Model of a railway network

Makushita

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A Railway Network

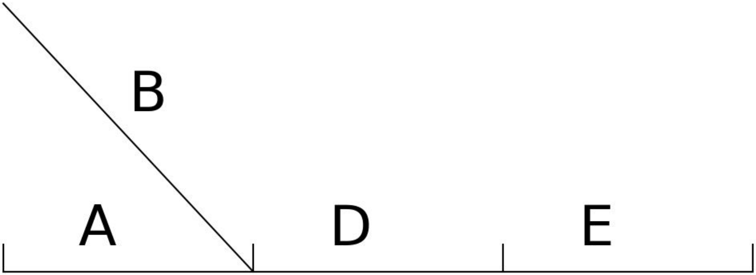


Figure: Junction of two tracks

A Railway Network

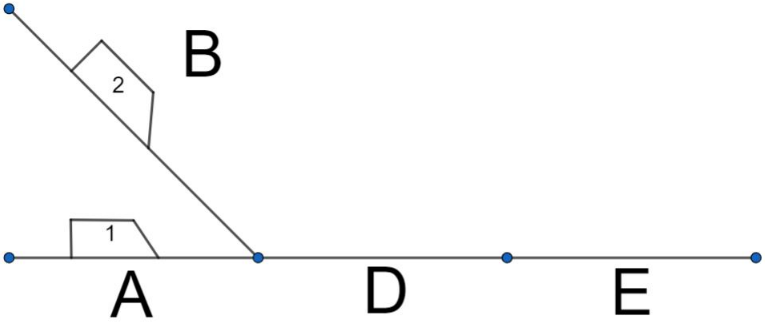


Figure: A Junction of two Tracks

A Railway Network

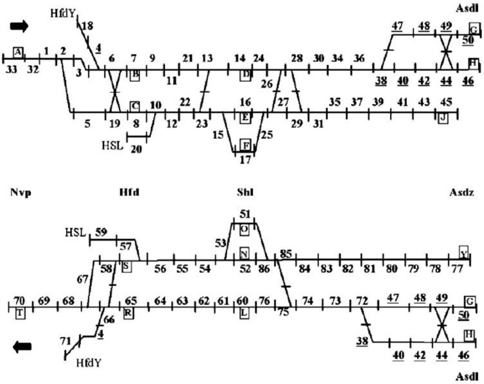


Figure: The Schiphol railway network, fig. by Andrea D'Ariano

Constraints

- No Overtaking
- Reordering
- Vision of the Choices and their Impacts

Petri Nets

$\mathcal{N} = (P, T, \rightarrow)$

P Places

T Transitions

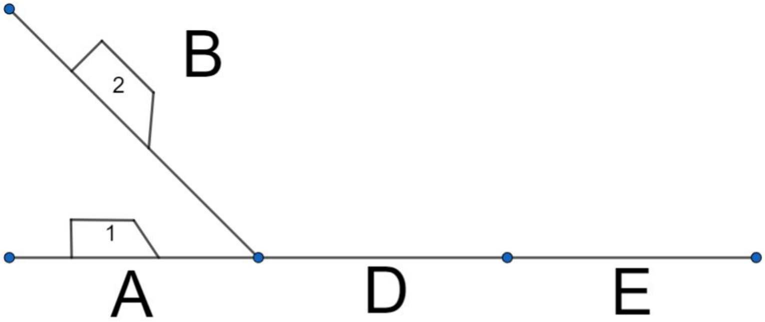
$\rightarrow \subseteq (P \times T) \cup (T \times P)$ Arcs

Petri Nets

Marking $M \subseteq P$

$t \in T$ is enabled iff ${}^\bullet t \subseteq M$
Firing of $t \in T$: $M' = (M \setminus {}^\bullet t) \cup t^\bullet$

Mutual Exclusion



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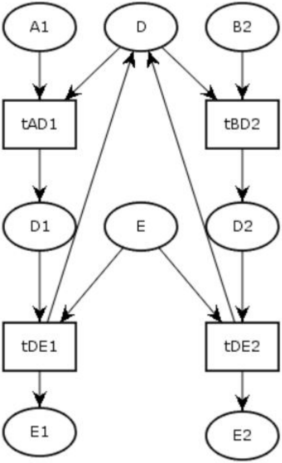
Mutual Exclusion



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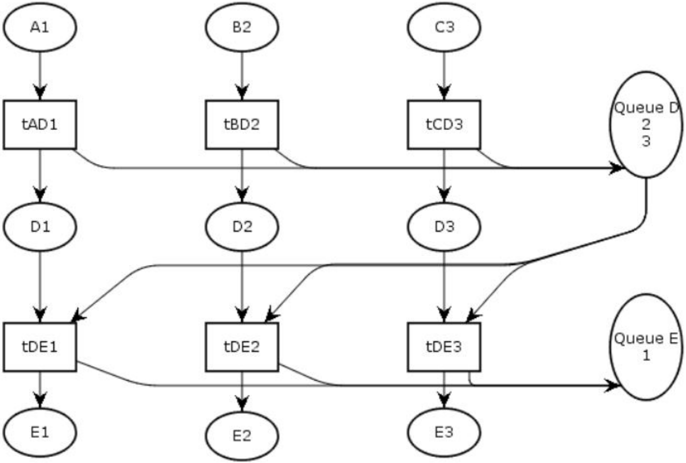
The Desired Semantics



The Desired Semantics

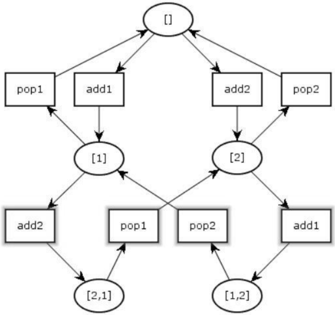
A transition t is enabled iff :

$$\begin{cases} {}^{\bullet}t \subseteq M \\ \forall\, bi \in {}^{\bullet}t,\ i \text{ is the first of Queue b} \end{cases}$$

Firing a transition t implies :

$$\begin{cases} M' = (M \setminus {}^{\bullet}t) \cup t^{\bullet} \\ \forall\, bi \in {}^{\bullet}t,\ i \text{ is removed from Queue b} \\ \forall\, bi \in t^{\bullet},\ i \text{ is added to the Queue b} \end{cases}$$

Naive Implementation

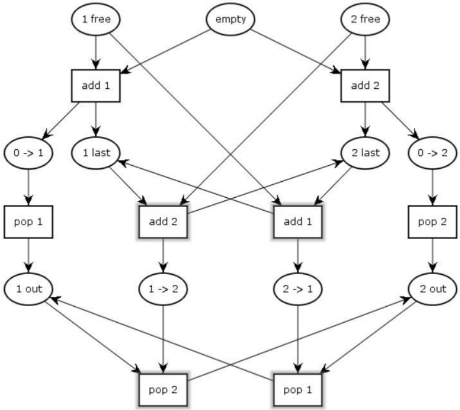


$$\sum_{k=0}^{n} \binom{n}{k} k! \sim e\, n! \text{ places}$$

Quadratic Implementation

$$Q = [q_1, q_2, q_3, \ldots, q_n]$$
$$(\perp, q_1),\ (q_1, q_2),\ (q_2, q_3),\ \ldots(q_{n-1}, q_n)$$

Better Queue with a Petri net



Comparison

n	first model	second model
2	13	20
3	46	38
4	193	62
5	976	92
6	5 869	128
10	29 592 301	332

Table: Number of Nodes

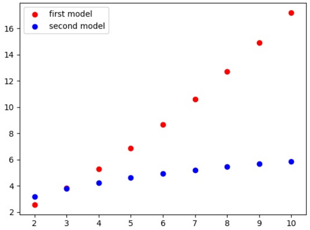
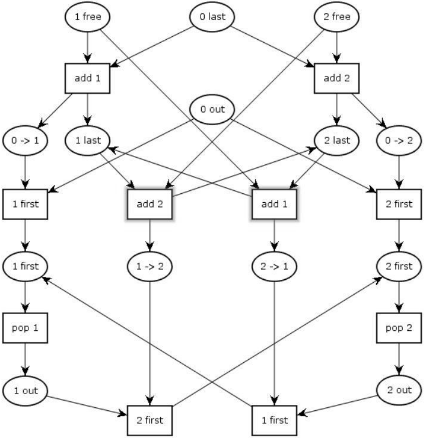


Figure: Log of the Number of Nodes

Final Implementation



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The Transitions

$$\begin{cases} b\ i \to j \\ b\ i\ out \\ b'\ k\ last \end{cases} \to \begin{cases} b'\ k \to j \\ b\ j\ out \\ b'\ j\ last \end{cases} \qquad O(n^3)$$

becomes

$$\begin{cases} b\ i \to j \\ b\ i\ out \end{cases} \to \begin{cases} b\ j\ first \end{cases} \qquad O(n^2)$$

and

$$\begin{cases} b\ j\ first \\ b'\ k\ last \end{cases} \to \begin{cases} b\ j\ out \\ b'\ k \to j \\ b'\ j\ last \end{cases} \qquad O(n^2)$$

Junction of 2 Trains

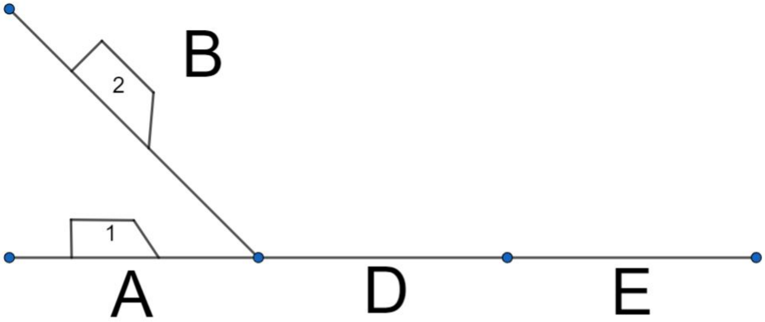

Figure: A Junction of two Tracks

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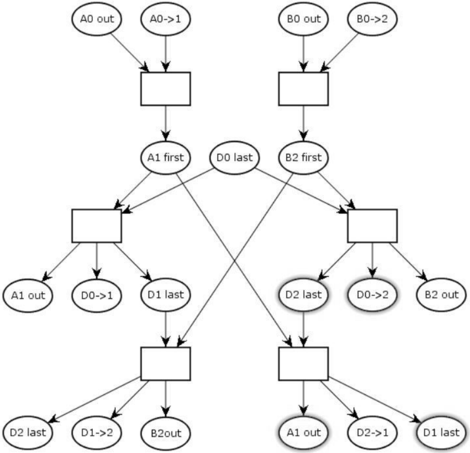
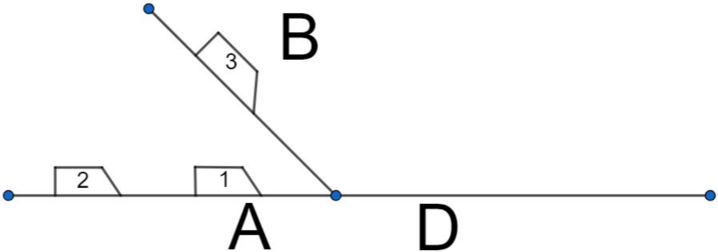
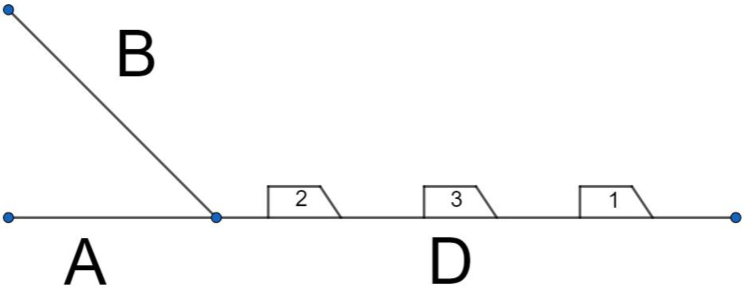


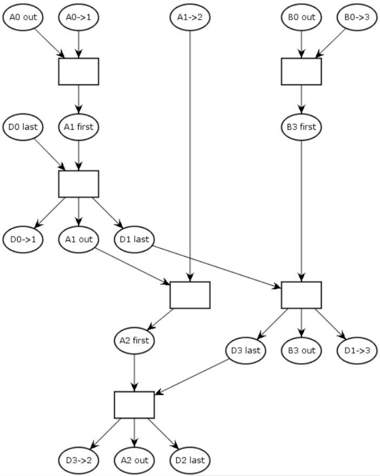
Junction of 3 Trains



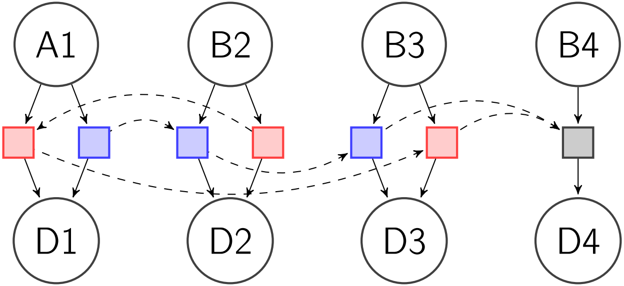
Junction of 3 Trains



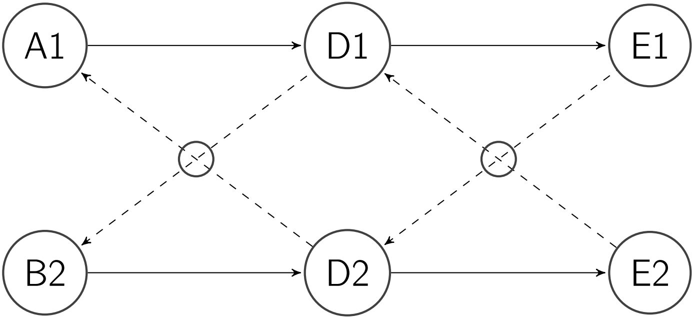
Junction of 3 Trains



Merged Processes



Alternative Graphs



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Definition

Disjoint union of places :
$P = P_1 \sqcup P_2$ and $P_0 = P_{1,0} \sqcup P_{2,0}$

Synchronization of transitions with the same label.

Product of Nets

$\mathcal{T} \times \mathcal{Q} \times \widetilde{\mathcal{O}}$

- $\mathcal{T}$ Trajectories
- $\mathcal{Q}$ Queues
- $\widetilde{\mathcal{O}}$ Extented Order
  $\Sigma_k = \{\sigma \in \mathcal{S}_n \,|\, \forall i,\ |\sigma(i) - i| \leq k\}$

Example of $\mathcal{T}$

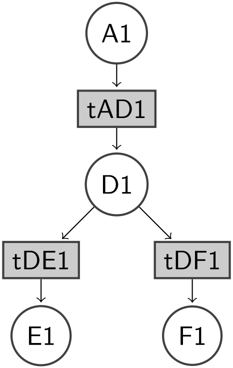
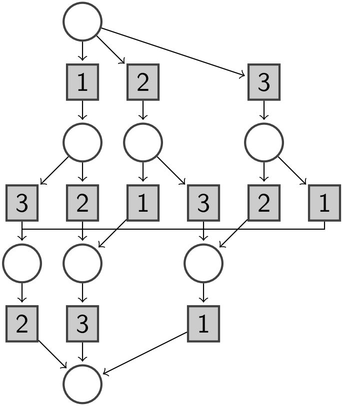


Figure: Trajectories of the train 1

Example of $\widetilde{\mathcal{O}}$



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Junction of 2 Trains

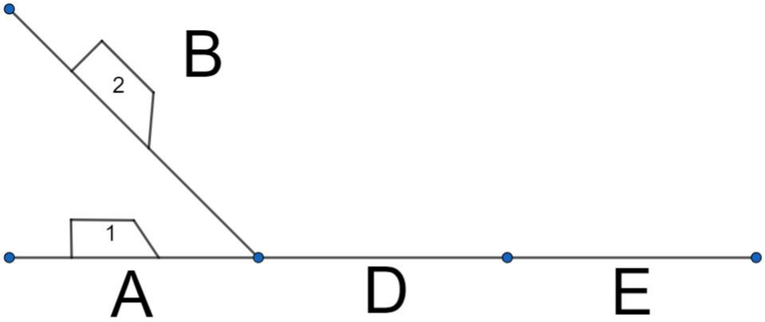
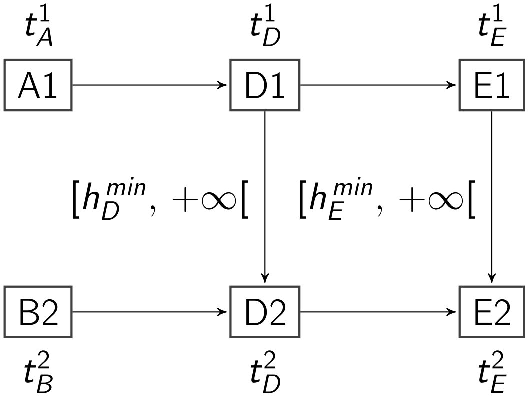


Figure: A Junction of two Tracks

Temporal Constraints Graph



Simple Temporal Problem

Constraints of the from :

$a_i \leq X_i \leq b_i$

$a_{i,j} \leq X_j - X_i \leq b_{i,j}$

Intervals of $(X_i)_{i \in I}$ obtainable in PTIME

Function to Optimize

- stick to nominal transit times

$$\Sigma \left|(t_Y^n - t_X^n) - d_X^{nom}\right|^2$$

- equalize headways

$$\Sigma \left|(t_X^{n+1} - t_X^n) - (t_X^n - t_X^{n-1})\right|^2$$

Convex solution set + Quadratic (convex) criterion

Algorithm

For each possible configuration :

- Compute the temporal constraints graph
- Solve the STP
- Optimize the criterion

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Future Work

- Share Computations between Configurations
- Limit the Configurations to study
- Different Model of a Junction
- Your Ideas ?

Thank you

Thank You for Listening !