Classification among Hidden Markov Models

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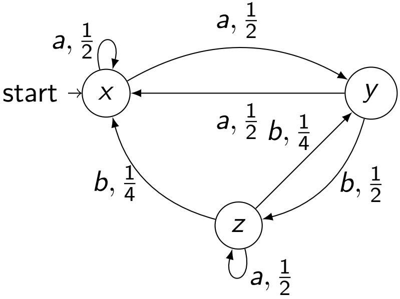
Summary

Introduction of the problem (1)

Framework

- Stochastic systems,
- Partial information.

$\Rightarrow$ Hidden Markov Models:



Introduction of the problem (2)

Classification

Given two systems A_1, A_2 and an observation w, decide which one produced it.

Encompasses diagnosis, opacity...

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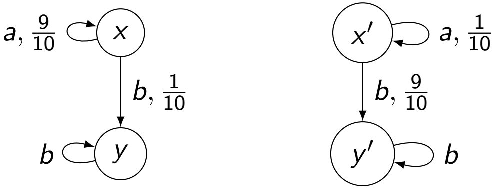
Can we classify...

- For sure?
- Almost sure?
- Limit sure?
- We cannot?



What is a classifier?

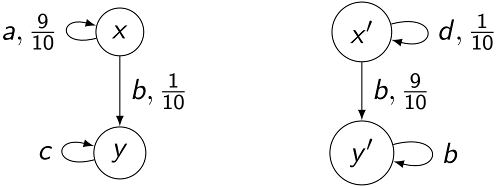
Function $f : \Sigma^* \to \{\perp, 1, 2\}$

What is a good classifier?

- Accurate?
- Reactive?
- No error?

Sure classification

Informally: ability to distinguish after some time.
Formally: $\forall w \in \Sigma^\infty, \exists v, w = vv', v \in L_1, v \notin L_2$.

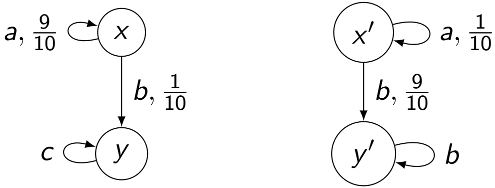


Theorem

Sure classification is decidable in PTIME, by deciding if $L_1^\infty \cap L_2^\infty = \emptyset$.

Almost sure classification

Informally: ability to distinguish after some time with probability 1.
Formally: $P(w \in \Sigma^\infty,\ \exists v, w = vv',\ v \in L_1,\ v \notin L_2) = 1$.

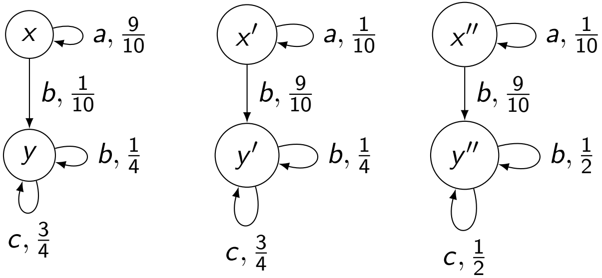


Theorem

Almost sure classification is PSPACE-complete, by deciding if $P(L_1^\infty \cap L_2^\infty) = 0$.

Limit sure classifiability

Informally: classify with arbitrarily high precision.
Formally: there is a classifier f that eventually answers correctly with probability $> 1 - \varepsilon$ for all $\varepsilon > 0$.



Main result on limit sure classifiability

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Limit sure classifiability is decidable in PTIME.

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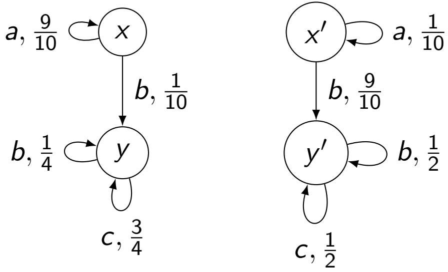
We want arbitrarily high precision:

- Transient components "do not matter",
- We mainly study BSCCs.



Study of BSCC

Two BSCCs are problematic if they:

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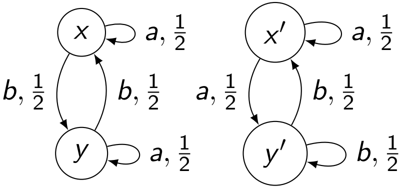
How to check this? State by state?



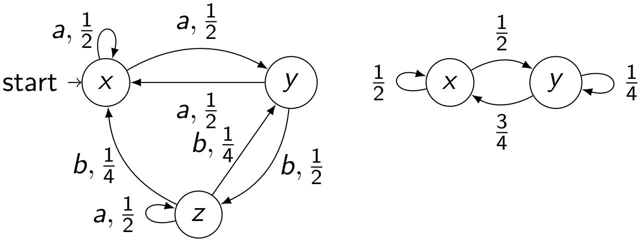
- $L_x \not\equiv L_{x'}$
- $L_x \not\equiv L_{y'}$
- ...

But $L_{\frac{1}{2}x+\frac{1}{2}y} \equiv L_{\frac{1}{2}x'+\frac{1}{2}y'}$.

How to be smart? (1)

Number of possible distributions: infinite!

Consider stationnary distributions σ_X on beliefs X.



For a belief X, σ_X is computable in PTIME.

Interest of stationary distributions

Theorem

The following are equivalent:

1. One cannot classify between A_1, A_2,
2. There exists an X in a BSCC of twin beliefs such that $(A_1, \sigma_X^1) \equiv (A_2, \sigma_X^2)$.

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One problem solved, but... still an exponential number of beliefs!

How to be smart? (2)

Have only a limited number of beliefs?
$A = A_1 \times A_2$, for a BSCC D_i of A and $(y_1, y_2) \in D_i$,

- $X_1 = \{x_1 \mid (x_1, y_2) \in D_i\}$,
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Theorem

NSC: check equivalence for such X_1, X_2.

- Polynomial number of such beliefs,
- Each check with Linear Programming: PTIME!

With tries?

User has a reset button:

- Can try again and again,
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Attack-classifiability

Decide if there exists a reset strategy such that:

1. It will finish with probability 1,
2. It is limit sure classifiable after the last reset.

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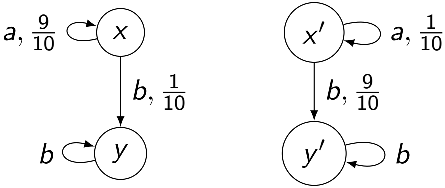
$1-\varepsilon$ attacker-classifiability

With ε fixed, decide if there exists a reset strategy such that:

1. It will finish with probability 1,
2. Classification will be correct with probability $1-\varepsilon$ after last reset.

An example



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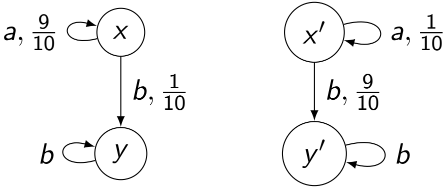

- Not attack classifiable,
- $\forall \varepsilon$, $1 - \varepsilon$ attack classifiable.

Results on these variants

Theorem

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Idea of proofs:

Attack-classifiability:

- Find subpart of the systems that are classifiable,
- Hardness: reduction from language inclusion for finite automata.

$1 - \varepsilon$ attacker-classifiability:

- Reduction from 0 and 1 isolation problem for PFA.

Summary

Classifiabilities

1. Sure: a word in only one language.
2. Almost Sure: a word in only one language with probability 1.
3. Limit Sure: probability of error decreases to 0.
4. Attack: Limit Sure with tries.
5. $1-\varepsilon$ Attack: decide with a fixed threshold of error with tries.

Class	Sure	Almost Sure	Limit Sure	Attack	$1-\varepsilon$ attack
Cplxt	PTIME	PSPACE	PTIME	PSPACE	undecidable

Strong links with:

- Distance 1 problem: determine if

$$sup_{W \in \Sigma^{\infty}} |P_1(W) - P_2(W)| = 1$$

- AFF-diagnosability and ε-diagnosability.

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Questions time!