Modeling knowledge, dynamics and probabilities

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Multi-agent systems
Definition

Agent
Entity capable of reasoning about its environment and about the other agents (humans, robots, *etc.*).

Multi-agent system
A set of agents that interact with each other and are situated in a common environment.

Multi-agent systems

High-order knowledge

Knowledge agents have about the knowledge of other agents.

- Important for reasoning about strategies.

Multi-agent systems

Dynamics

Actions modifying the state of the system.

- Announcements;
- Playing cards;
- Drawing cards. . .

Multi-agent systems

Probabilities

Express best strategies instead of always winning strategies.

Modeling by Dynamic Epistemic Logic

Dynamic Epistemic Logic (DEL)

- Models:
  - High-order knowledge (“Agent *a* knows that *b* knows that . . . ”).
  - Dynamic actions.

But not probabilities...

Overview

Outline

Plan

Epistemic logic

Syntax

$$\varphi ::= p \mid \varphi_1 \vee \varphi_2 \mid \neg\varphi \mid K_A\varphi$$

Propositional logic + A knows φ.

$\boxed{1}_A \wedge \boxed{2}_B$

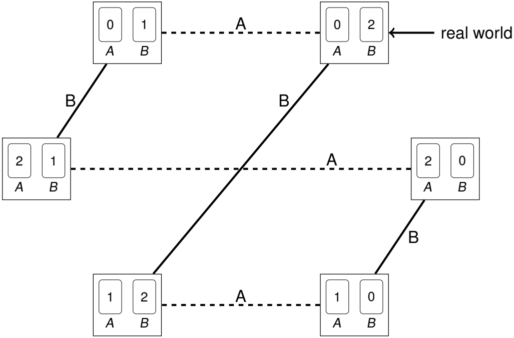
Agent A has card 1 and agent B has card 2.

$K_A \neg K_B \boxed{1}_A$

Agent A knows that agent B does not know that agent A has card 1.

Semantics: Kripke models M

Example of a 3-card game.

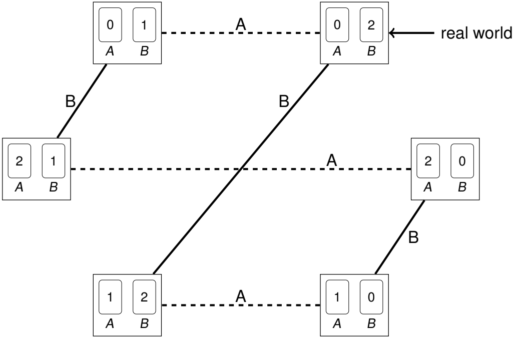


Kripke model

- Nodes: possible worlds.
- Edges: indistinguishability relations for each agent.

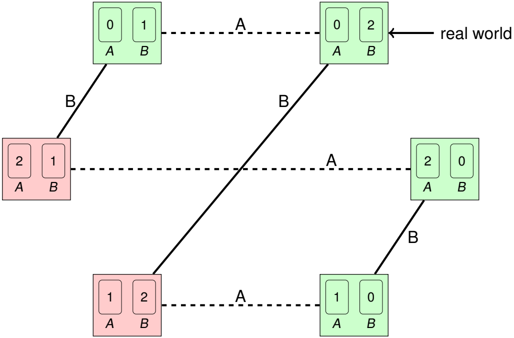
Semantics of epistemic logic

Semantics of φ: set of worlds, defined by induction on φ.



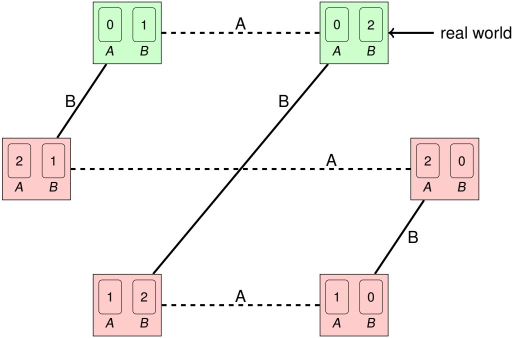
Truth value of Boolean formula β

Set of worlds whose valuation satisfy β.
Example: $\beta = \boxed{0}_A \vee \boxed{0}_B$



Truth value of $K_A\varphi$

Set of worlds such that all A-successors satisfy φ.
Example: $K_A(\boxed{0}_A \vee \boxed{0}_B)$.



Plan

Dynamic epistemic logic

Syntax

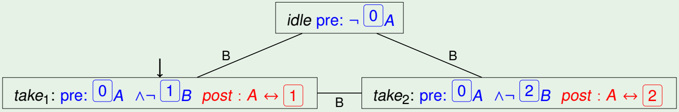
$\varphi ::= p \mid \varphi_1 \vee \varphi_2 \mid \neg\varphi \mid K_A\varphi \mid \quad \langle E \rangle \varphi$

Epistemic logic + φ is true after executing event model E

Event model

- Nodes: possible events with:
  - a precondition: necessary condition to execute the action.
  - a postcondition: effect of the action.
- Edges: indistinguishability relations.

Switching agent A's card with the draw when agent A has card 0.

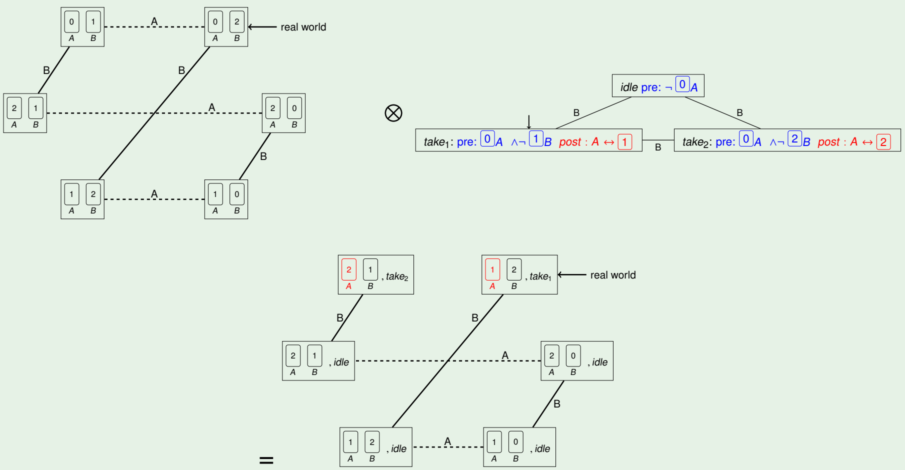


post : $A \leftrightarrow \boxed{1}$ is a macro for $\boxed{0}_A \leftarrow \bot$, $\boxed{1}_A \leftarrow \top$.

Effect of an event model E

Synchronous product with the current Kripke model M, noted $M \otimes E$.

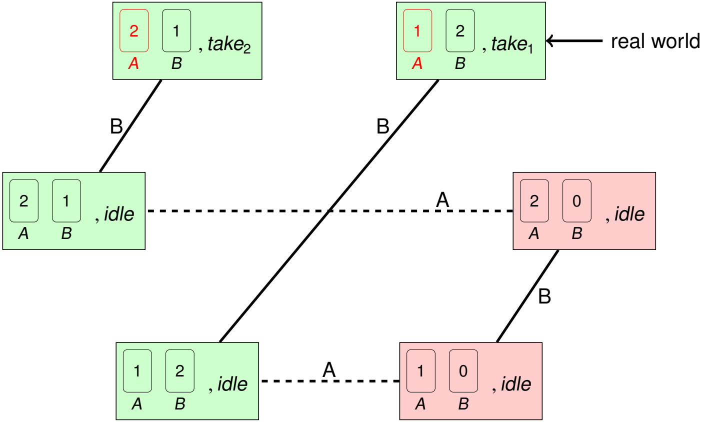
$M \otimes E_{show}$



Semantics of $\langle E \rangle \varphi$

Set of worlds in $M \otimes E$ that satisfy φ.

Example: $\langle E_{show} \rangle (K_B \boxed{0}_A \vee K_B \boxed{1}_A \vee K_B \boxed{2}_A)$



Outline

Plan

Probabilistic epistemic logic

Syntax

$$\varphi ::= p \mid \varphi_1 \vee \varphi_2 \mid \neg\varphi \mid K_A\varphi \quad \mid \quad P_A\varphi \geq q$$

Epistemic logic + Probability of φ is $\geq q$ for agent A.

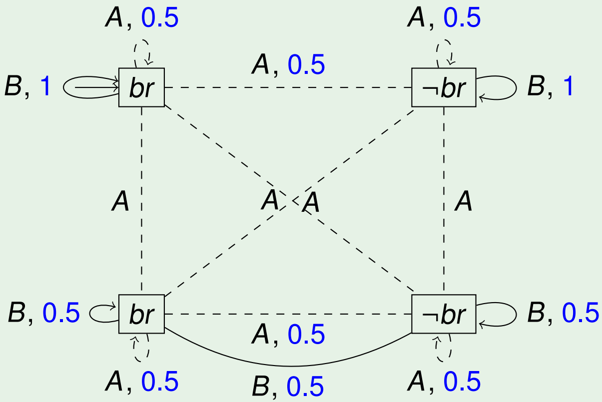
Example: $P_A(\neg K_B \boxed{1}_A) \geq \frac{1}{2}$

The probability for Agent A that that agent B does not know that agent A has card 1 is greater than $\frac{1}{2}$.

Semantics: probabilistic Kripke model

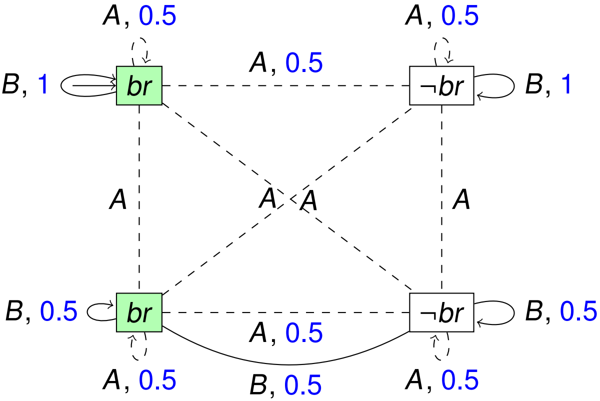
Example: broken coin

br: the coin is broken.



Semantics: probabilistic Kripke model

Example: model checking $\neg K_A(P_B br \geq 1)$.
Set of worlds where *br* is true.



Semantics: probabilistic Kripke model

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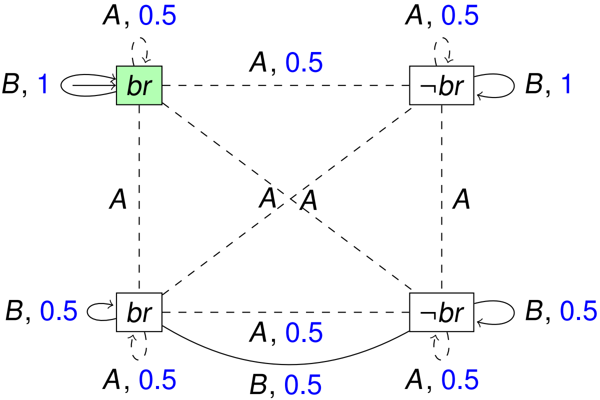

Semantics: probabilistic Kripke model

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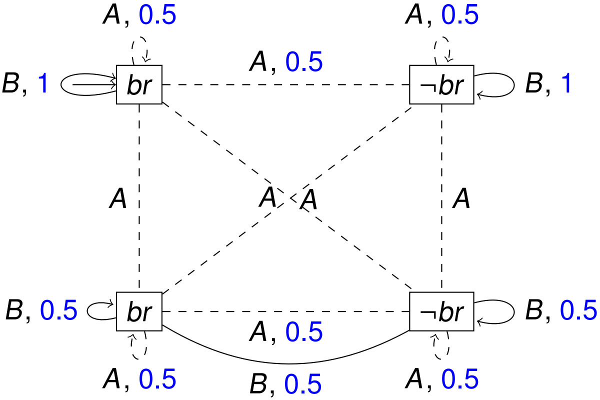

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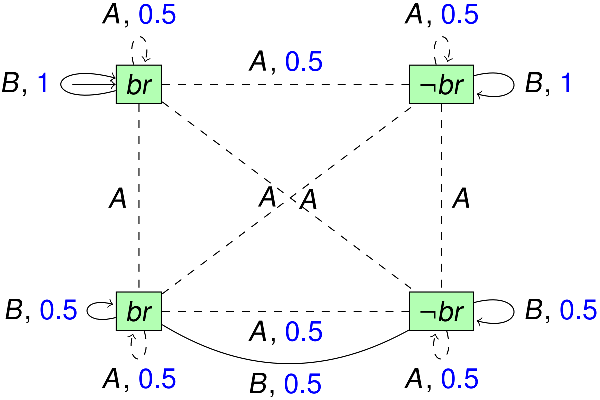
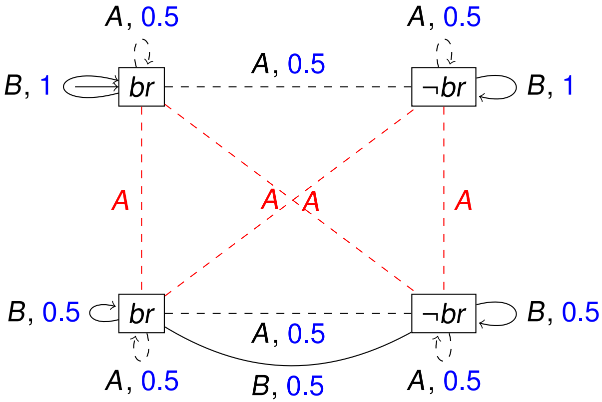


Why distinguishing knowledge from probabilities?

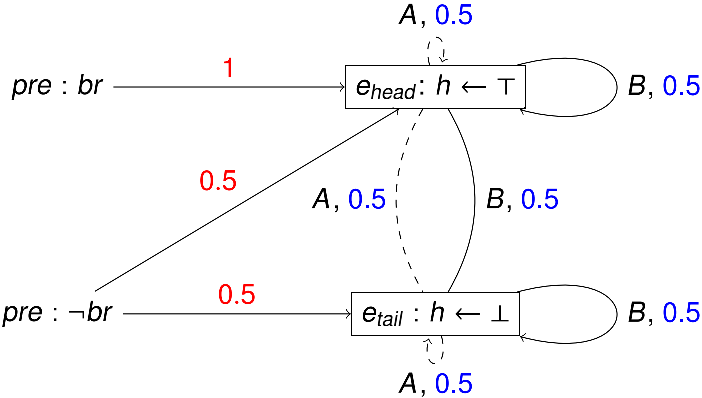
Probabilities $\subseteq$ Knowledge but not the other way around...



It does not make sense to assign probability to "*B* is a magician".

Plan

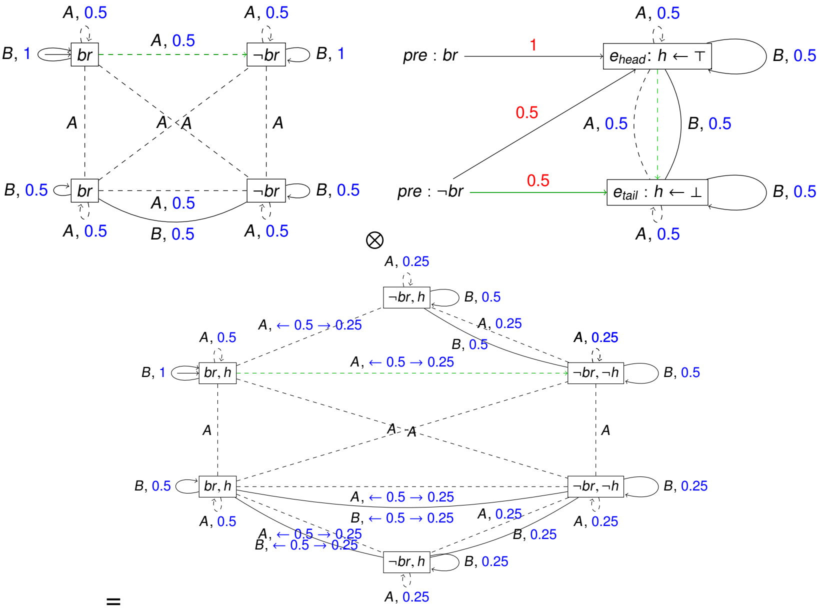
Example: flipping the coin



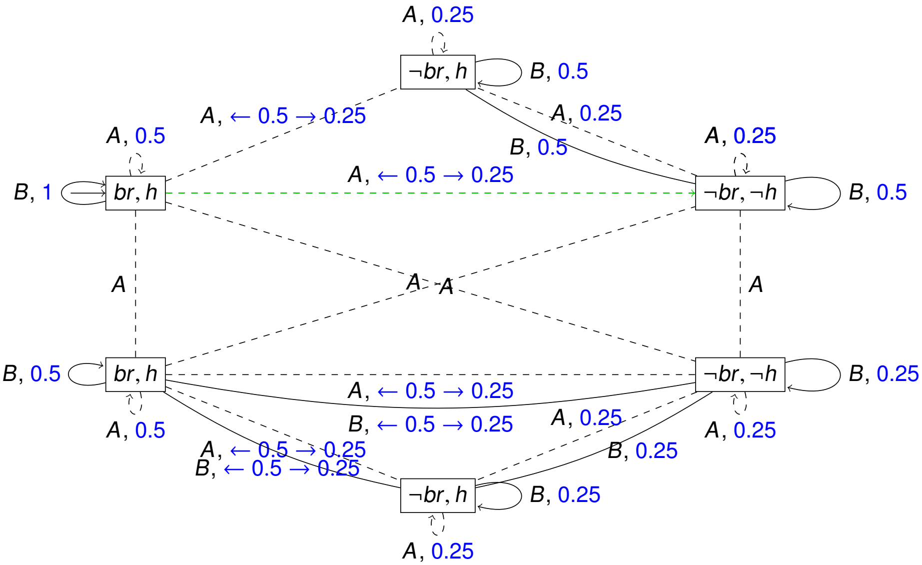
Objective probabilities

Subjective probabilities

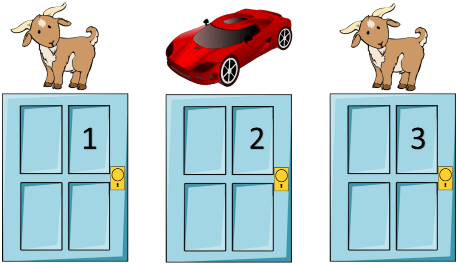
Application of a probabilistic action



Zoom on the product



Another example: the Monty Hall problem



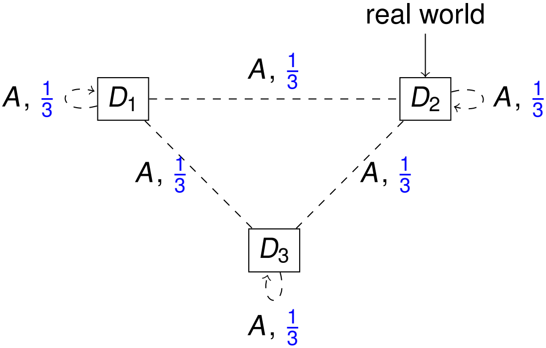
Definition

A candidate participates in a TV show where he can win a car. There are three doors in front of him. Behind one of there is the car, behind the other two a goat.
The candidate chooses a door and then the presenter opens a door with a goat behind. The candidate is able to change his choice at this point, what should he do?

What do you think?

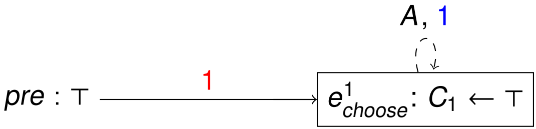
Modeling the Monty Hall problem: the initial Kripke model

Proposition D_i: the car is behind door i.

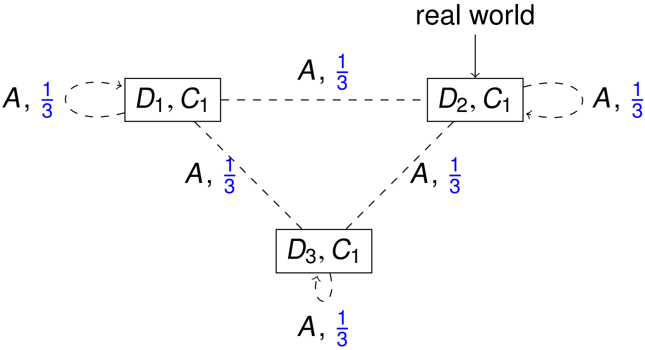


Modeling the Monty Hall problem: choosing door 1

Proposition C_i: the candidate chooses door i.

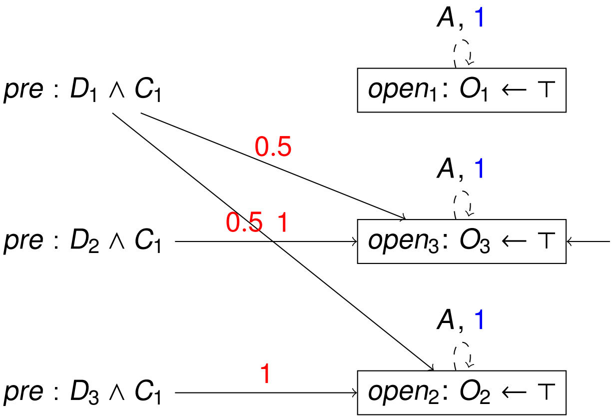


Modeling the Monty Hall problem: Kripke model after the choice

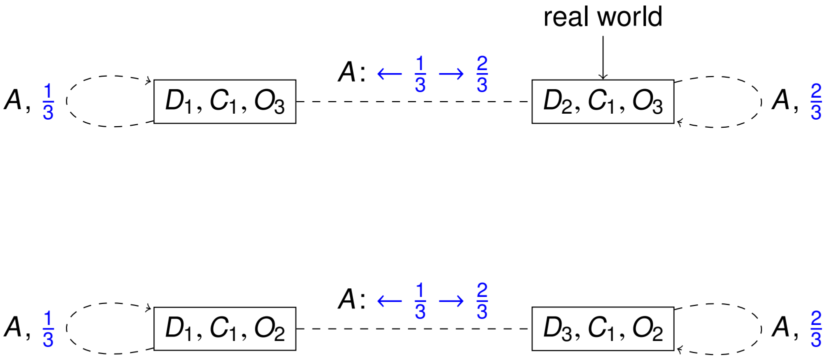


Modeling the Monty Hall problem: the presenter opens a door given that the candidate chose door 1.

Proposition O_i: the presenter opens door i.



Modeling the Monty Hall problem: after the presenter opened the door



Best move for the candidate

He should change his choice!

Outline

Conclusion

Probabilistic Dynamic Epistemic Logic (PDEL)

Models:

- High-order knowledge (agent *a* knows that agent *b* knows. . .);
- Dynamics (flip a coin);
- Probabilities.

Complexity of model checking PDEL

Should be PSPACE-complete (same complexity as non probabilistic DEL).

Ongoing work

- Proof of complexity for model checking.
- Use for defining best strategies in bounded games.
- Symbolic methods.

Thank you for your attention! Questions?