Correct Handling of Floating-Point Computations in Symbolic Execution

Bernard Botella
THALES A.S.

Arnaud Gotlieb
IRISA / INRIA

Claude Michel
INRIA Sophia-Antipolis

Introduction

☞ Symbolic Execution (SE): Evaluation of statements with symbolic values along a control flow path

☞SE is exploited in several applications :

- automatic test data generation [King 75, Clarke TSE'76, Meudec STVR'01]
- feasible path analysis [Goldberg et al. ISSTA'94]
- program proving [Chen et al. ISSTA'02]
- model checking [Khushid et al. TACAS'03]

...

☞Currently, floating-point variables are handled as reals in SE applications

```
float foo( float x) {
      float y = 1.0e12 ;
1.    if( x < 10000.0 )
2.          z = x + y;
3.    if( z > y)
4.          …
```

Is the path 1-2-3-4 feasible ?

Path conditions:

$x < 10000.0$

$x + 1.0e12 > 1.0e12$

On the reals : $x \in (0,10000)$

On the floats : no solution !

Conversely,

```
float foo( float x) {
        float y = 1.0e12 ;
1.      if( x > 0.0 )
2.              z = x + y;
3.      if( z == y)
4.              …
```

Is the path 1-2-3-4 feasible ?

<u>Path conditions:</u>

$x > 0.0$

$x + 1.0e12 = 1.0e12$

On the reals : no solution

On the floats: $x \in (0, 32767.99\ldots)$

Outline

- IEEE-754 and restrictions
- Simple Symbolic Execution
- Solving path conditions over the floats
- Current implementation FPSE
- Further work

Floating-point numbers (IEEE-754)

- float : (s,f,e) a bit pattern of 32, 64 or more bits

- $0 < e < e_{max}$: Normalized

significand (23, 52 bits or extended)

$$(-1)^s \; 1.f \; 2^{(e - bias)}$$

sign (1 bit)

exponent (8, 11 bits or extended)

- e = 0 : Denormalized $(-1)^s \; 0.f \; 2^{(-bias + 1)}$
  +0.0, -0.0
- $e = e_{max}$: +INF, -INF, NaNs

- 4 rounding modes (near,up,down,chop),5 types of fp_exception
  Monotony of rounding required

Accuracy requirement of IEEE-754

For `add`, `sub`, `mult`, `div`, `sqrt`, `rem` and `conv`: the floating-point result of an operation between floating-point numbers must be the rounding result of the exact operation over the reals

Ex : 999999995904_f `add` 10000_f yields to 999999995904_f
$= near(999999995904_f + 10000_f)$

- Poor (but well-done) approximation of the reals
  - finite set not uniformly distributed over the reals
  - associativity and distributivity are lost in general,…

- Absorption ($X_f + \varepsilon == X_f$), Cancellation ($X_f - Y_f == K.(X-Y)$ et $K>>1$)

Context of this work

- Programs strictly conform to IEEE-754

```
E  ::= E add E |E subs E |E mult E |E div E
    |E == E |E != E |E > E | E >= E
    |(float) E |(double) E | Var | Constants
```

- No extended-formats, only the **near-to-even** rounding mode, no exception, no NaNs
- Decomposition preserves the order of evaluation
- Temporary results are stored in known formats (requires to set up specific options when compiling)

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Simple Symbolic Execution [Clarke 76]

<u>Notations :</u> Control Flow Graph (N,A,e,s)

X vector of symbolic input

<u>Definition (*Symbolic State*) :</u>

$(Path, State, PC)$ where

$Path = n_i\text{-}..\text{-}n_j$ is a (partial) path of the CFG

$State = \{<v,\varphi>\}_{v \in Var(P)}$ φ is an algebraic expr. over X

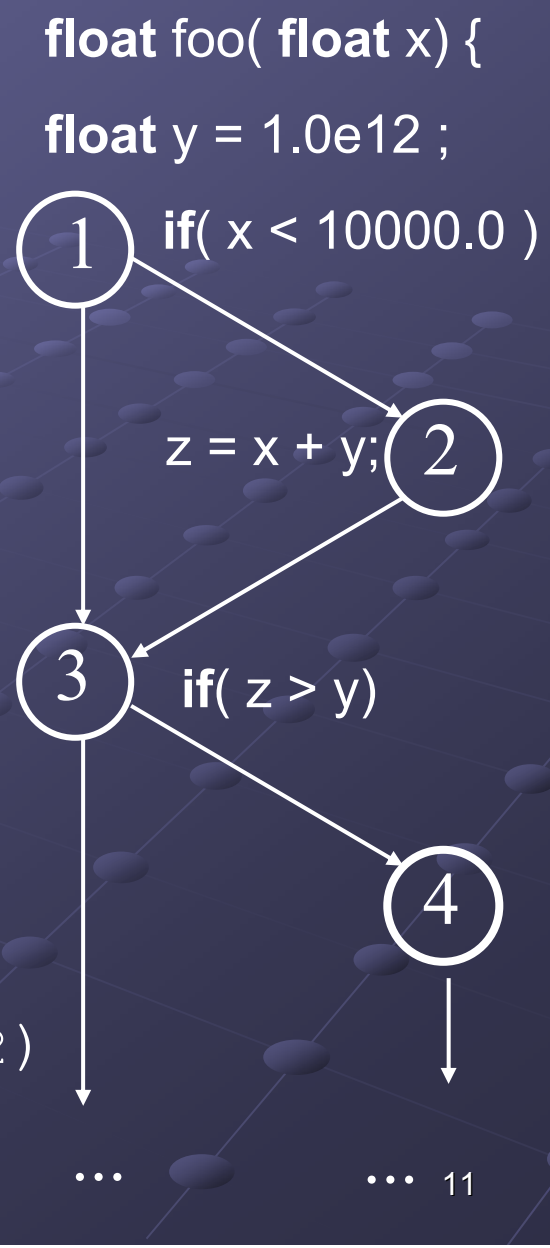
$PC = c_1,...,c_n$ a finite conjunction of conditions over X or a temporary assignments

(Path,State,PC) : examples

(1,{<x,X>,<y,1.0e12>,<z,⊥>},true)

(1-2-3,
{<x,X>,<y,1.0e12>,<z,X+1.0e12>},
X < 10000.0)

(1-2-3-4,
{<x,X>,<y,1.0e12>,<z,X+1.0e12>},
X<10000.0, T := X add 1.0e12, T > 1.0e12)



Symbolic state : features

- `(Path,State,PC)` is computed either by a forward or a backward analysis over the vertex of `Path`
- Let S_{PC} be the solution-set of `PC`
  then $\forall X \in S_{PC}$, `Path` is sensitized by `X`
- When $S_{PC}=\emptyset$ then `Path` is non-feasible
  undecideable in the general case [Weyuker 79]

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Non-linear constraint solving over R with interval propagation [Cleary 87,Davis 87]

- Real var x abstracted by an interval with FP bounds I_x
- Interval Arithmetic over the reals

$I_x = [a,b]$ and $I_y = [c,d]$ then

$I_{x+y} = [down(a+c), up(b+d)]$ $I_{x-y} = [down(a-d), up(b-c)]$

$I_{exp(x)} = [down(exp(a)), up(exp(b))]$ ...

- Relations over intervals / decomposition into proj. fcts

$[z = x + y]$ leads to

$$\begin{cases} I_{z'} \leftarrow I_{x+y} \cap I_z \\ I_{x'} \leftarrow I_{z-y} \cap I_x \\ I_{y'} \leftarrow I_{z-x} \cap I_y \end{cases}$$

⌘ Constraint propagation

Example : $y = \log(x)$, $x+y = 0$

4 projection functions

$$\begin{cases} I_{x'} \leftarrow I_{\exp(y)} \cap I_x & ① \\ I_{y'} \leftarrow I_{\log(x)} \cap I_y & ② \\ I_{x'} \leftarrow I_{-y} \cap I_x & ③ \\ I_{y'} \leftarrow I_{-x} \cap I_y & ④ \end{cases}$$

$X \in$	$[-\infty,+\infty]$	$[0,+\infty]$	$[0, 1]$	$[0.56, 1]$	$[0.56, 0.57]$
$Y \in$	$[-\infty,+\infty]$	$[-\infty, 0]$	$[-1, 0]$	$[-1, -0.56]$	$[-0.57, -0.56]$

If there is a solution x, then $x \in [0.56, 0.57]$

Conservative over-estimation of the solution-set –
Formally speaking, *2-b(w)-consistency* is achieved

A few existing solvers based on IP

- BNR-Prolog [Older &Vellino 93]
- Interlog [Botella & Taillibert 93]

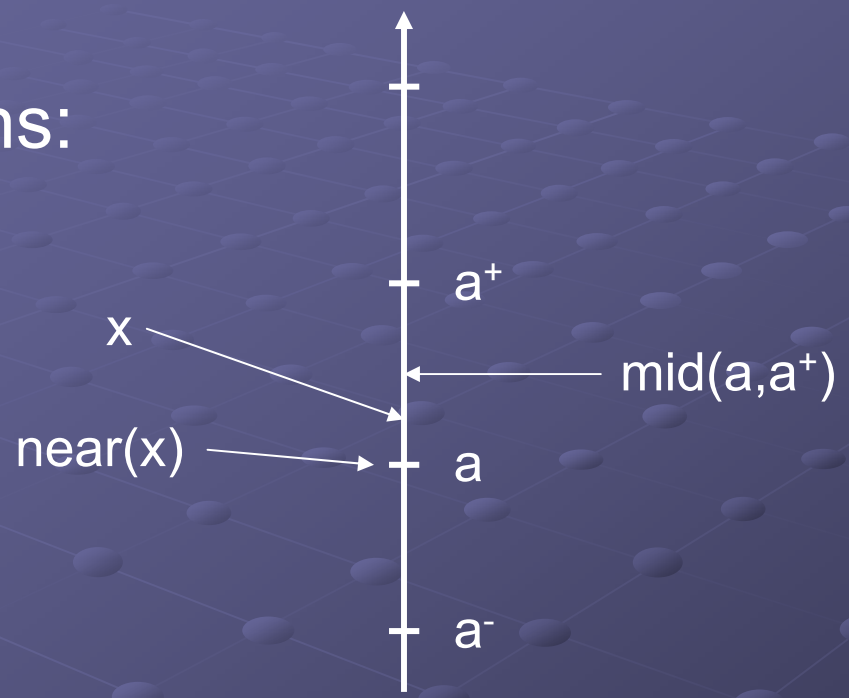
  K_b_consistencies [Lhomme 93]
  Dynamic optimizations [Lhomme Gotlieb Rueher 96,98]

- Newton [Benhamou et al.94]
- Ilog Solver [Puget 94]
- Prolog IV [Colmerauer 94]
- Numerica [Van Hentenryck 97]

...

Our approach to solve path conditions : Interval propagation over floating-point variables

- Notations:



Recall that [a `add` b] denotes [near(a + b)]

- Path conditions are made of constraints and assignments

Our approach: floating-point projections

⌘Direct and indirect projections for the assignment:

[r := a `add` b] leads to
- proj(r, r:= a `add` b) (direct)
- proj(a, r:= a `add` b) (1st indirect)
- proj(b,r := a `add` b) (2nd indirect)

⌘ **Direct projections** (over numeric fp numbers):

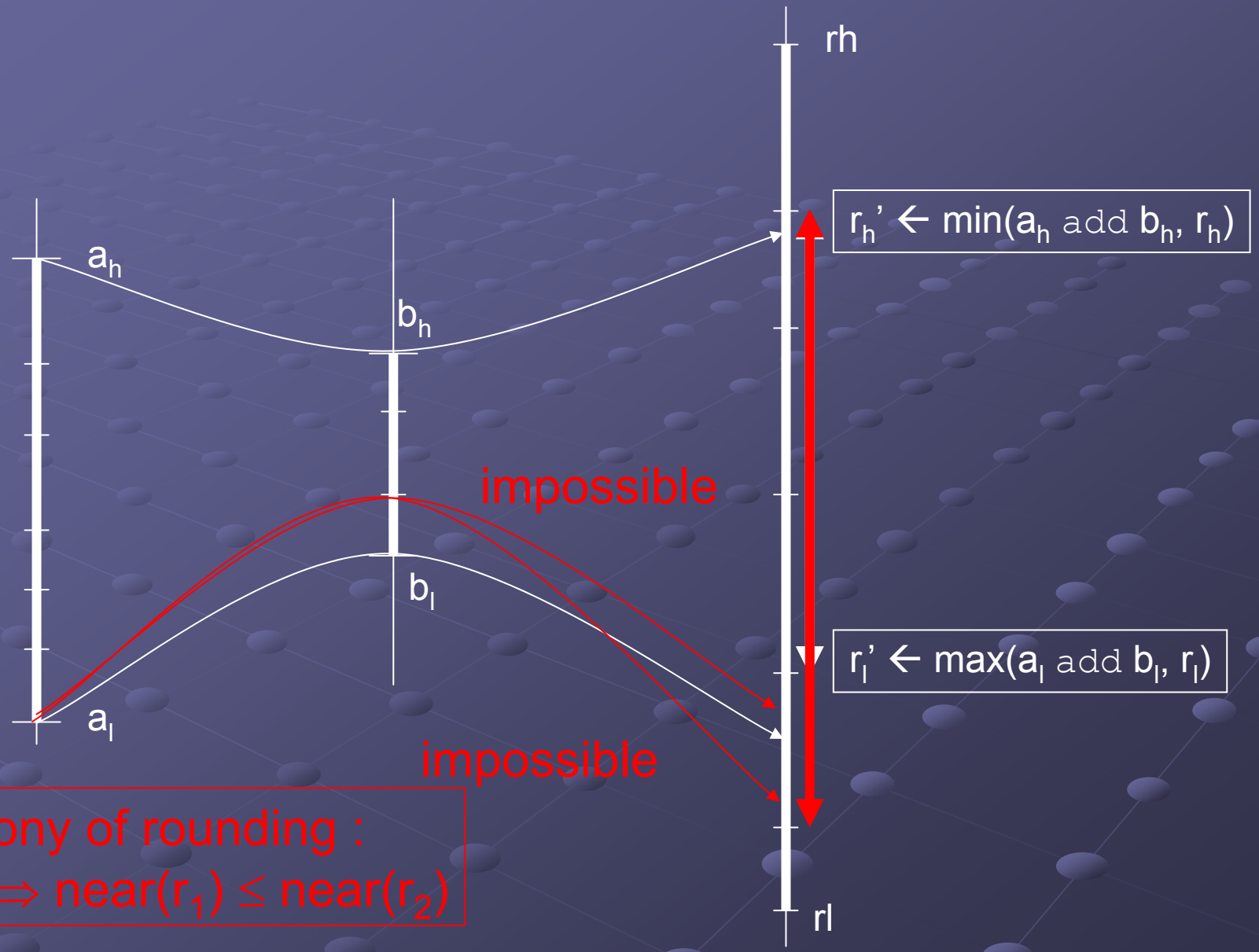
If $I_r = [r_l,r_h]$, $I_a = [a_l,a_h]$ and $I_b = [b_l,b_h]$ then

[r := a `add` b] $[r_l',r_h'] \leftarrow [a_l \texttt{ add } b_l, a_h \texttt{ add } b_h] \cap [r_l,r_h]$

[r := a `subs` b] $[r_l',r_h'] \leftarrow [a_l \texttt{ subs } b_h, a_h \texttt{ subs } b_l] \cap [r_l,r_h]$

...

Ex: Direct projection `[r := a add b]`



Monotony of rounding :
$r_1 \leq r_2 \Rightarrow near(r_1) \leq near(r_2)$

More complex : indirect projections

If $I_r = [r_l,r_h]$, $I_a = [a_l,a_h]$ and $I_b = [b_l,b_h]$ then

<u>1st indirect projection of `[r := a add b]`</u>

$[a_l',a_h'] \leftarrow [\mathrm{mid}(r_l,r_l^-)\ \mathtt{subs}\ b_h, \mathrm{mid}(r_h,r_h^+)\ \mathtt{subs}\ b_l] \cap [a_l,a_h]$

<u>1st indirect projection of `[r := a subs b]`</u>

$[a_l',a_h'] \leftarrow [\mathrm{mid}(r_l,r_l^-)\ \mathtt{add}\ bl, \mathrm{mid}(r_h,r_h^+)\ \mathtt{add}\ b_h] \cap [a_l,a_h]$

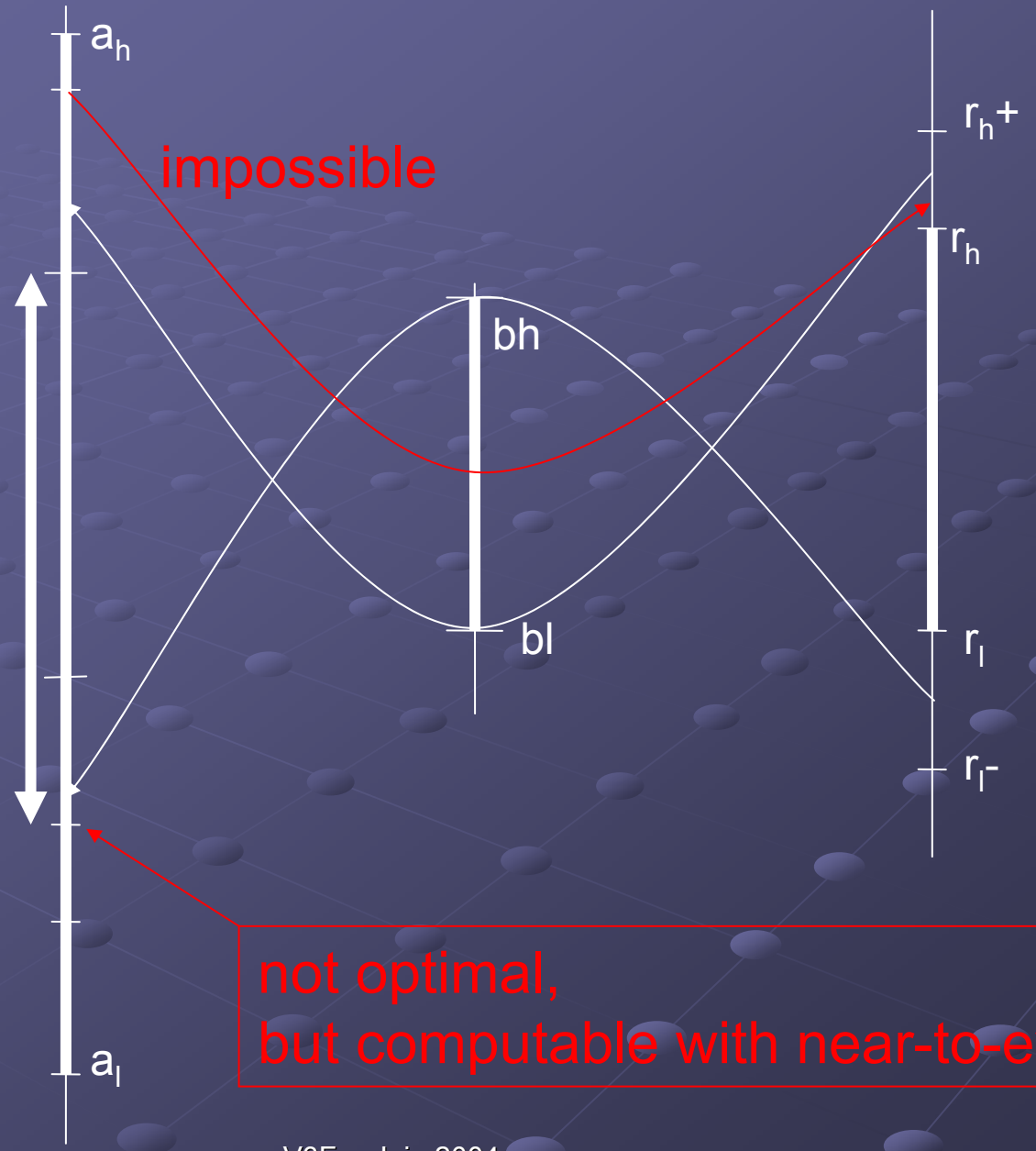
<u>2nd indirect projection of `[r := a subs b]`</u>

$[b_l',b_h'] \leftarrow [a_l\ \mathtt{subs}\ \mathrm{mid}(r_h,r_h^+), a_h\ \mathtt{subs}\ \mathrm{mid}(r_l,r_l^-)] \cap [b_l,b_h]$

Ex: 1st indirect projection `[r := a add b]`

$a_h' \leftarrow \min(\mathrm{mid}(r_h, r_h^+)$ `subs` $b_l, a_h)$

$a_l' \leftarrow \max(\mathrm{mid}(r_l, r_l^-)$ `subs` $b_h, a_l)$



not optimal,
but computable with near-to-even

Handling comparisons and conversions

<u>Comparisons (1st proj) :</u>

$[a_l', a_h'] \leftarrow [\max(a_l, b_l), \min(a_h, b_h)]$ when `[a==b]`

$[a_l', a_h'] \leftarrow [\max(a_l, b_l)^+, a_h]$ when `[a > b]`

$[a_l', a_h'] \leftarrow [\text{if}(a_l = b_l = b_h) \text{ then } a_l^+ \text{ else } a_l,$
$\text{if}(a_h = b_l = b_h) \text{ then } a_h^- \text{ else } a_h]$ when `[a!=b]`

<u>Floating-point conversions:</u>

when `[ r := (float)a ]`

$[r_l', r_h'] \leftarrow [\max_f((\text{float})a_l, r_l), \min_f((\text{float})a_h, r_h)]$ (direct proj.)

$[a_l', a_h'] \leftarrow [\max_d(al, \text{mid}(r_l, r_l^-)), \min_d(a_h, \text{mid}(r_h, r_h^+))]$ (indirect)

Handling zeros and infinities

Based on an extended arithmetic defined by specific tables:

values of `a` in 1st indirect projection of `[r := a add b]`

$_b \backslash^r$	-INF	-0.0	+0.0	Nv	+INF
-INF	Nv, -INF,±0.0	--	--	--	--
-0.0	-INF	-0.0	+0.0	Nv	+INF
+0.0	-INF	--	±0.0	Nv	+INF
Nv	Nv,-INF	--	Nv,±0.0	Nv,±0.0	Nv,+INF
+INF	--	--	--	--	Nv,+INF,± 0.0

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FPSE: Floating-Point Symbolic Execution

- Implemented as part as the INKA project (by Thales A.S.)
  Bernard : full design and implementation
  Me : a few improvments on fp projection functions

- Handles C computations on Sparc and Intel

- Written in Prolog (constraint propagation engine)
  and C (floating-point projection functions)

- Integrated with a collaborative integer constraint solver
  (SICStus clpfd library)

A straightforward collaboration principle

- Synchronous trigger mechanism between FPSE and clpfd
- Communicates through alarms (fd_global mechanism) when int-to-float conversion constraints are encountered

An example (extracted from www.astree.ens.fr/)

```
/* double-error.c */
int main () {
double x;
float y,z,r;
x=1125899973951488.0;
y = x + 1;
z = x - 1;
r = y - z;
printf("%f\n", r);
}
%134217728.000000
```

```
test24 :-
 solveur:init_env(E),
 flottant:news([Y,Z,R],float(32),['y','z','r'],E),
 flottant:news([X,C,T1,T2],double(64),['x','c','t1','t2'],E),

 flottant:affect(in(1125899973951488.0, 1125899973951488.0),X),
 flottant:affect(in(1.0, 1.0),C),

 flottant:affect('+',X,C,T1),
 flottant:affect(conv(double(64),float(32)),T1,Y),
 flottant:affect('-',X,C,T2),
 flottant:affect(conv(double(64),float(32)),T2,Z),

 flottant:affect('-',Y,Z,R),
 solveur:solve(E),

 flottant:fprint([R]).
```

Very, very first experimental results (extracts)

Programs	Path conditions	in	clpr	clpq	FPSE
[Goldberg 91] ((2.e-30+1.e30)-1.e30) -1.e-30	T_2:=2.e-30 add 1.e30, T_1:=T_2 subs1.e30, T_3:=T_1subs1.e-30, X==T_3	X	0.0	1/9999..	-1.00000000 317107e-30
[Goldberg 91] D := B^2 - 4AC A:=1.22,B:=3.34C:=2.28	T_1:=BmultB,T_2:=AmultC T_3:=4. mult T_2 D:=T_1 subs T_3	D	0.029200..	73/2500 =0.0292	0.029199600 219726562
[Goldberg 91] D := B^2 - 4AC A:=1.22,B:=3.34,D:=+0.	T_1:=BmultB,T_2:=AmultC T_3:=4. mult T_2 D:=T_1 subs T_3	C	2.285983.7	27889/ 12200 =2.285...	Interval of 8 single floats + 2.285982..4
X < 10000., T_1:= X add 1.e12, T_1 > 1.e12		X	(-0.,10000)	(0,10000)	infeasible
X > 0.0, T_1:=X add 1.e12, T_1==1.e12		X	infeasible	infeasible	[1.4012..e45, 32768.0
[Howden 82] T_1 := A mult B, X_1 := T_1 add 2., X_1 > 100., X_2 := 100. subs X_1, X_3 := X_2 subs 50., X_3 > 50.		A B	infeasible	infeasible	infeasible

Programs intended to run on Pentium 4 over the singles, compiled by Visual C++ with /Op

Problems (1)

- FPSE must be runned on the same machine than the tested program to get the same rounding algorithms

- NaNs (for example INF – INF) are currently not handled Prolog's fail as result

- **In practice**, it is required to check a given result as:
  lack of documentation for the compiler design,
  hardware optimizations (such as the fused mult-add a*b+c),
  possible change of rounding modes, ..

Problems (2)

Weaknesses of fp_constraints to prune the domains

example adapted from [KPV TACAS'03] :

```
int f(float x,float y)
if(x > y) {
  x = x+y;
  y = x-y;
  x = x-y; }
if(x - y > 0)
      ...
```

```
test12(T) :-
        solveur:init_env(E),
        flottant:news([X,Y,X1,X2,Y1,T1,C1],T,_,E),
        entier:new(R,int(32),'r',E),
        flottant:affect('>',X,Y,R),
        flottant:affect('+',X,Y,X1),
        flottant:affect('-',X1,Y,Y1),
        flottant:affect('-',X1,Y1,X2),
        flottant:affect('-',X2,Y1,T1),
        flottant:affect(const('0.0'),C1),
        flottant:affect('>',T1,C1,R),  R = 1,
        flottant:affect(in(+1.0E+30, +1.001E+30),X),
        flottant:affect(in(+1.0E+30, +1.001E+30),Y),
        solveur:solve(E),
        flottant:fprint([X,Y]).

The results provided by FPSE~:
float(32):x in 1.00000009060533E+30..1.001E+30
float(32):y in 1.0E+30..1.000999947816015E+30
```

Contributions and further work

Contribution: New results to solve accurately path conditions over floating-point computations

- Symbolic values
- Comparisons and conversions

Implementation is in progress: FPSE

Application: Structural test data generation

Extensions: NaNs

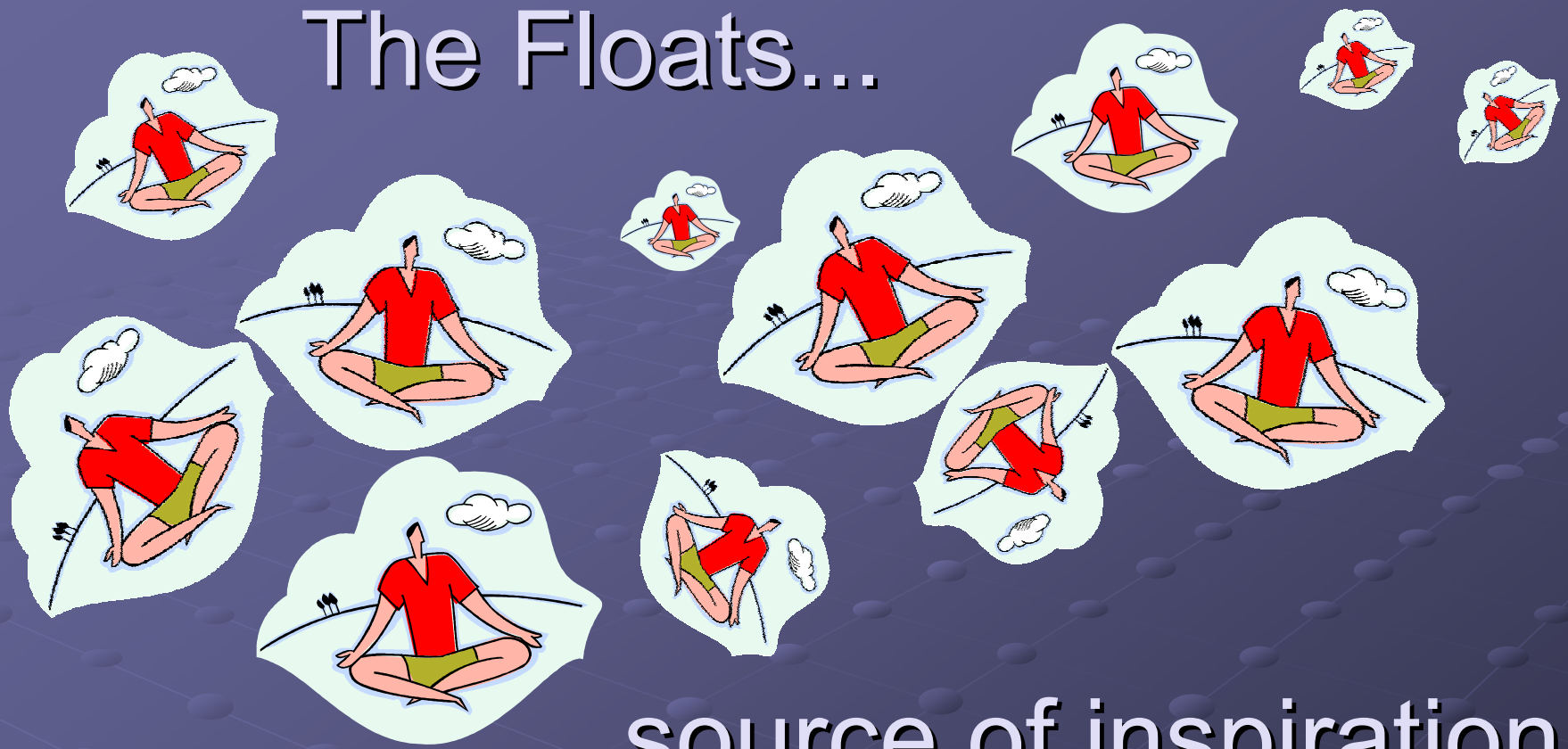
Transcendental functions (`sqrt,sin,exp, ...`)

Problems: Improving the pruning capacity of projection functions

Labelling process (heuristics)

Experimental evaluation

The Floats...



...source of inspiration ?

1. B. Botella, A. Gotlieb, C. Michel
   «Correct handling of floating-point computations in symbolic execution »

   RR-INRIA 5150, mar. 2004, 24pp

2. B. Botella, A. Gotlieb

   «Documentation of a floating-point interval constraint solver » -- V3F Internal report