Model of a railway network

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Makushita Model of a railway network

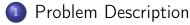
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A Railway Network

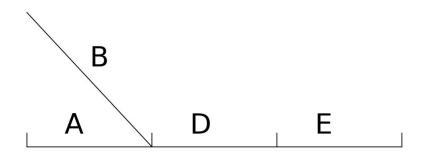


Figure: Junction of two tracks

Makushita Model of a railway network

A Railway Network

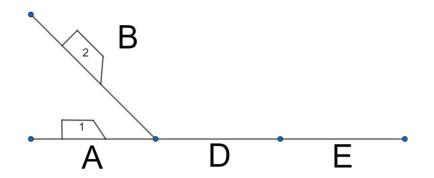


Figure: A Junction of two Tracks

Makushita Model of a railway network

Problem Description Implementation of a Queue Model of a Junction

Model of a Junction Product of Nets Temporal Constraints Conclusion

A Railway Network





Figure: The Schiphol railway network, fig. by Andrea D'Ariano

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- No Overtaking
- Reordering
- Vision of the Choices and their Impacts

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Petri Nets

$$\mathcal{N} = (P, T, \rightarrow)$$

$$P \text{ Places}$$

$$T \text{ Transitions}$$

$$\rightarrow \subseteq (P \times T) \cup (T \times P) \text{ Arcs}$$

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Marking $M \subseteq P$

$t \in T$ is enabled iff $\bullet t \subseteq M$ Firing of $t \in T$: $M' = (M \setminus \bullet t) \cup t^{\bullet}$

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Mutual Exclusion

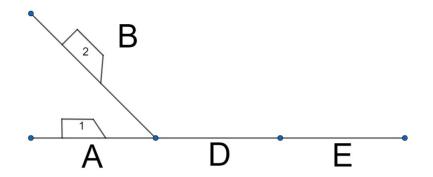
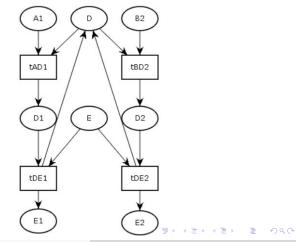


Figure: A Junction of two Tracks

Makushita Model of a railway network

Mutual Exclusion



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Model of a railway network

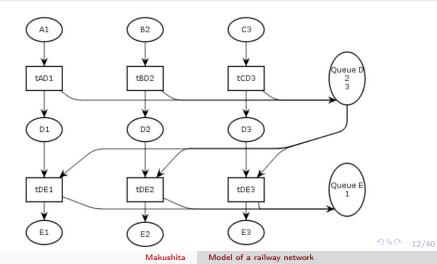
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The Desired Semantics



The Desired Semantics

A transition t is enabled iff :

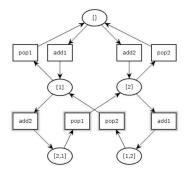
$$\begin{cases} {}^{\bullet}t \subseteq M \\ \forall \ bi \in {}^{\bullet}t, \ i \ \text{is the first of Queue b} \end{cases}$$

Firing a transition *t* implies :

$$\begin{cases} M' = (M \setminus {}^{\bullet}t) \cup t^{\bullet} \\ \forall \ bi \in {}^{\bullet}t, \ i \ \text{is removed from Queue b} \\ \forall \ bi \in t^{\bullet}, \ i \ \text{is added to the Queue b} \end{cases}$$

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Naive Implementation



 $\sum_{k=0}^{n} \binom{n}{k} k! \sim e n!$ places

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Model of a railway network

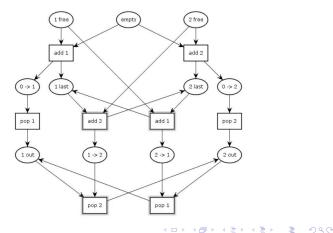
Quadratic Implementation

$$egin{aligned} Q &= [q_1, q_2, q_3, \, ..., q_n] \ (ota, q_1), \, (q_1, q_2), \, (q_2, q_3), \, ...(q_{n-1}, q_n) \end{aligned}$$

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Better Queue with a Petri net



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Comparison

n	first model	second model
2	13	20
3	46	38
4	193	62
5	976	92
6	5 869	128
10	29 592 301	332

Table: Number of Nodes

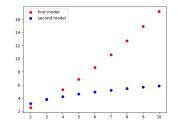


Figure: Log of the Number of Nodes

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Final Implementation

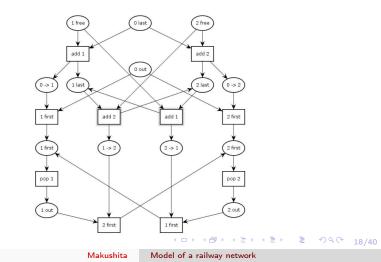


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The Transitions

and

$$\begin{cases} b \ i \to j \\ b \ i \ out \\ b' \ k \ last \end{cases} \rightarrow \begin{cases} b' \ k \to j \\ b \ j \ out \\ b' \ j \ last \end{cases} O(n^3)$$
becomes
$$\begin{cases} b \ i \to j \\ b \ i \ out \end{cases} \rightarrow \begin{cases} b \ j \ first \\ b' \ k \ last \end{cases} \rightarrow \begin{cases} b \ j \ first \\ b' \ k \ hast \end{cases} O(n^2)$$
and
$$\begin{cases} b \ j \ first \\ b' \ k \ last \end{cases} \rightarrow \begin{cases} b \ j \ out \\ b' \ k \to j \\ b' \ last \end{cases} O(n^2)$$

$$b' \ k \to j \ O(n^2)$$

$$b' \ j \ last \longrightarrow \mathbb{R} \rightarrow \mathbb{R}$$
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Junction of 2 Trains

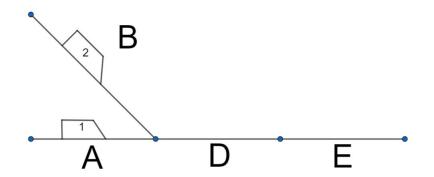
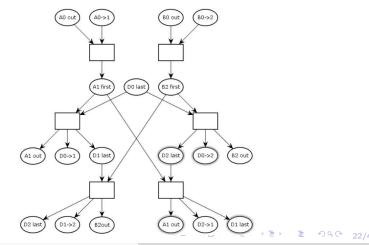


Figure: A Junction of two Tracks

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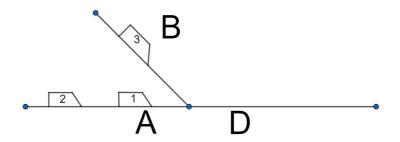
Junction of 2 Trains



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Model of a railway network

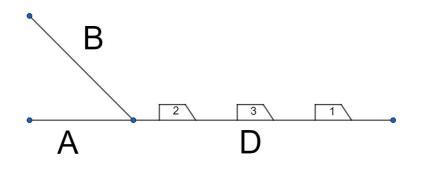
Junction of 3 Trains



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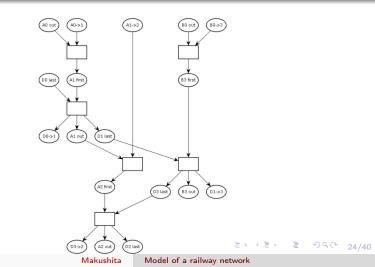
Junction of 3 Trains



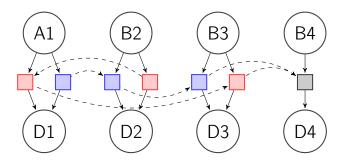
Makushita Model of a railway network

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Junction of 3 Trains

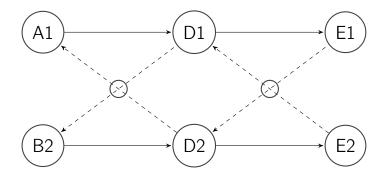


Merged Processes



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Alternative Graphs



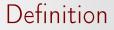
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Disjoint union of places : $P = P_1 \sqcup P_2$ and $P_0 = P_{1,0} \sqcup P_{2,0}$

Synchronization of transitions with the same label.

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Product of Nets

$\mathcal{T}\times\mathcal{Q}\times\widetilde{\mathcal{O}}$

- \mathcal{T} Trajectories
- \mathcal{Q} Queues
- $\widetilde{\mathcal{O}}$ Extented Order
 - $\Sigma_k = \{ \sigma \in \mathcal{S}_n \, | \, \forall i, \, |\sigma(i) i| \le k \}$

Example of ${\mathcal T}$

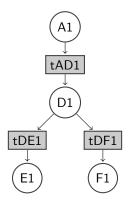
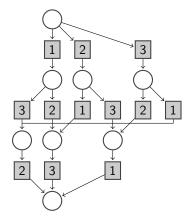


Figure: Trajectories of the train 1

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Example of $\widetilde{\mathcal{O}}$



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Junction of 2 Trains

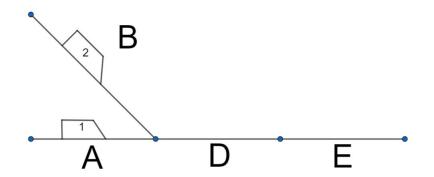
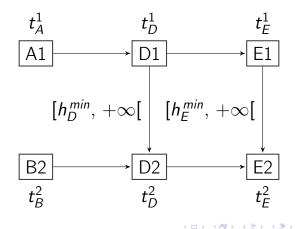


Figure: A Junction of two Tracks

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Temporal Constraints Graph



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Simple Temporal Problem

Constraints of the from : $a_i \leq X_i \leq b_i$ $a_{i,j} \leq X_j - X_i \leq b_{i,j}$

Intervals of $(X_i)_{i \in I}$ obtainable in PTIME

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Function to Optimize

• stick to nominal transit times

$$\Sigma |(t_Y^n - t_X^n) - d_X^{nom}|^2$$

• equalize headways

$$\Sigma |(t_X^{n+1} - t_X^n) - (t_X^n - t_X^{n-1})|^2$$

Convex solution set + Quadratic (convex) criterion



For each possible configuration :

- Compute the temporal constraints graph
- Solve the STP
- Optimize the criterion

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Conclusion

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- Share Computations between Configurations
- Limit the Configurations to study
- Different Model of a Junction
- Your Ideas ?



Thank You for Listening !

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