the model adopted for their decoding.

The results on Fig. 5 confirm that the bit planes are non-uniformly distributed, besides their distribution varies from bit plane to bit plane. This justifies that the non-uniform modeling is a better model than the uniform one, and that we have to estimate the parameter of each bit plane separately. Comforted by the huge non-uniformity of the bit planes, we address the problem of adapting the SW decoder so as to take into account their non-uniformity. The channel between the correlated bit planes is modeled as Laplacian; we use the soft information from Block 6 to initialize the intrinsic of the proposed LDPC decoder (section 3.1.1).

4.3 Experimental results

We place the proposed LDPC decoder in Block 8a. The results presented in Fig. 6 and Fig. 7 are obtained for all the WZ frames of the video sequences Hall monitor, Foreman, Soccer, with a GOP length of 2. Key frames are encoded with H.264/AVC Intra (main profile), and the quantization parameters for each point are chosen so that the average quality (PSNR) of the WZ frames is similar to the quality of the KF. All rate and distortion results refer only to the luminance. The proposed SW decoder that uses the non-uniformity is compared to the standard DISCOVER’s SW decoder.

The channel is assumed to be additive

First, we assume that the Laplacian channel is additive, and we take into account the non-uniformity for the decoding of all the bit planes. The results are presented in Fig. 6.

The rate gain occurs only for the sequence Soccer at the highest PSNR, it is 1.88kbps (−0.65%). This low rate gain contrasts with the huge non-uniformity of the bit planes.
5. CONCLUSION

We have proposed a novel predictice channel for the modeling of the correlation between two distributed sources. We have shown that the achievable compression rate increases with the non-uniformity when the channel is additive; when the channel is predictive, there is no rate gain exploiting the non-uniformity. These theoretical results were verified with synthetic sources. The predictive channel better describes the behavior of the noise that is observed between the non-uniform WZ and SI bit planes produced by a distributed video codec. To handle the non-uniformity and the two channel models, we have proposed an adapted asymmetric SW decoder using LDPC codes. The Bernoulli parameter of the source is estimated iteratively at the decoder, along with the decoding. The decoder was finally incorporated into the video codec, exploiting the non-uniformity of the WZ bit planes and the correlation models. We presented results that prove the efficiency of the proposed SW decoder, in terms of improving its PSNR versus rate performance.

REFERENCES