Distributed algorithms: a challenging playground for model checking

Nathalie Bertrand Inria

OPODIS - December 13th 2021

based on joint work with Igor Konnov, Marijana Lazić, Bastien Thomas and Josef Widder

Model checking in a nutshell

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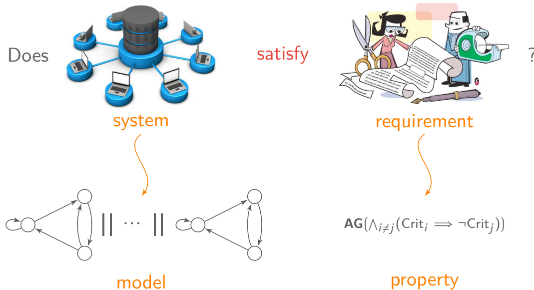

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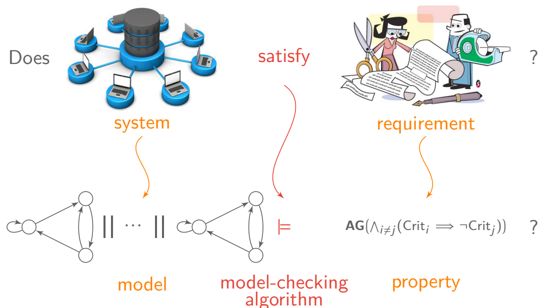

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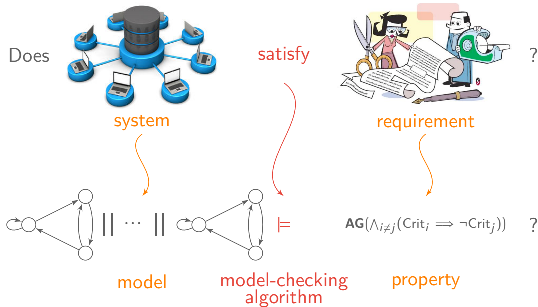

⊕ generic, successfully applied to hardware/software verification
embedded softwares, real-time systems, controllers in avionics, telecommunications, planning, etc.

⊖ undecidable in general, scalability issues

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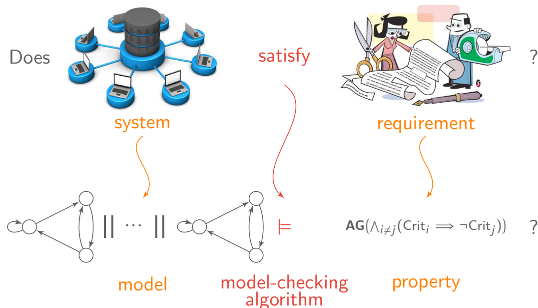

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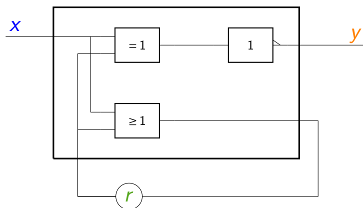
2 Turing awards

- Pnueli, 1996: temporal logic; program and systems verification
- Clarke, Emerson and Sifakis, 2007: model checking as highly effective verification technology

Hardware model checking: a basic example

output control function $\lambda_y = \neg(x \oplus r)$
register evaluation $\delta_r = x \vee r$



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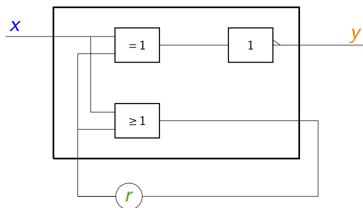

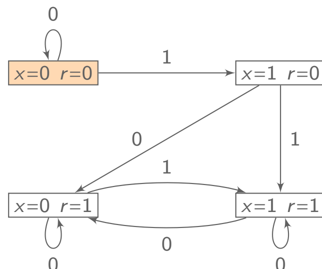

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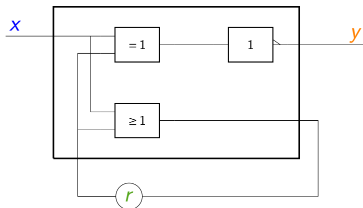

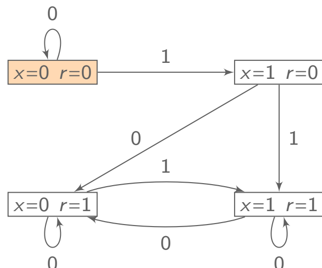

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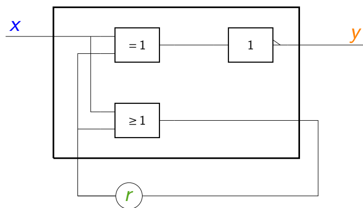

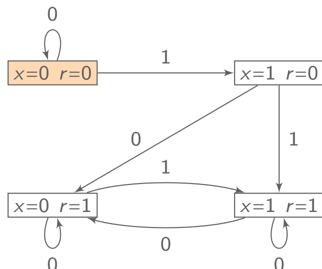

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automaton is exponential in number of variables

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1  low:=0; high:=n-1;
2  while low <= high {mid := (low + high)/2;
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5       else if x > t[mid]
6            then low:=mid+1;
7            else return mid;}
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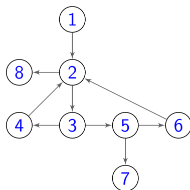

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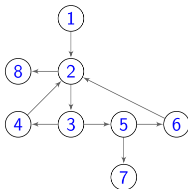

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reduces to maximal paths in CFG reach 7 or 8

→ CFG model is not sufficient to prove termination

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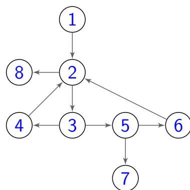

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- ⊕ FIFO queues: communication channels
- ⊕ pushdown automata: recursion stack

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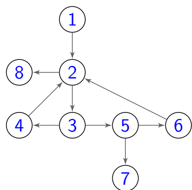

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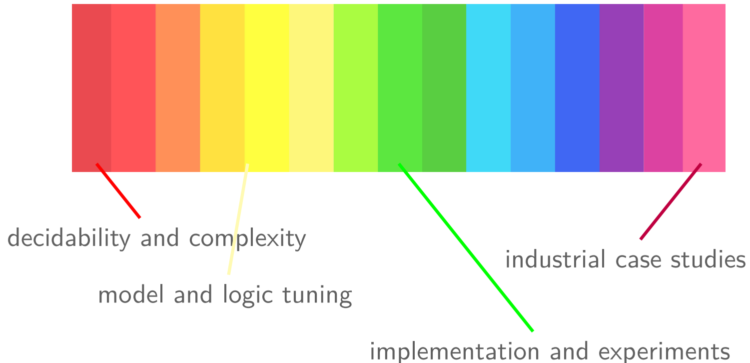
the more expressive the model, the more complex the verification

Model checking community

Wide spectrum of activities: from theory to applications



Outline

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Parameterized verification for distributed algorithms

Two frameworks for threshold-based fault-tolerant algorithms
- Threshold automata (with random choices)
- Predicate abstraction

Conclusion

Peterson's mutual exclusion algorithm

[Peterson Information Processing Letters 1981]

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loop forever;
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.   /* non-critical actions */
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b_i := ⊤ ; x := 1 − i;
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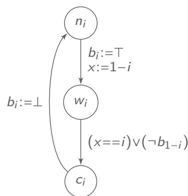

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the processes are not in their critical section simultaneously

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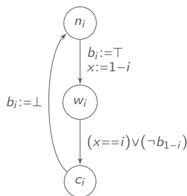

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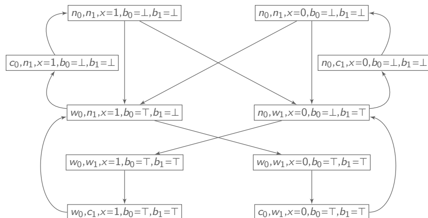
Product transition system representing all possible interleavings



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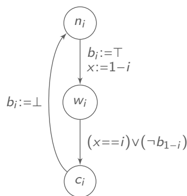

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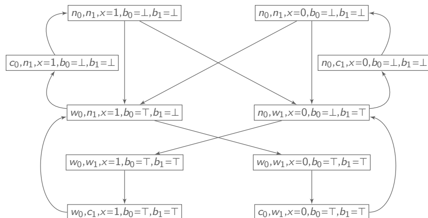

checking correctness reduces to

no state $(c_0, c_1, _, _, _)$ is reachable

→ exhaustive exploration of executions

Limitations of standard model checking techniques

⚠ **state-space explosion**: product transition system is exponential in number of processes, and of variables
→ tools hardly scale to large number of processes or real-life examples

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- BDD encodings
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⚠ models with **fixed number of processes**

e.g. randomized consensus for 10 participants with model checker PRISM

→ correctness should be proven for arbitrary number of processes

Parameterized verification: to infinity and beyond!

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$$\forall n \quad \underbrace{C \,||\cdots||\, C}_{n \text{ times}} \,||\, S \quad \models \quad \varphi$$

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⚠ model checking infinitely many instances at once

Good news! it may be computationally easier to prove correctness "for all n" than "for n a large fixed value".

Parameterized verification for distributed algorithms

Starting from...

- algorithm pseudo-code

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bool v := input_value({0, 1});
int  r := 1 ;
while (true) do
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 wait for n − t messages (R,r,∗);
 if received (n + t) / 2 messages (R,r,w)
 then send (P,r,w,D) to all;
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  validity, agreement
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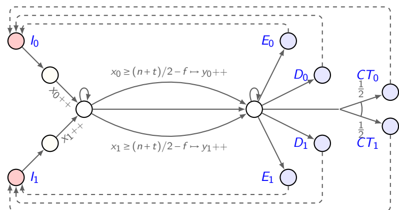

- formulas

$\forall k, k' \in \mathbb{N}, \quad \mathbf{A}\left(\mathbf{F}\,\boldsymbol{\kappa}[D_V, k] > 0 \rightarrow \mathbf{G}\,\boldsymbol{\kappa}[D_{1-V}, k'] = 0\right)$

$\forall k \in \mathbb{N}\, \mathbf{A}\left(\mathbf{F}\,\boldsymbol{\kappa}[I_V, 0] = 0 \quad \rightarrow \quad \mathbf{G}\,\boldsymbol{\kappa}[D_V, k] = 0\right)$

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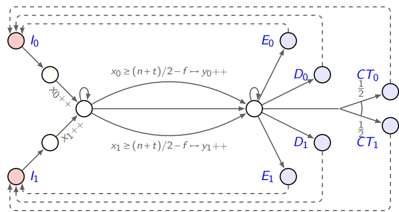

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- **model checking algorithms**
- **prototype implementation**

Which distributed algorithms?

A variety of settings to explore

- **timing model**: asynchronous, synchronous, etc.
- **communication paradigm**: shared variable, broadcast, etc.
- **failure model**: no failures, crash, Byzantine processes
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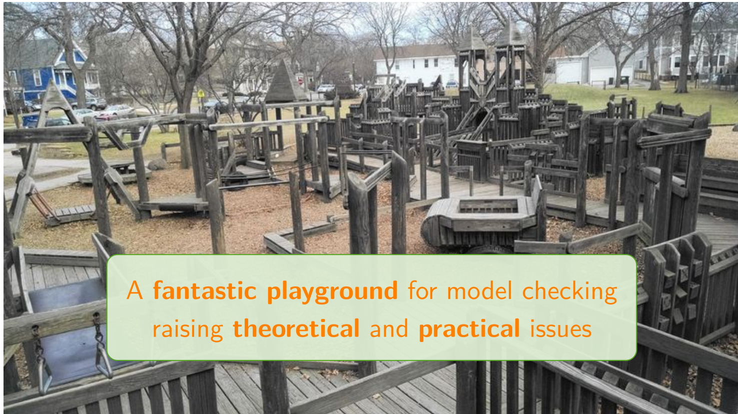

A **fantastic playground** for model checking raising **theoretical** and **practical** issues

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for fault-tolerant threshold-based distributed algorithms

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- guards as linear constraints on variables and parameters

[Konnov Veith Widder CAV'15, Konnov Lazić Veith Widder POPL'17]

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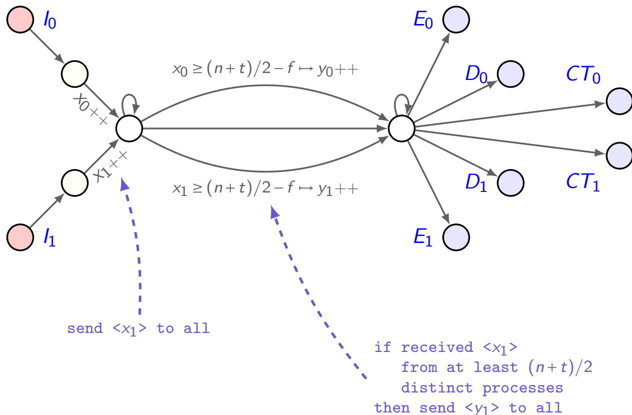

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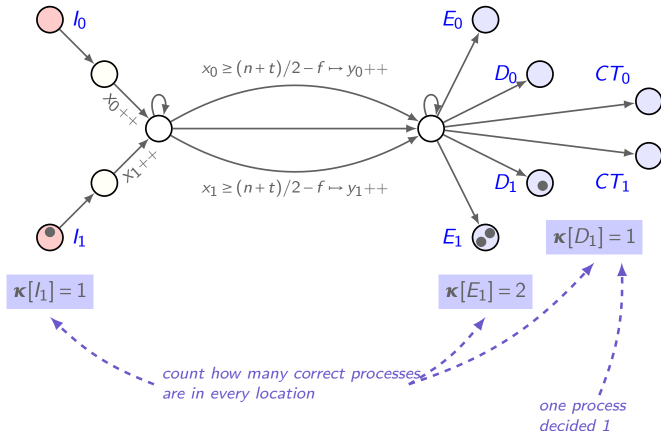

Specifying and verifying correctness

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- atomic propositions: whether counter value is 0 or not

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Given a threshold automaton TA, a specification φ in $\mathsf{ELTL}_{\mathsf{FT}}$, and a resilience condition RC
one can decide whether for all parameters satisfying RC, $\mathrm{Sys(TA)} \models \varphi$

Tool support: ByMC at `forsyte.at/software/bymc/`

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How to handle randomization?

Ben Or's randomized algorithm for consensus [Ben Or PODC'83]

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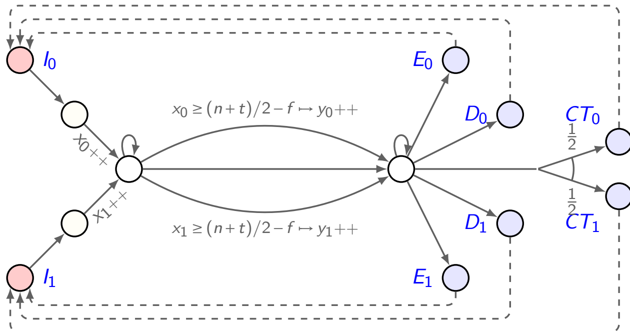
    messages (P,r,w,D) then {
  v := w;                  /∗ enough support −> update estimate ∗/
  if received at least (n + t) / 2
   messages (P,r,w,D)
  then decide w;                     /∗ strong majority −> decide ∗/
 } else v := random(0, 1) ;      /∗ unclear −> coin toss      ∗/
 r := r + 1 ;
od
```

Modeling challenges

- unboundedly many rounds
- probabilistic choices for local/global coin tosses

Probabilistic threshold automata

Ben Or's randomized consensus algorithm



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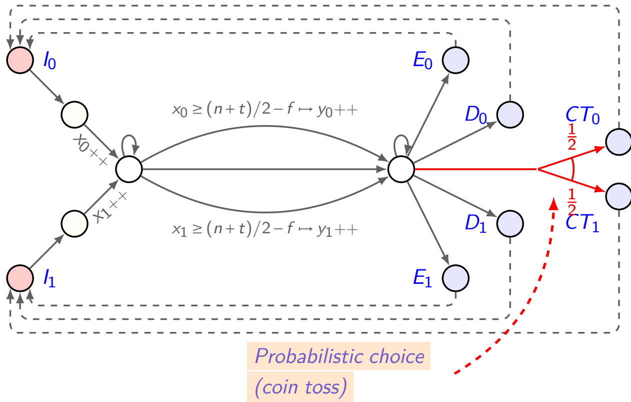

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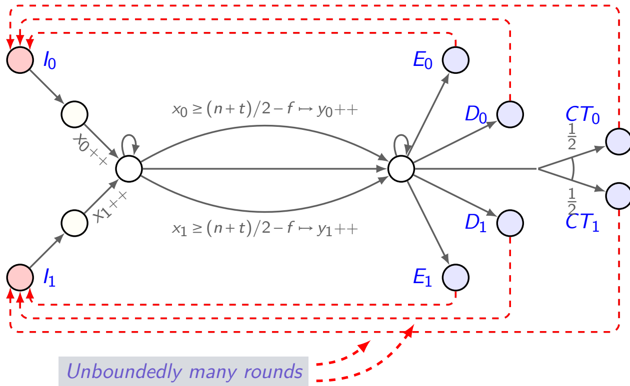

Specifying correctness

Agreement: No two correct processes decide differently (safety)

$$(\forall k \in \mathbb{N}_0)\,(\forall k' \in \mathbb{N}_0)\ \mathbf{A}\left(\mathbf{F}\ \boldsymbol{\kappa}[D_v, k] > 0 \quad \rightarrow \quad \mathbf{G}\ \boldsymbol{\kappa}[D_{1-v}, k'] = 0\right)$$

Validity: Any decided value was proposed initially (safety)

$$(\forall k \in \mathbb{N}_0)\ \mathbf{A}\left(\mathbf{F}\ \boldsymbol{\kappa}[I_{1-v}, 0] = 0 \quad \rightarrow \quad \mathbf{G}\ \boldsymbol{\kappa}[D_{1-v}, k] = 0\right)$$

Almost sure termination: under every adversary, with probability 1 every correct process eventually decides (prob. liveness)

$$\mathbb{P}_{\mathtt{a}}\left(\ \bigvee_{k\in\mathbb{N}_0} \bigvee_{v\in\{0,1\}} \mathbf{G} \bigwedge_{\ell\in\mathscr{L}\setminus\{D_v\}} \boldsymbol{\kappa}[\ell, k] = 0\right) = 1$$

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Verification challenges

- specifications over multiple rounds
- probabilistic guarantees

Verifying correctness

Safety properties

- must hold on **all** executions
  - probabilistic choices can be transformed into non-determinism
  - reduction to non-probabilistic threshold automata
- communication-closure
  - reduction to single round threshold automaton by reordering actions
- round switch invariants
  - reduction to single round specification

[**B.** Konnov Lazić Widder Concur'19]

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  - restriction to *round-rigid* adversaries to reorder actions within rounds
  - sufficient conditions on single round threshold automaton that imply almost sure termination

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sufficient conditions only, approach mimicks manual proof

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Experimental evaluation

- 6 classical randomized distributed algorithms
- several one-round safety and liveness properties for each

Algorithm	Verif time per property
- Ben-Or's Byzantine random. consensus	≤ 1 sec
- Ben-Or's crash random. consensus	≤ 1 sec
- Ben-Or's clean crash random. consensus	≤ 1 sec
- Bracha's randomized consensus	≤ 1 sec
- Raynal's k-set agreement	3–40 sec
- Song's and van Renesse's BOSCO	3 hours on a cluster

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Threshold-based round-based algorithms

Phase King algorithm [Berman Garay, Mathematical Systems Theory 1993]

```
int id := identifier({0 ... n−1});
bool v := input_value({0, 1});
for r=0 to t do
    broadcast (r,id,v);
    receive all (r,_,_);
    if # of (r,_,0) received > n/2 + t /* majority of 0 */
       v := 0;
       else if # of (r,_,1) received > n/2 + t /* majority of 1 */
            v := 1;
            else v := v' where (r,r,v') received; /* new value is king value */
```

Threshold-based round-based algorithms

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- processes send their local state as message contents
- unbounded number of rounds (parameter t)
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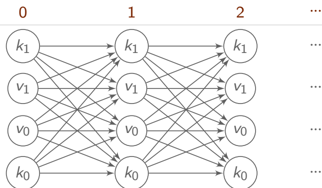


e.g. guard from v_1 to v_0

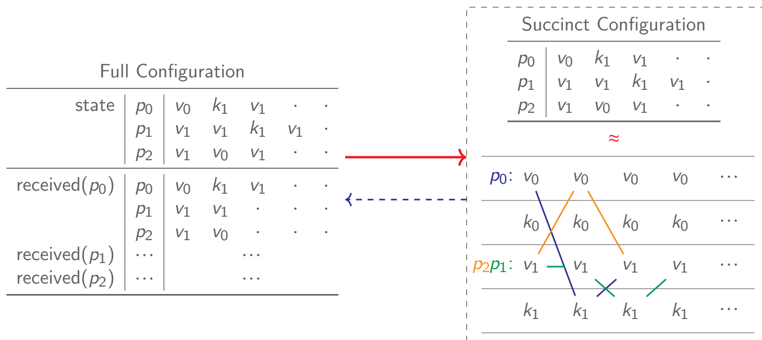
$v_0 + f > n/2 + t \vee (k_0 > 0 \wedge v_1 \leq n/2 + t)$

Abstraction steps

Full Configuration

state	p_0	v_0	k_1	v_1	·	·
	p_1	v_1	v_1	k_1	v_1	·
	p_2	v_1	v_0	v_1	·	·
received(p_0)	p_0	v_0	k_1	v_1	·	·
	p_1	v_1	v_1	·	·	·
	p_2	v_1	v_0	·	·	·
received(p_1)	$\cdots$			$\cdots$		
received(p_2)	$\cdots$			$\cdots$		

Abstraction steps



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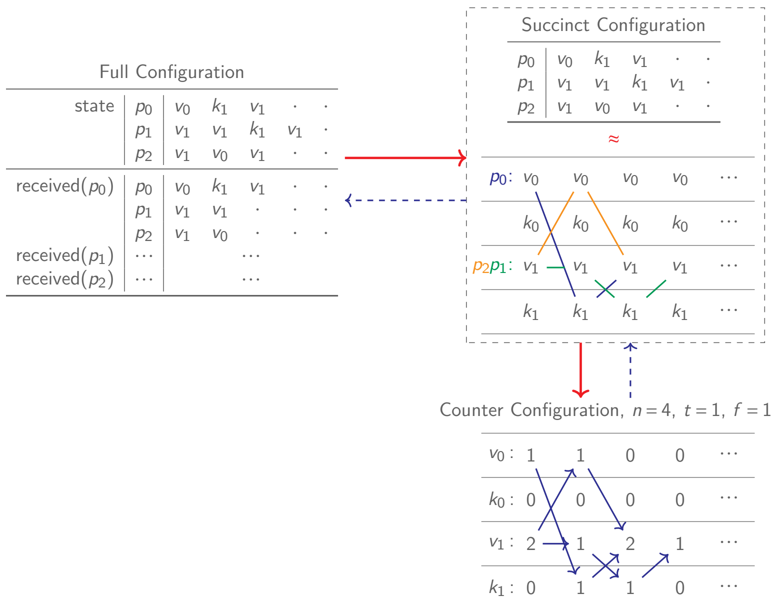

Abstraction steps

Full Configuration

state	p_0	v_0	k_1	v_1	·	·
	p_1	v_1	v_1	k_1	v_1	·
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received(p_1)	⋯			⋯		
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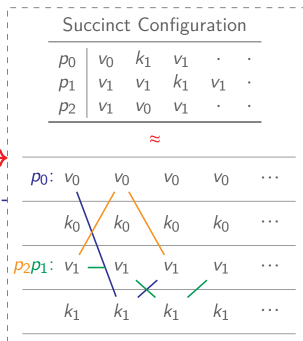

Guard Configuration

$v_0 > 0$	T	T	F	F	⋯
$k_0 > 0$	F	F	F	F	⋯
$v_1 > 0$	T	T	T	T	⋯
$k_1 > 0$	F	T	T	F	⋯
$2(v_0 + k_0 + f) > n + 2t$	F	F	F	F	⋯
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$2(v_1 + k_1) > n + 2t$	F	F	F	F	⋯
$v_0 + k_0 + v_1 + k_1 + f \geq n$	T	T	T	F	⋯

Counter Configuration, $n = 4$, $t = 1$, $f = 1$

v_0:	1	1	0	0	⋯
k_0:	0	0	0	0	⋯
v_1:	2	1	2	1	⋯
k_1:	0	1	1	0	⋯

Guard automaton

$v_0 > 0$	T	T	F	F	F	$\cdots$
$k_0 > 0$	F	F	F	F	F	$\cdots$
$v_1 > 0$	T	T	T	T	F	$\cdots$
$k_1 > 0$	F	T	T	F	F	$\cdots$
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guard configuration

[**B.** Thomas Widder Concur'20]

Guard automaton

	γ_0	γ_1	γ_2	γ_3	γ_4	
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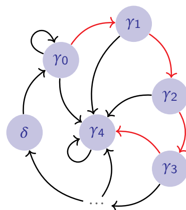

guard automaton

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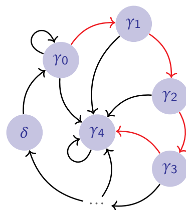

guard automaton

- fixed set of predicates
- transitions in guard automaton via predicate abstraction (SMT solver)

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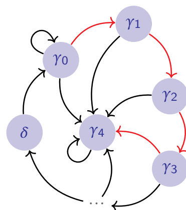

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The language of the guard automaton **overapproximates** the set of all executions of the layered threshold automaton.

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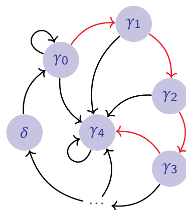

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⚠ Incomplete method yet **sufficient to prove correctness** of Phase King algorithm + **analysis can be refined** by adding more predicates

[B. Thomas Widder Concur'20]

How good are guard automata and predicate abstraction?

Drawbacks and advantages compared to threshold automata approach

- ⊖ layered assumption on algorithm structure
- ⊖ formalization using domain theory concepts
- ⊖ randomization not supported

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- ⊕ formal relation between distributed algorithms and model semantics
- ⊕ allows to check for liveness properties on infinite executions
- ⊕ enables refinement
- ⊕ stand alone implementation

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Parameterized verification for distributed algorithms

Two frameworks for threshold-based fault-tolerant algorithms
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Parameterized verification of distributed algorithms

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- apply to **simple standard** distributed algorithms
- provide **automated correctness** proofs
  in contrast to error-prone manual proofs and non-exhaustive simulation
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  one model-checking algorithm for a class of distributed algorithms

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Two available tools to try out: ByMC, Peregrine

Still many challenges for model checking

Current interests

- compositional analysis of threshold automata to improve scalability
- shared-memory consensus algorithms [Aspnes JACM 2002]

Mid- to long-term objectives

- model extraction from pseudo-code
- quantitative analysis of randomized consensus algorithms
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