Fairness in LTL

Recall: action-based fairness

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fairness assumption for TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$:

$$\mathcal{F} = (\mathcal{F}_{ucond}, \mathcal{F}_{strong}, \mathcal{F}_{weak})$$

where $\mathcal{F}_{ucond}$, $\mathcal{F}_{strong}$, $\mathcal{F}_{weak} \subseteq 2^{Act}$

$\mathcal{F}_{ucond}$	unconditional fairness assumption
$\mathcal{F}_{strong}$	strong fairness assumption
$\mathcal{F}_{weak}$	weak fairness assumption

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- for all $A \in \mathcal{F}_{weak}$:

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satisfaction relation for LT-properties under fairness:

$\mathcal{T} \models_{\mathcal{F}} E$ iff for all $\mathcal{F}$-fair paths π of $\mathcal{T}$:
$trace(\pi) \in E$

Process fairness is LTL-definable

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$$\varphi ::= \mathit{true} \mid a \mid \varphi_1 \wedge \varphi_2 \mid \neg\varphi \mid \bigcirc \varphi \mid \varphi_1 \mathsf{U} \varphi_2$$

eventually $\Diamond\varphi \stackrel{\text{def}}{=} \mathit{true} \mathsf{U} \varphi$

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infinitely often $\Box\Diamond\varphi$

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e.g., unconditional fairness $\Box\Diamond \mathit{crit}_i$

strong fairness $\Box\Diamond \mathit{wait}_i \longrightarrow \Box\Diamond \mathit{crit}_i$

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LTL fairness assumptions

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... are conjunctions of LTL formulas of the form:

- unconditional fairness $\Box\Diamond\phi$
- strong fairness $\Box\Diamond\phi_1 \rightarrow \Box\Diamond\phi_2$
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$$s \models_{fair} \varphi \quad \text{iff} \quad \text{for all } \pi \in Paths(s): \text{ if } \pi \models fair \text{ then } \pi \models \varphi$$

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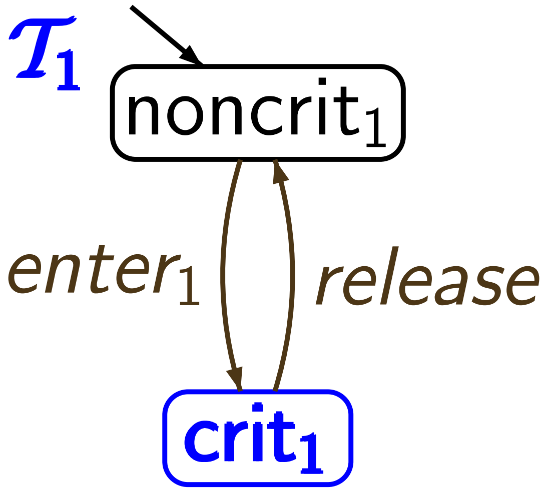
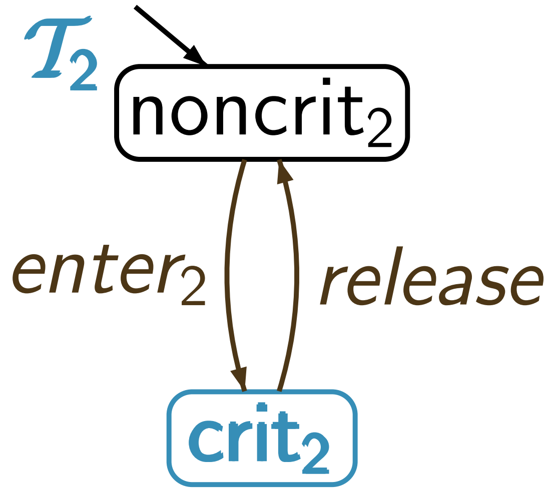
&\text{iff} \quad s \models fair \rightarrow \varphi
\end{aligned}
$$

Randomized arbiter for MUTEX





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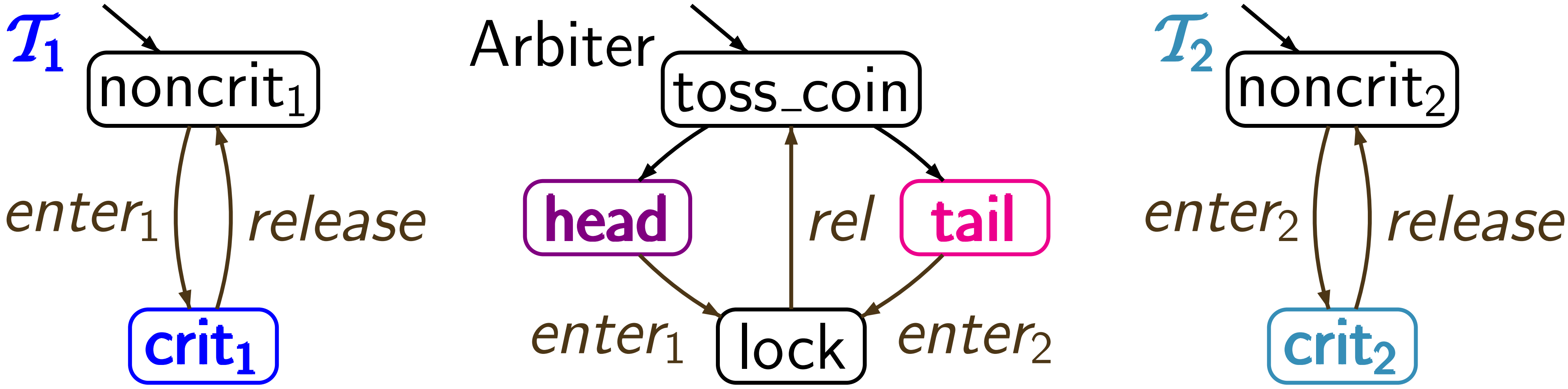
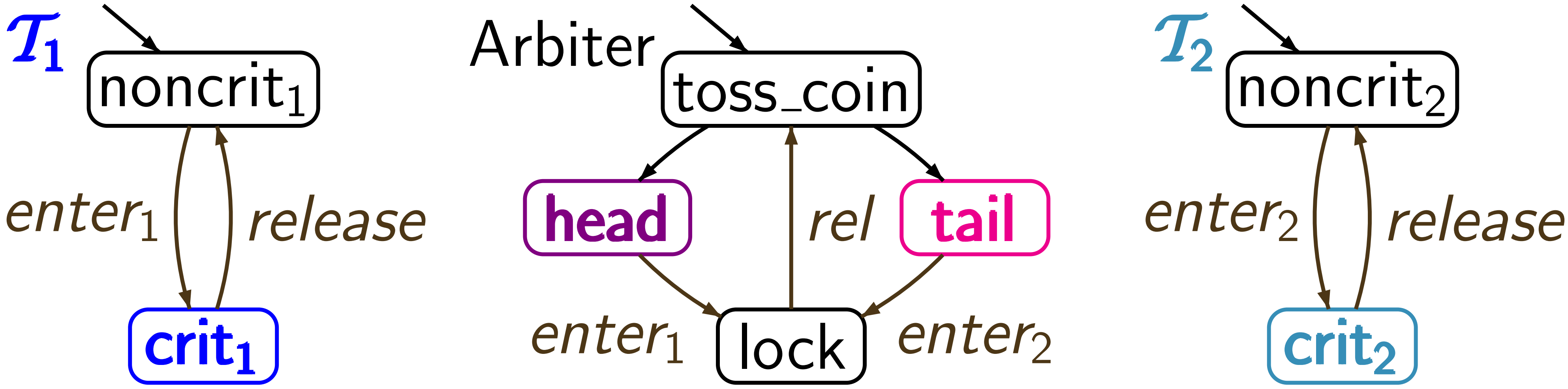
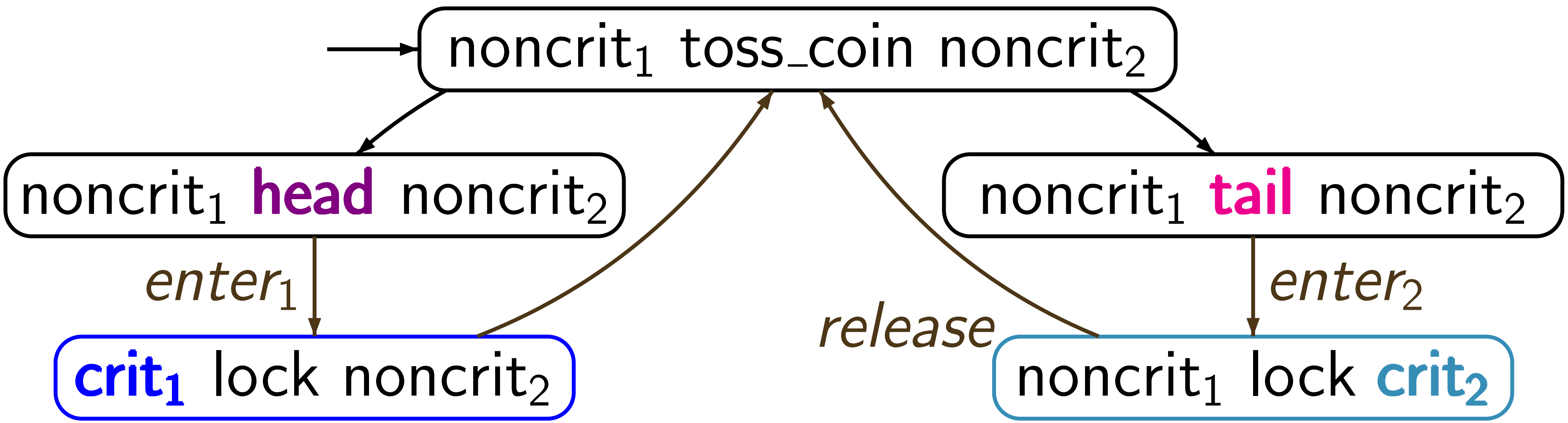


Randomized arbiter for MUTEX



$(\mathcal{T}_1 \,|||\, \mathcal{T}_2) \,\|\, \mathit{Arbiter}$



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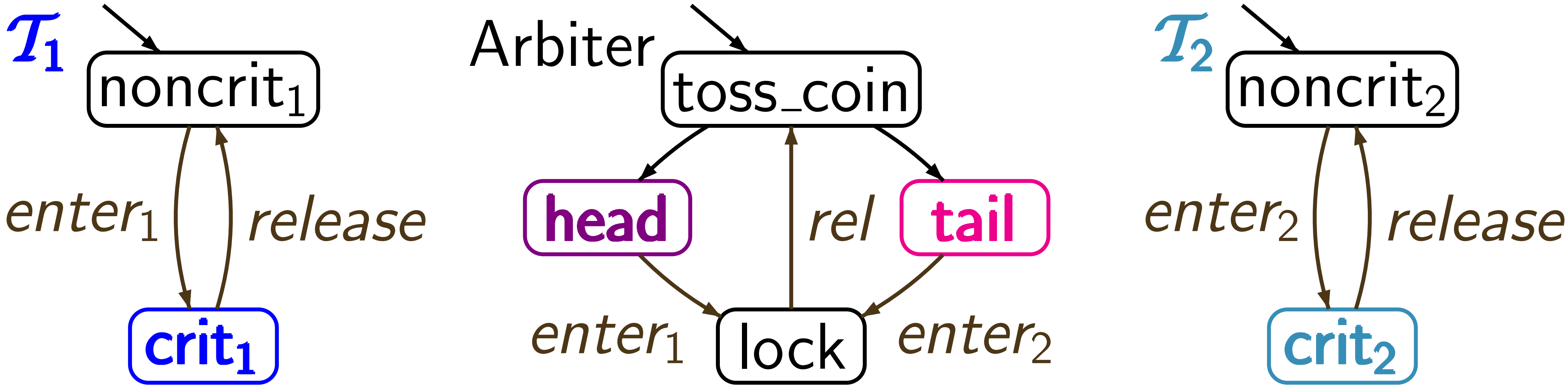
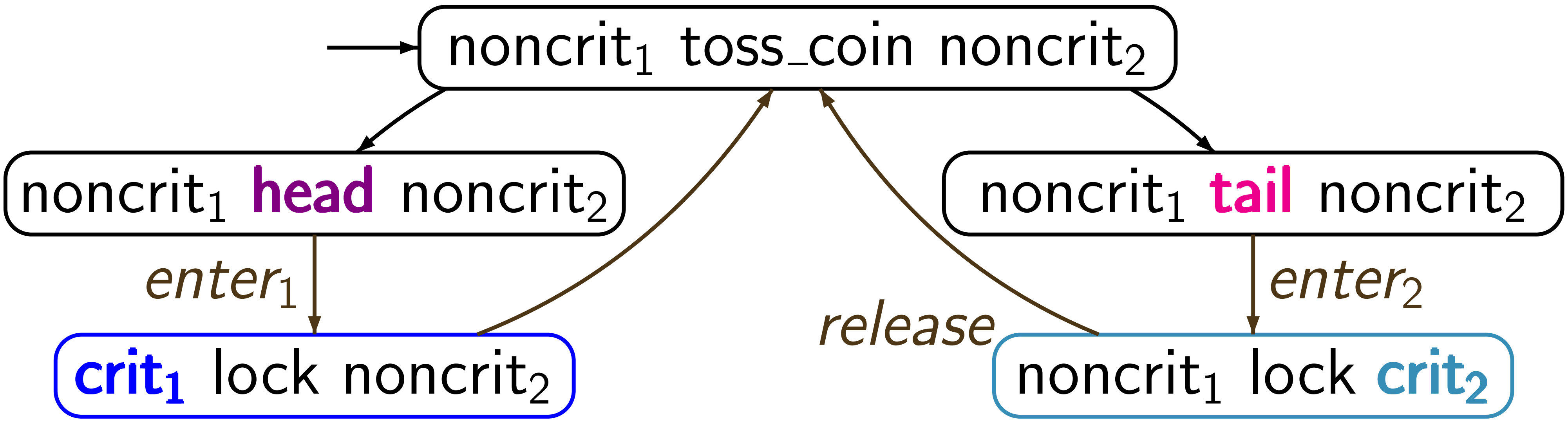


$$(\mathcal{T}_1 \;|||\; \mathcal{T}_2) \parallel Arbiter \not\models \Box\Diamond crit_1 \wedge \Box\Diamond crit_2$$



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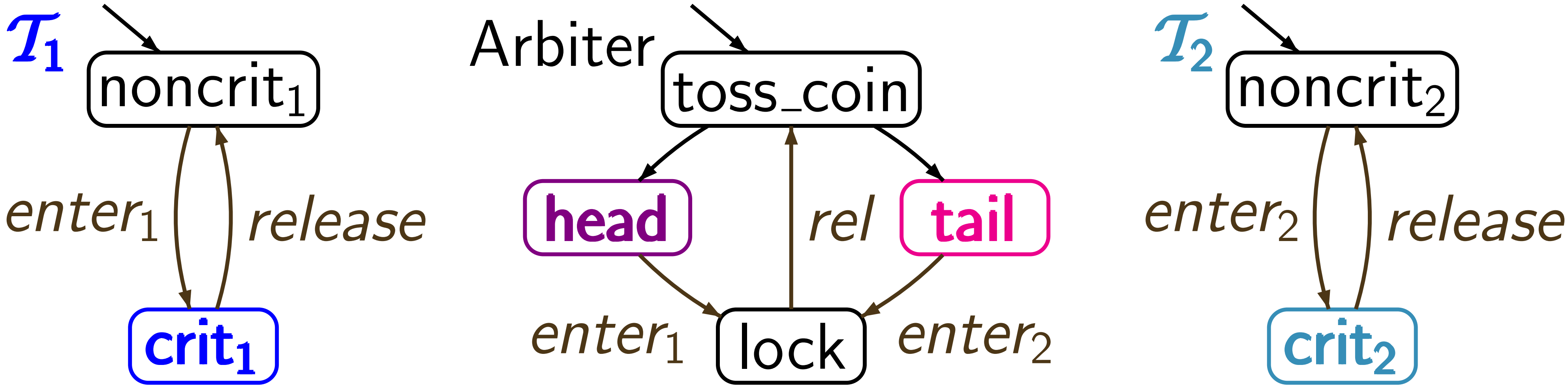

unconditional LTL-fairness:

$$fair \;=\; \Box\Diamond \textbf{head} \;\wedge\; \Box\Diamond \textbf{tail}$$

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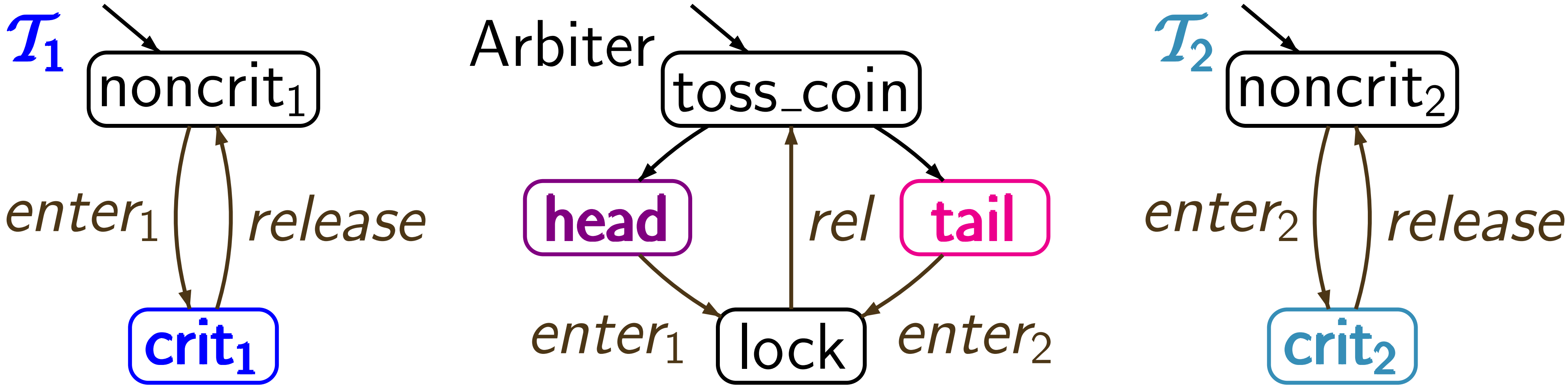

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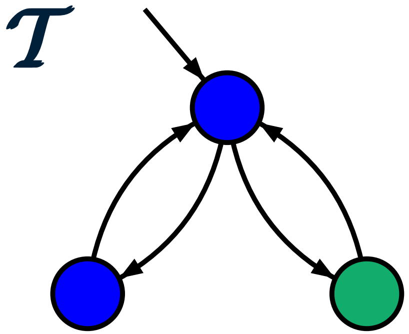
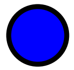
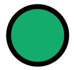
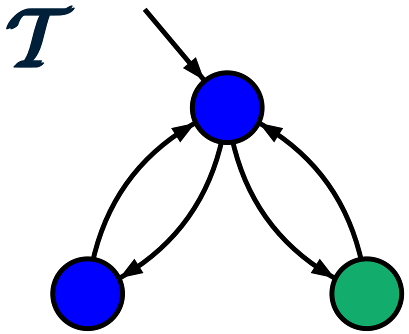


LTL fairness assumption

$fair = \Diamond\Box a \rightarrow \Box\Diamond b$

 $\widehat{=} \{a\}$  $\widehat{=} \{b\}$

Correct or wrong?



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● $\widehat{=} \{a\}$ ● $\widehat{=} \{b\}$

$\mathcal{T} \models_{fair} \bigcirc b$ **?**

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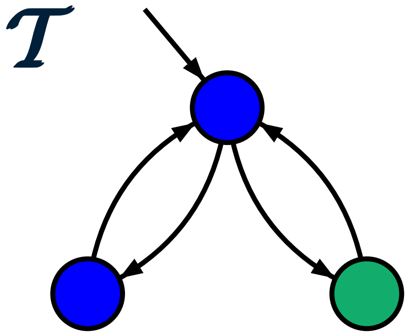

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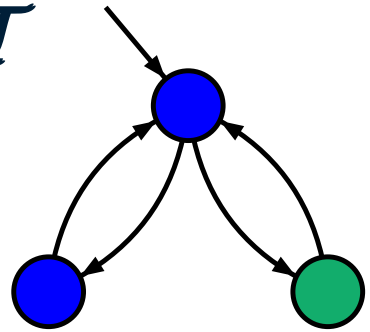
Correct or wrong?

$\mathcal{T}$



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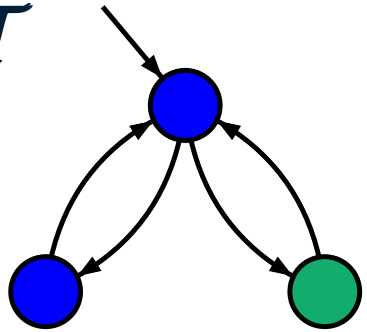

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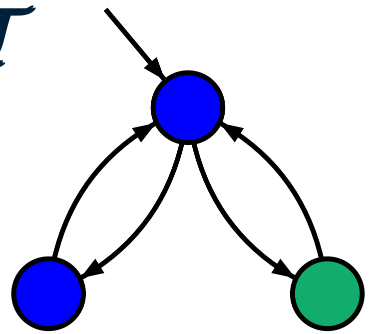
$\mathcal{T} \models_{fair} a \,\mathsf{U}\, b$ $\checkmark$

Correct or wrong?

$\mathcal{T}$



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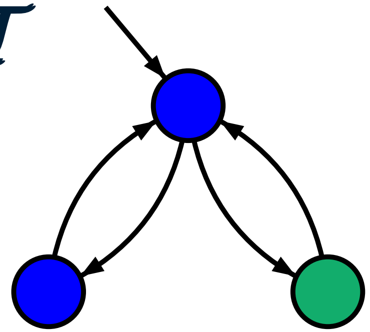
$\mathcal{T} \models_{fair} a \,\mathsf{U}\, b$ $\surd$

$\mathcal{T} \models_{fair} a \,\mathsf{U}\, \Box(b \leftrightarrow \bigcirc a)$ **?**

Correct or wrong?

$\mathcal{T}$



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$\mathcal{T} \models_{fair} a \, \mathsf{U} \, b$ √

$\mathcal{T} \not\models_{fair} a \, \mathsf{U} \, \Box(b \leftrightarrow \bigcirc a)$

as ●→●→●→●→●→●→ ... is fair

LTL-fairness assumptions

- can be necessary to prove liveness properties, e.g., mutual exclusion with arbiter/semaphore

$$\mathcal{T}_{sem} \not\models \Box\Diamond crit_1 \wedge \Box\Diamond crit_2$$

$$\mathcal{T}_{sem} \models_{fair} \Box\Diamond crit_1 \wedge \Box\Diamond crit_2$$

for appropriate fairness condition

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for appropriate fairness condition, e.g.,

$$fair = \bigwedge_{i=1,2} \big((\Box\Diamond wait_i \rightarrow \Box\Diamond crit_i) \wedge (\Diamond\Box noncrit_i \rightarrow \Box\Diamond wait_i) \big)$$

LTL-fairness assumptions

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$$\mathcal{T}_{sem} \models_{fair} \Box\Diamond crit_1 \wedge \Box\Diamond crit_2$$

for appropriate fairness condition

- can be verifiable system properties

e.g., Peterson algorithm guarantees strong fairness

$$\mathcal{T}_{Pet} \models \Box\Diamond wait_1 \rightarrow \Box\Diamond crit_1$$

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  for appropriate fairness condition

- can be verifiable system properties, e.g.,

$$\mathcal{T}_{Pet} \models \Box\Diamond wait_1 \rightarrow \Box\Diamond crit_1$$

- are irrelevant for verifying safety properties

$$\mathcal{T} \models \varphi_{safe} \quad \text{iff} \quad \mathcal{T} \models_{fair} \varphi_{safe}$$

if *fair* is realizable

Correct or wrong?

Each strong **LTL** fairness assumption

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is realizable for each TS over $AP = \{a, b, \ldots\}$.

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recall: a fairness condition is called realizable if for each reachable state s there exists a fair path starting in s

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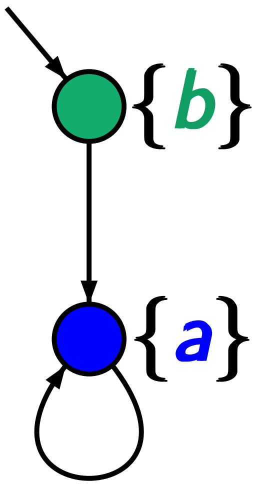
wrong



$fair = \Box\Diamond a \rightarrow \Box\Diamond b$

is not realizable

Action-based fairness $\rightsquigarrow$ LTL-fairness

Action-based fairness $\rightsquigarrow$ LTL-fairness

idea: use new atomic propositions ***enabled***(A) and ***taken***(A) and extend the labeling function:

$$enabled(A) \in L(s) \quad \text{iff} \quad s \xrightarrow{\alpha} \ldots \text{ for some } \alpha \in A$$

$$taken(A) \in L(s) \quad \text{iff} \quad \text{for all transitions } \ldots \xrightarrow{\alpha} s: \alpha \in A$$

Action-based fairness $\rightsquigarrow$ LTL-fairness

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- unconditional A-fairness: $\Box\Diamond taken(A)$
- strong A-fairness: $\Box\Diamond enabled(A) \rightarrow \Box\Diamond taken(A)$
- weak A-fairness: $\Diamond\Box enabled(A) \rightarrow \Box\Diamond taken(A)$

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$$taken(A) \in L(s) \quad \text{iff} \quad \text{for } \boxed{\text{all}} \text{ transitions } \ldots \xrightarrow{\alpha} s\text{: } \alpha \in A$$

problem: each state s can have several incoming transitions

$$t \xrightarrow{\alpha} s, \quad u \xrightarrow{\beta} s, \ \ldots$$

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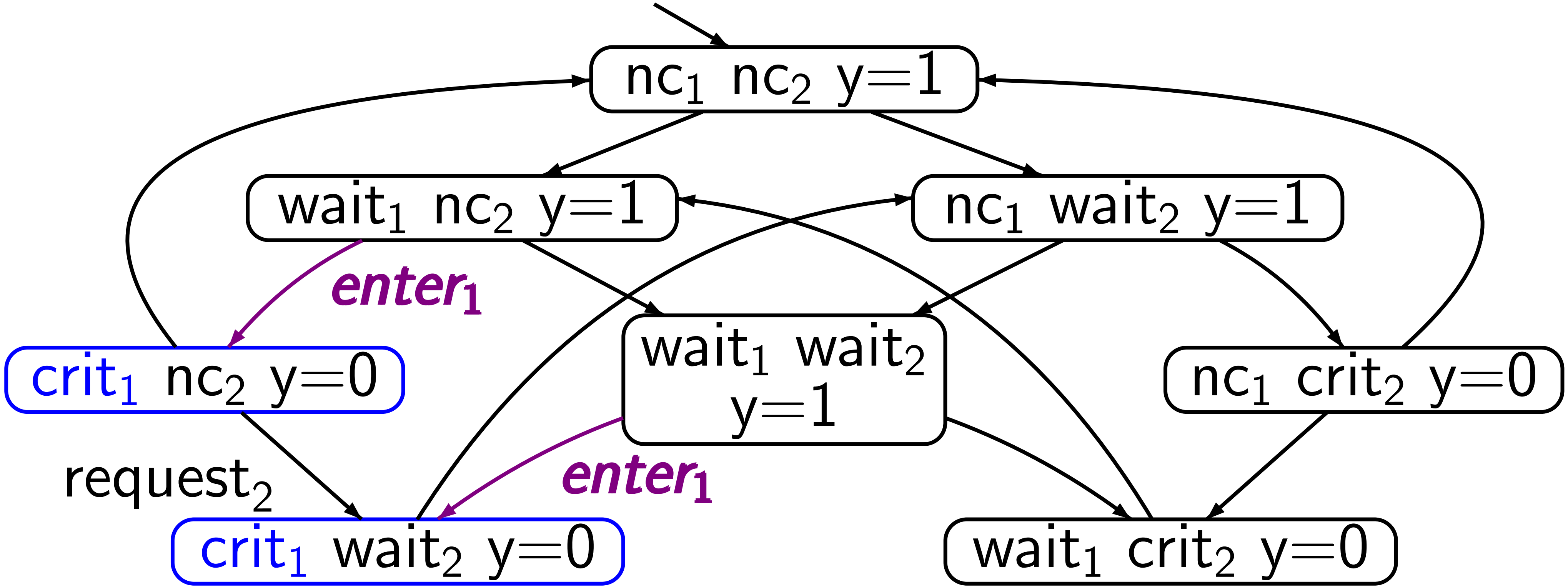
alternative 1: ad-hoc choice of "***taken***-predicate"

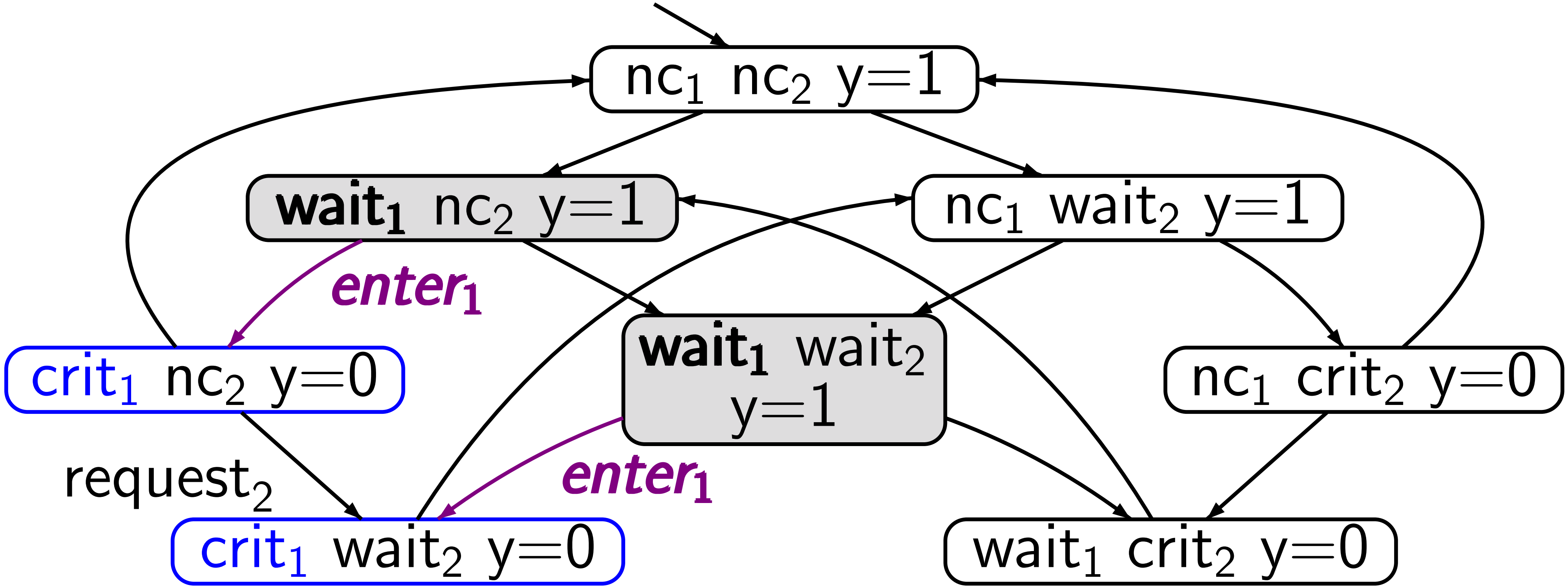
alternative 2: modify the given transition system by adding an action component to the states

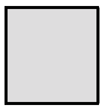
Ad-hoc: action fairness ⇝ LTL-fairness



TS for mutual exclusion with semaphore

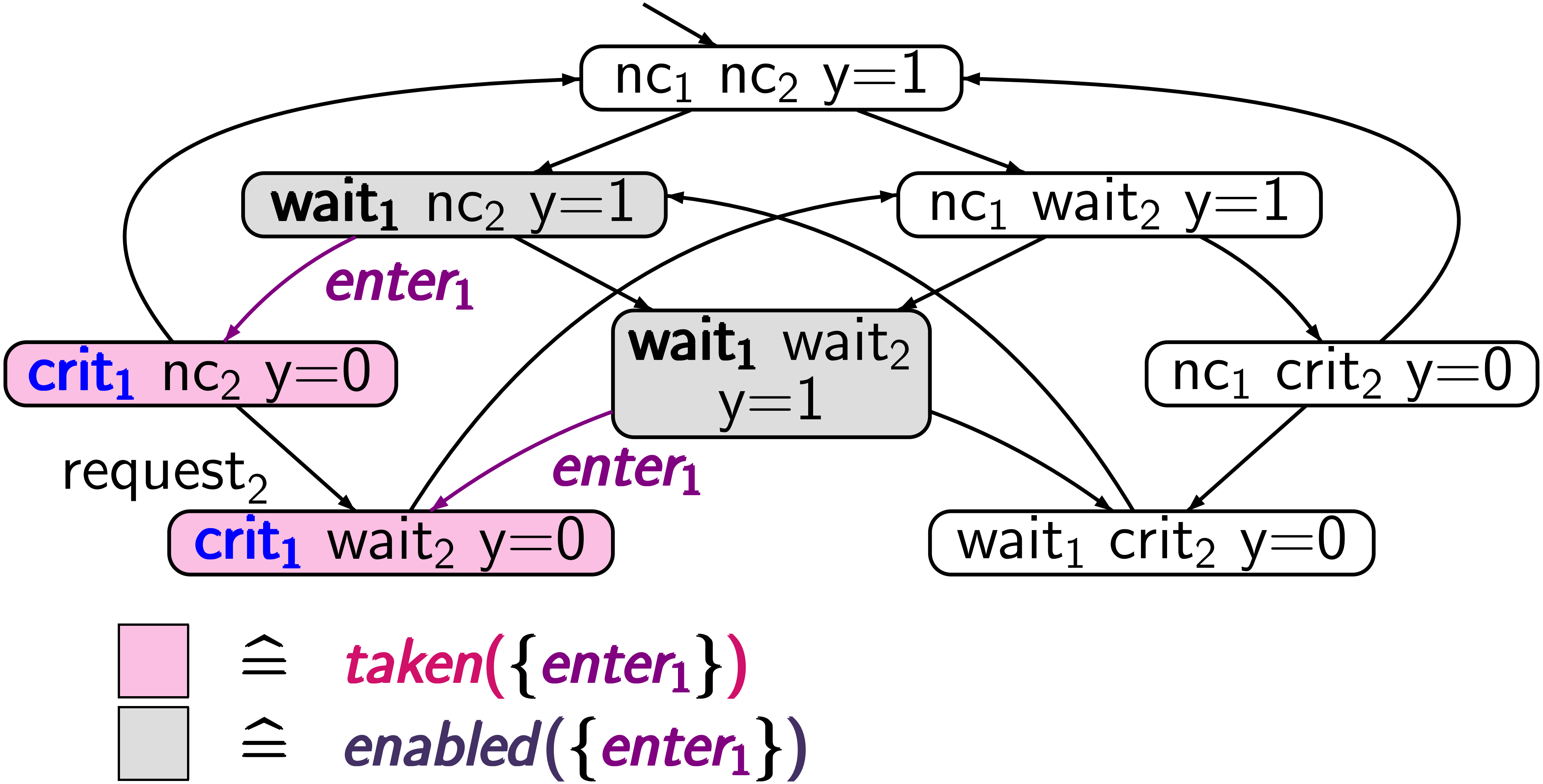
Ad-hoc: action fairness ⇝ LTL-fairness



 $\widehat{=}$ *enabled*({*enter*$_1$})

TS for mutual exclusion with semaphore

Ad-hoc: action fairness ⇝ LTL-fairness



TS for mutual exclusion with semaphore

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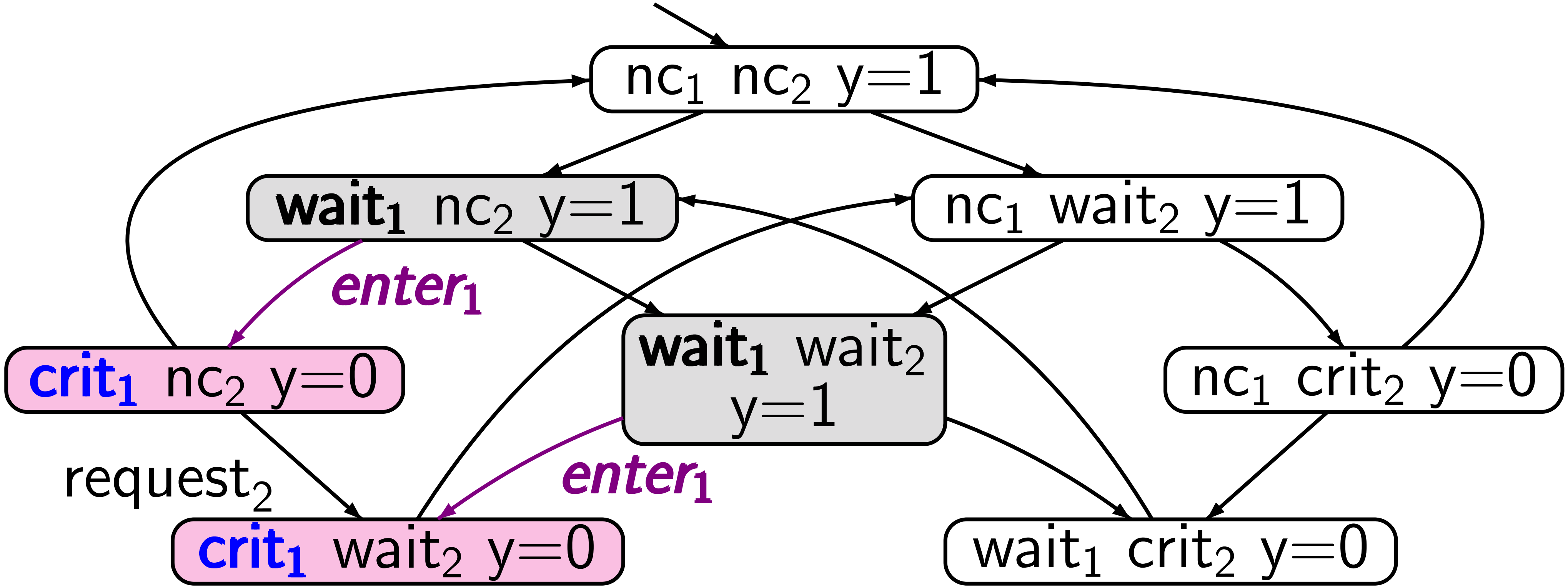
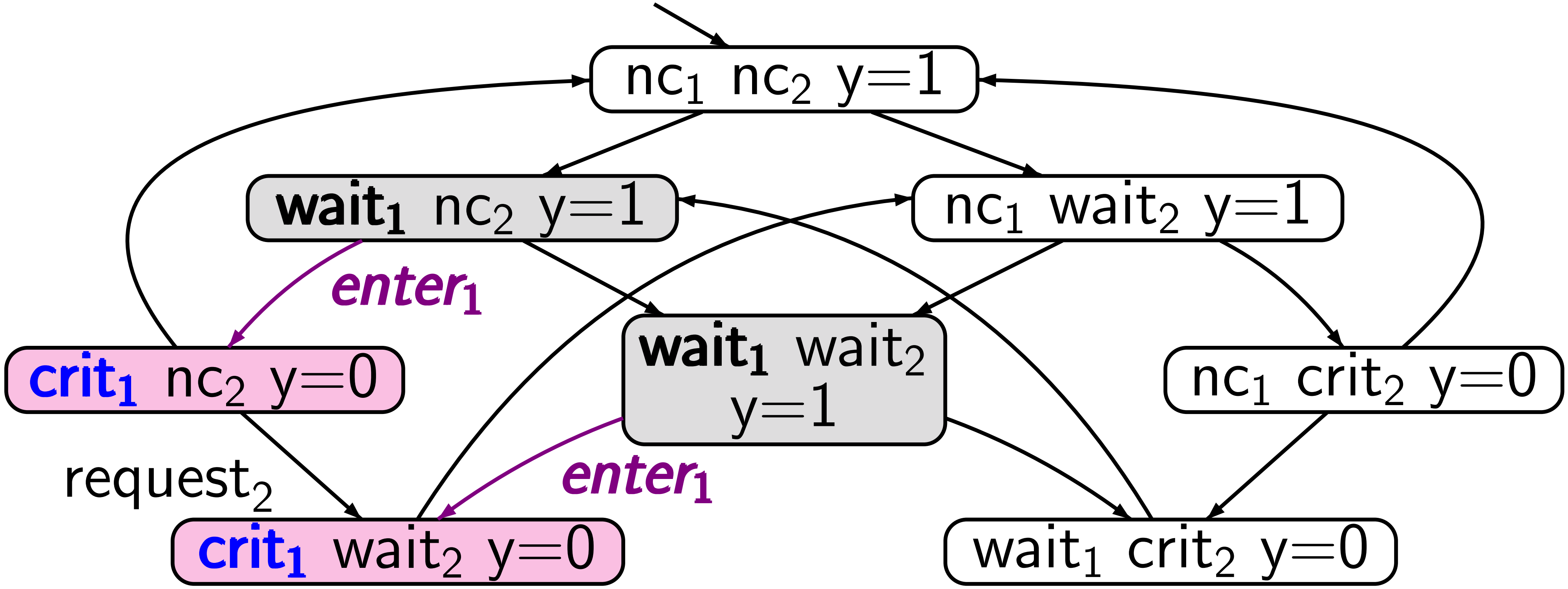


(pink)	$\widehat{=}$	*taken*({*enter*$_1$})	$\widehat{=}$	**crit$_1$**
(grey)	$\widehat{=}$	*enabled*({*enter*$_1$})	$\widehat{=}$	**wait$_1$ $\wedge$ $\neg$crit$_2$**

Ad-hoc: action fairness ⇝ LTL-fairness

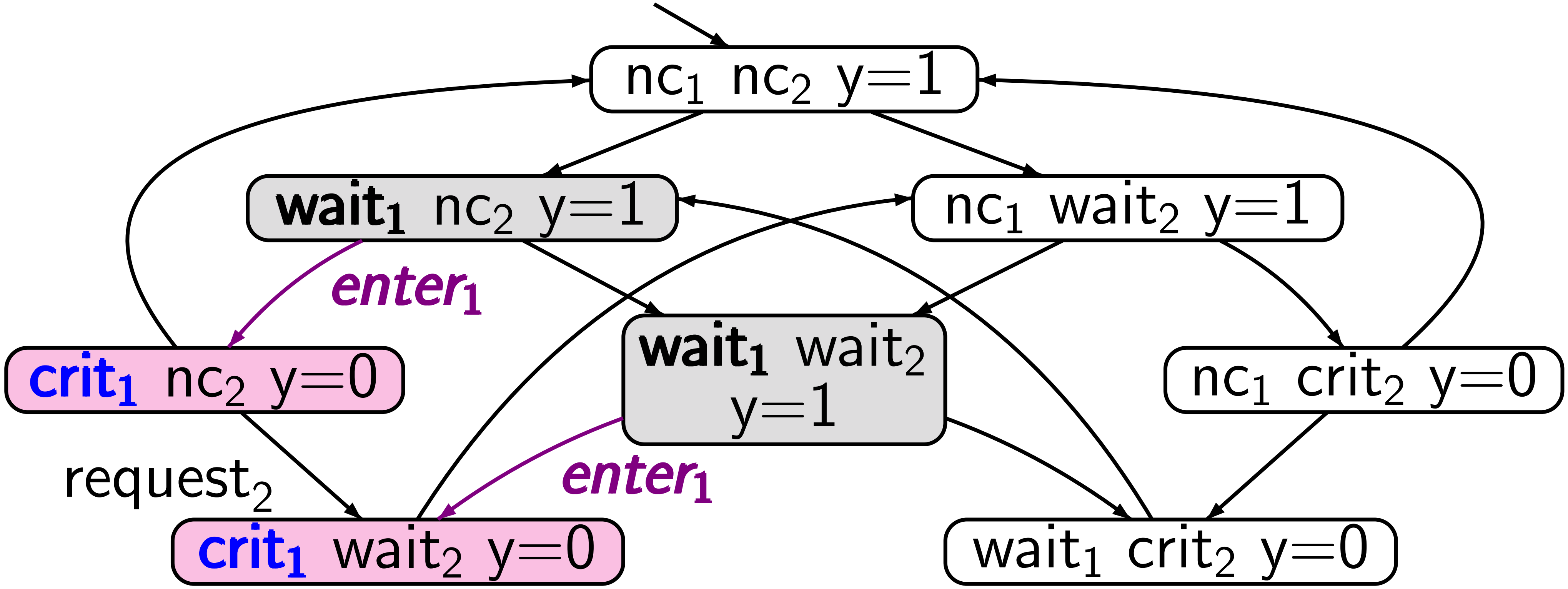


(pink)	$\widehat{=}$	*taken*({*enter*₁})	$\widehat{=}$	**crit₁**
(grey)	$\widehat{=}$	*enabled*({*enter*₁})	$\widehat{=}$	$\mathbf{wait_1} \wedge \neg \mathbf{crit_2}$

strong {*enter*$_1$}-fairness: LTL formula

$$\Box\Diamond\, \mathit{enabled}(\{\mathit{enter}_1\}) \rightarrow \Box\Diamond\, \mathit{taken}(\{\mathit{enter}_1\})$$

Ad-hoc: action fairness ⇝ LTL-fairness



(pink) $\widehat{=}$ *taken*({*enter*$_1$}) $\widehat{=}$ **crit$_1$**

(grey) $\widehat{=}$ *enabled*({*enter*$_1$}) $\widehat{=}$ **wait$_1$ $\wedge$ ¬crit$_2$**

$$\Box\Diamond\, enabled(\{enter_1\}) \rightarrow \Box\Diamond\, taken(\{enter_1\})$$

$$\widehat{=}\ \Box\Diamond\, (wait_1 \wedge \neg crit_2) \rightarrow \Box\Diamond\, crit_1$$

Action-based fairness $\leadsto$ LTL-fairness

idea: use new atomic propositions ***enabled***(A) and ***taken***(A) and extend the labeling function:

$$enabled(A) \in L(s) \quad \text{iff} \quad s \xrightarrow{\alpha} \ldots \text{ for some } \alpha \in A$$

$$taken(A) \in L(s) \quad \text{iff} \quad \text{for all transitions } \ldots \xrightarrow{\alpha} s\text{:} \quad \alpha \in A$$

alternative 1: **ad-hoc choice** of "***taken***-predicate"

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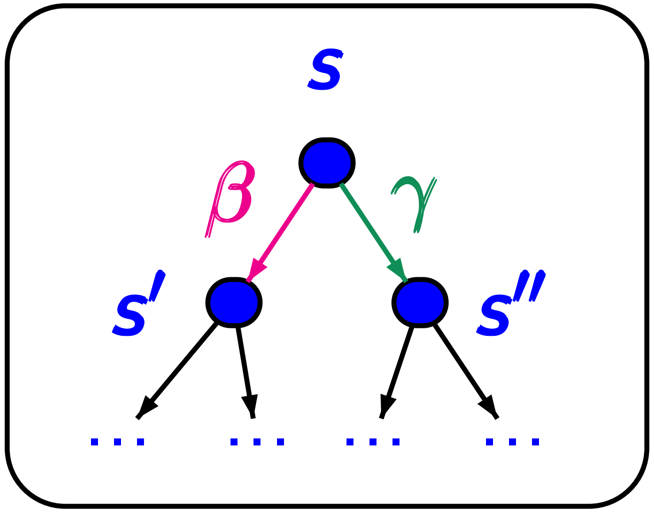
$\mathcal{T} = (S, Act, \rightarrow, \ldots)$



Action-based fairness ⇝ LTL-fairness

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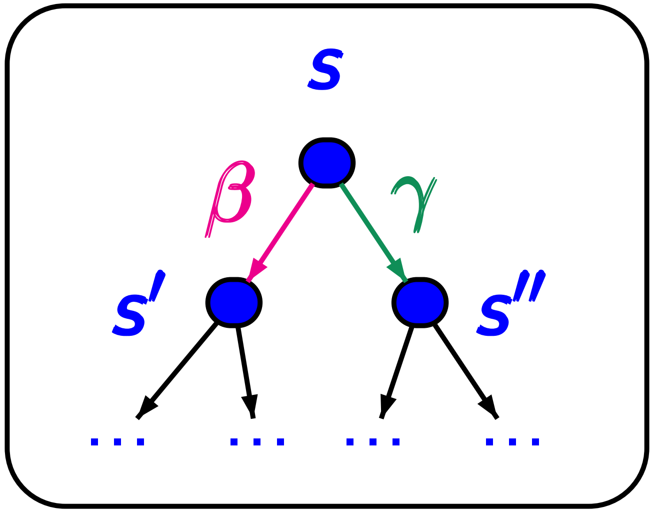

transition system
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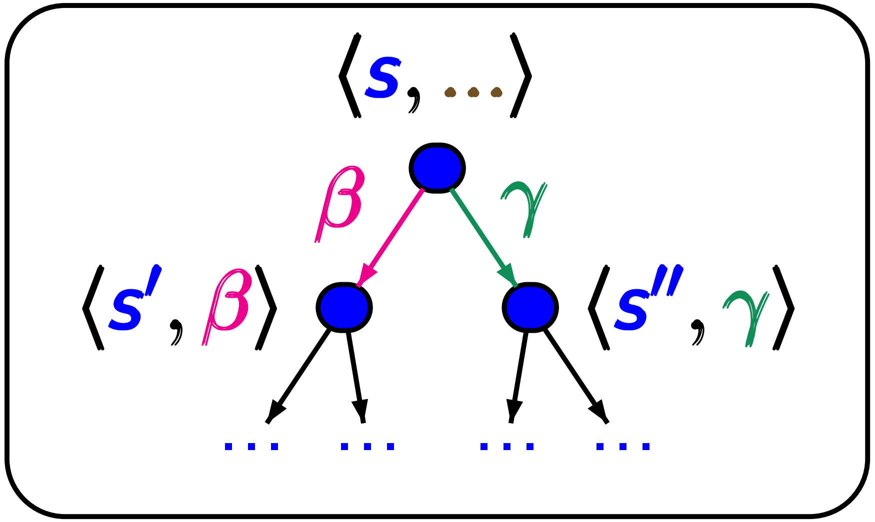

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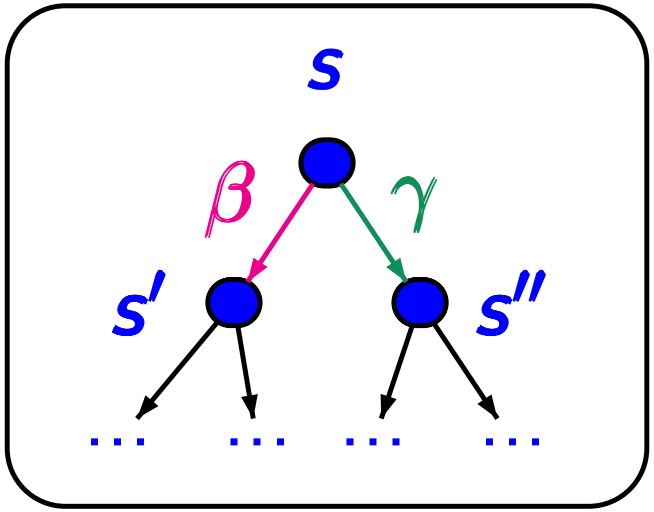

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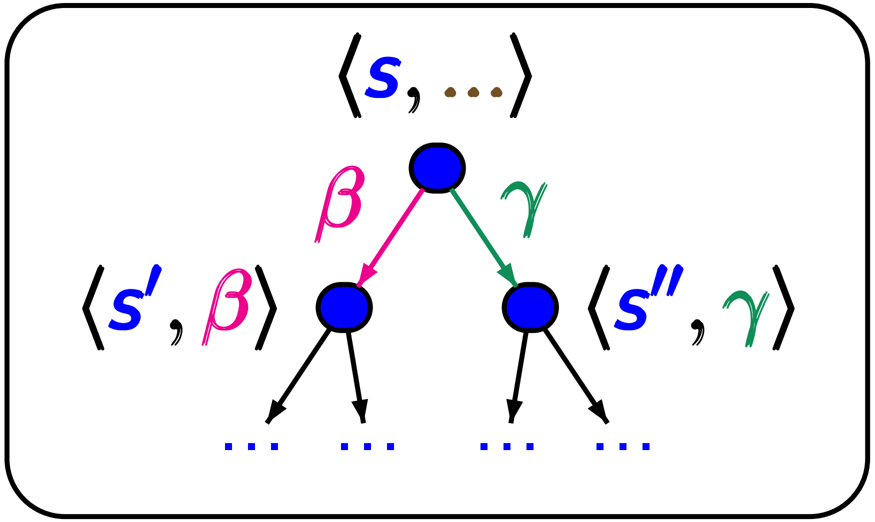

strong A-fairness
for $A \subseteq Act$

strong **LTL**-fairness
$\Box\Diamond enabled(A) \rightarrow \Box\Diamond taken(A)$

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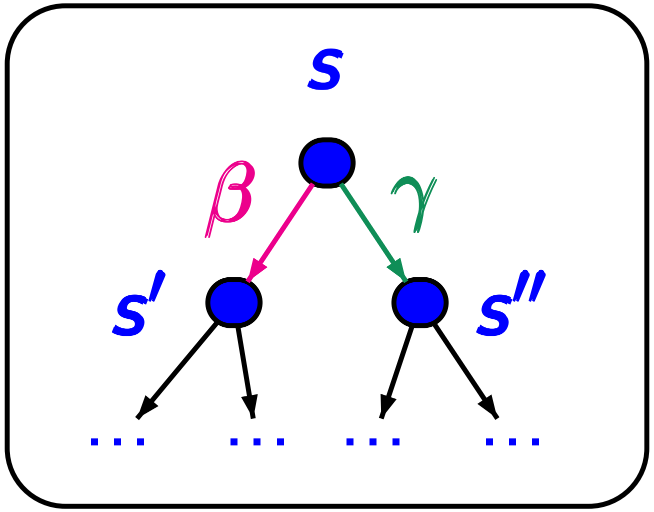

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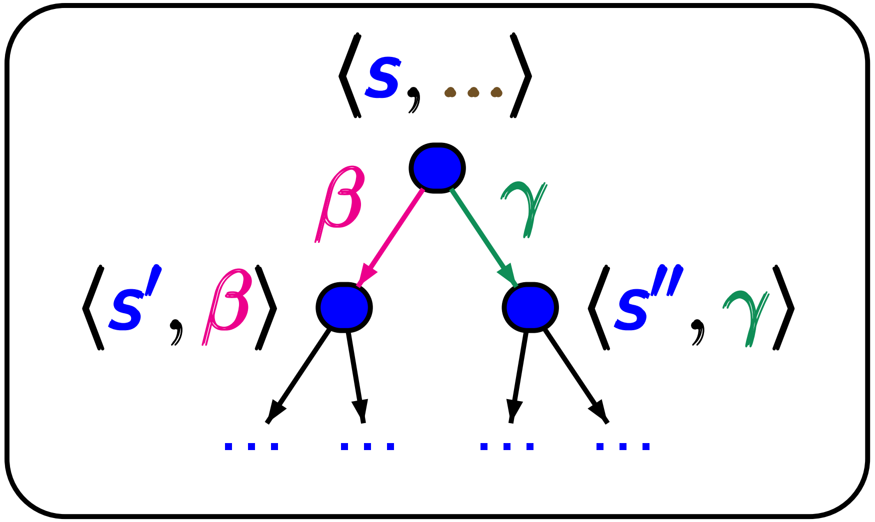

strong A-fairness
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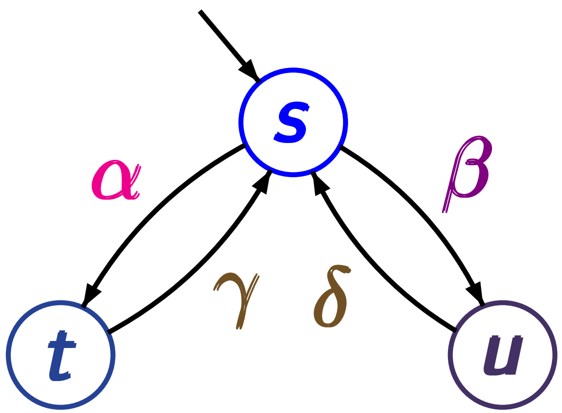
strong **LTL**-fairness
$\Box\Diamond enabled(A) \rightarrow \Box\Diamond taken(A)$

$enabled(A) \in L'(\langle s, \alpha \rangle)$ iff $s \xrightarrow{\beta} \ldots$ for some $\beta \in A$

$taken(A) \in L'(\langle s, \alpha \rangle)$ iff $\alpha \in A$

Example: action fairness $\rightsquigarrow$ LTL-fairness

action-based fairness $\rightsquigarrow$ LTL-fairness



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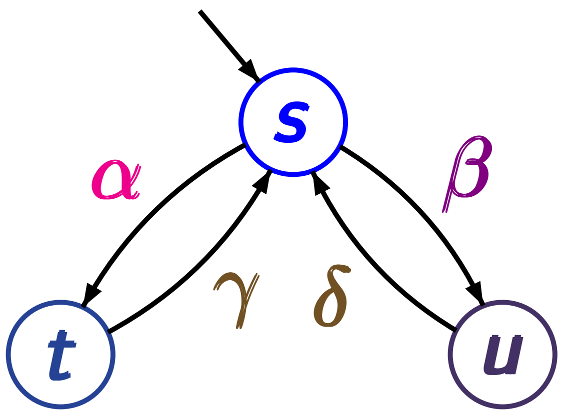

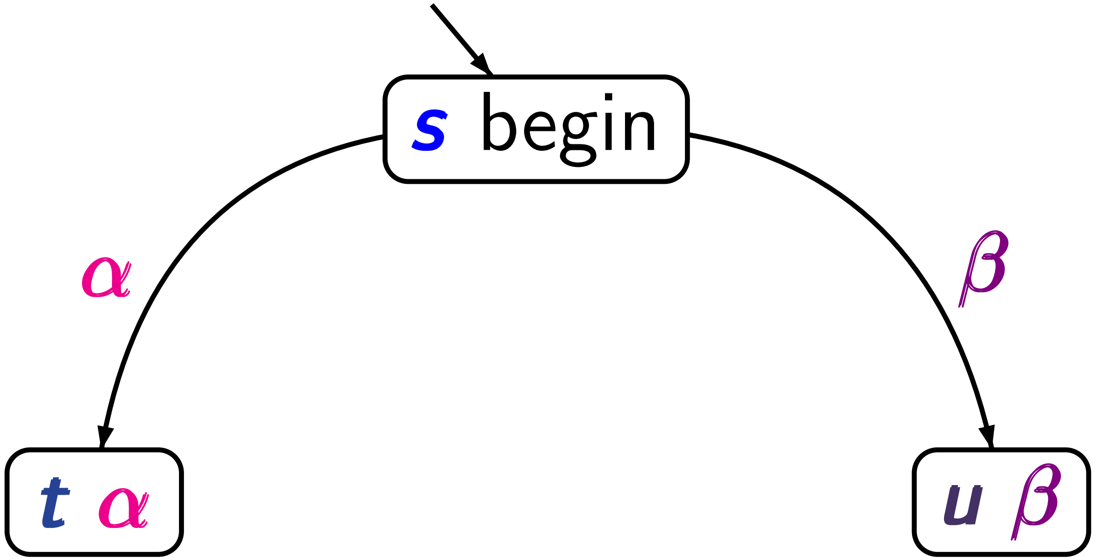

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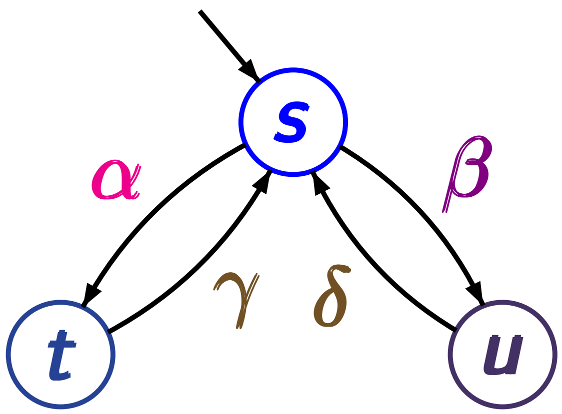

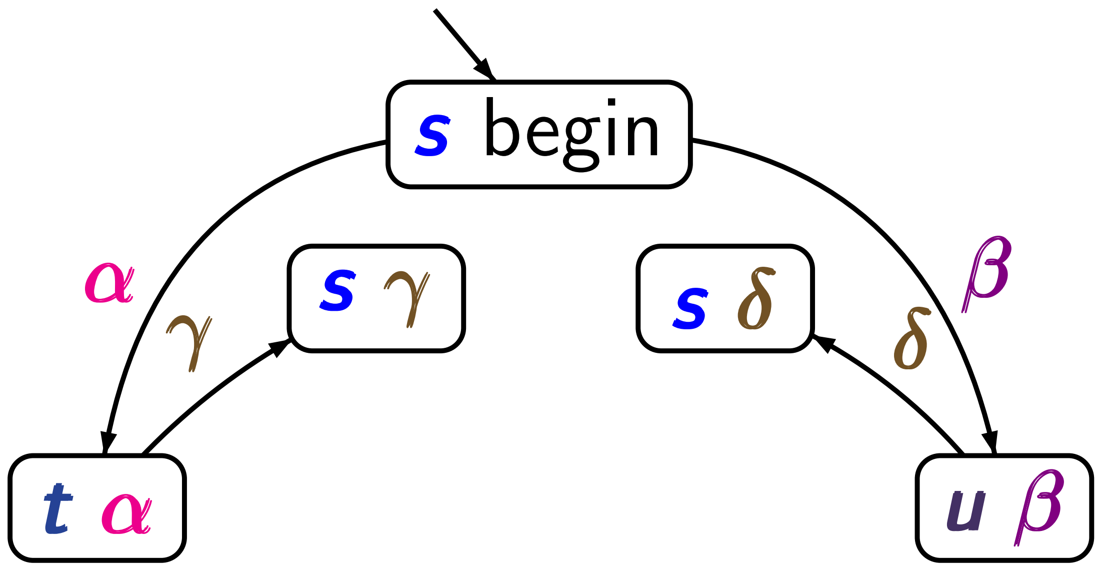

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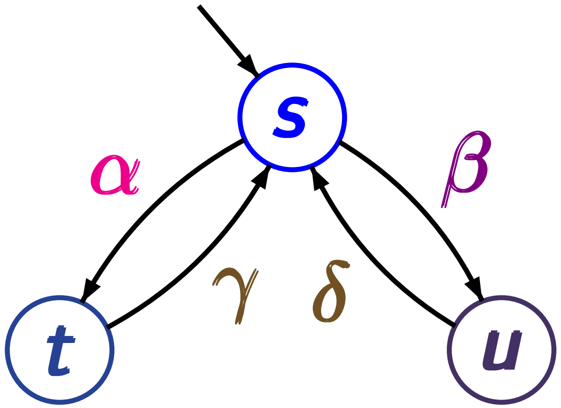

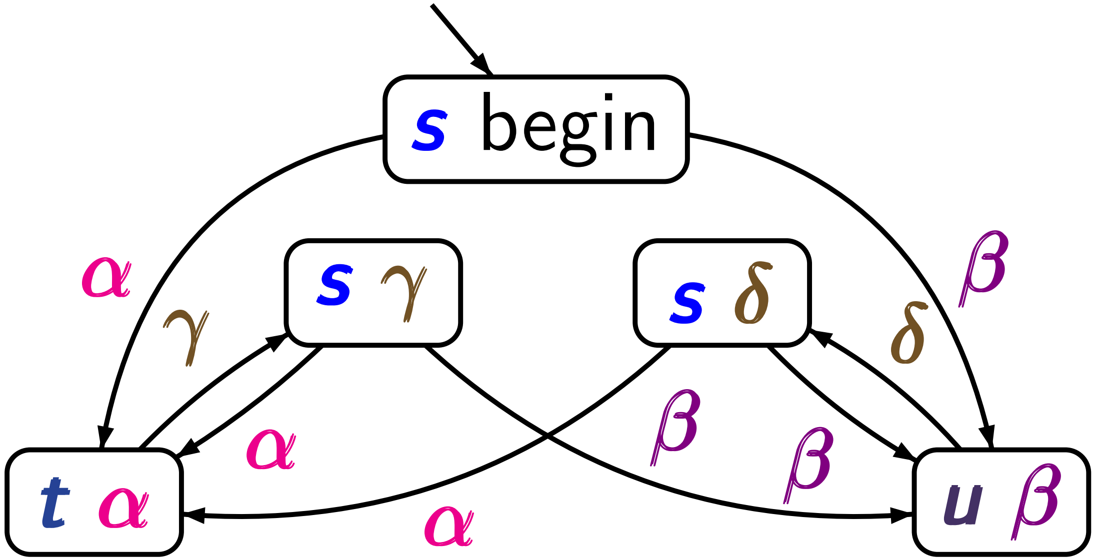

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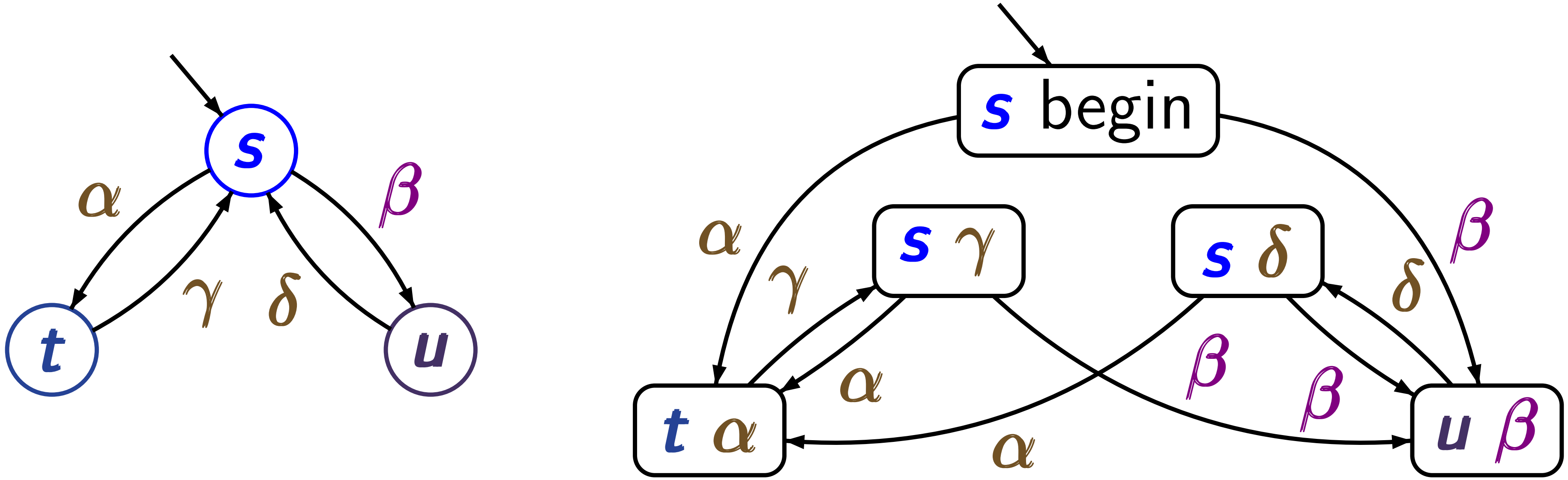
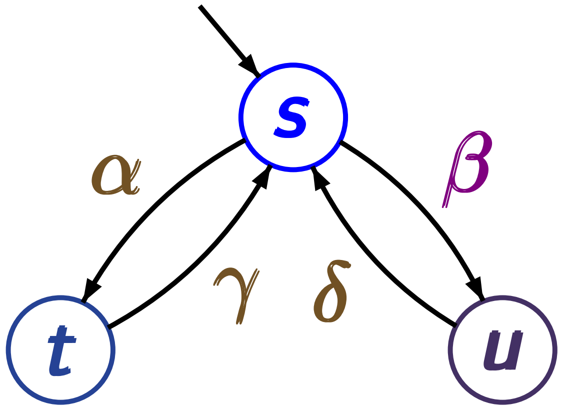
action-based fairness ⇝ LTL-fairness

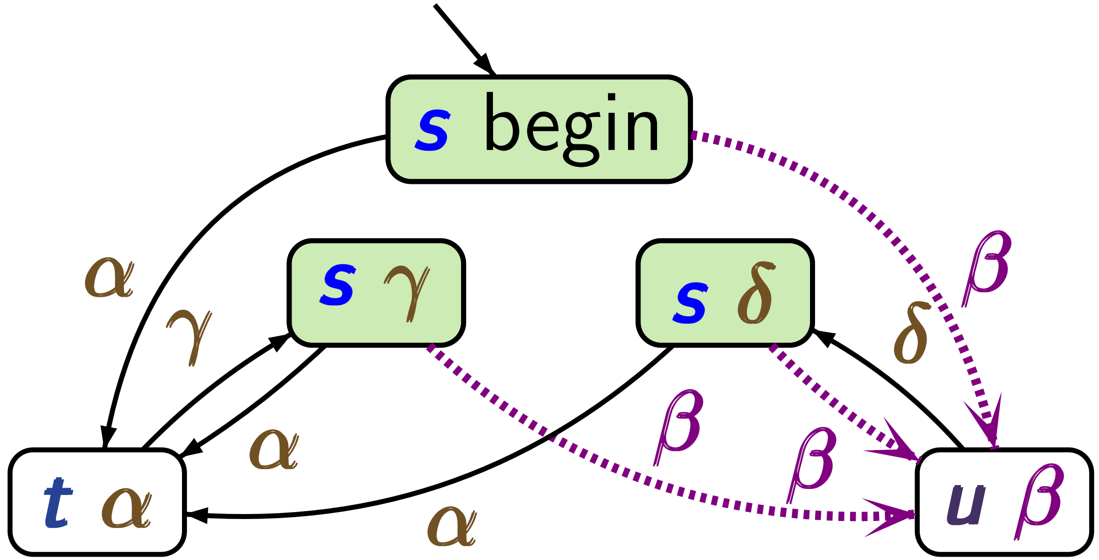


strong fairness for $\{\beta\}$:

$$\square\lozenge\, \textit{enabled}(\beta) \rightarrow \square\lozenge\, \textit{taken}(\beta)$$

action-based fairness ⇝ LTL-fairness





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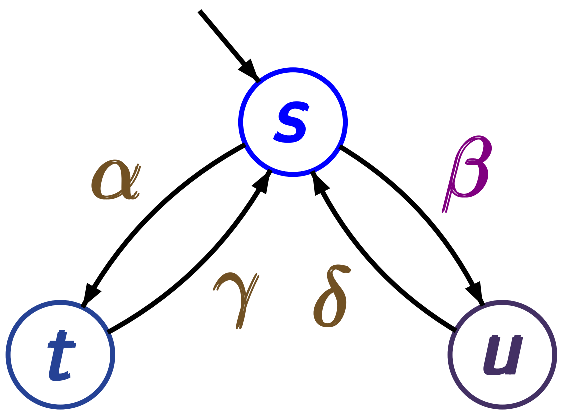

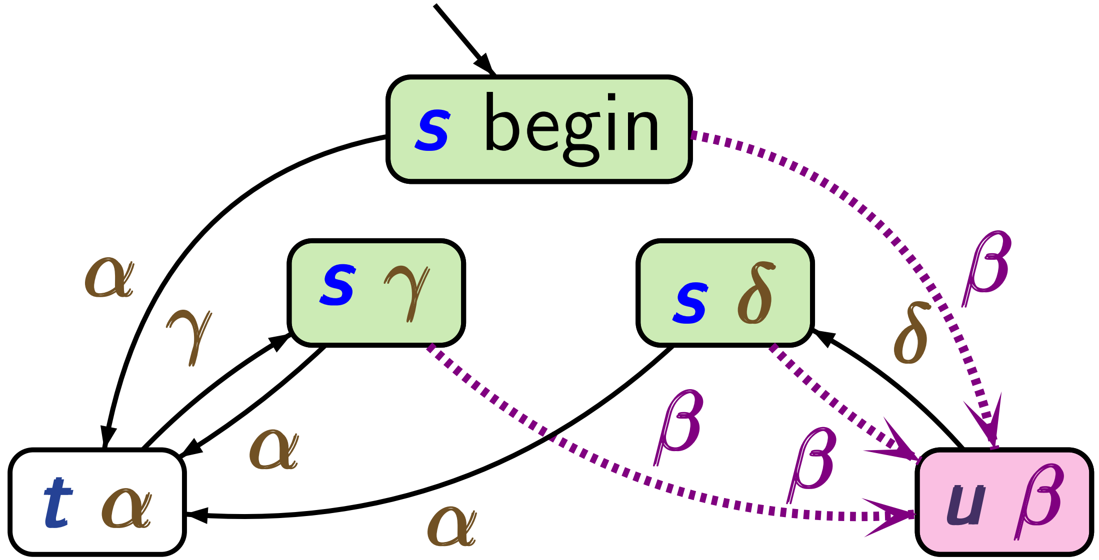

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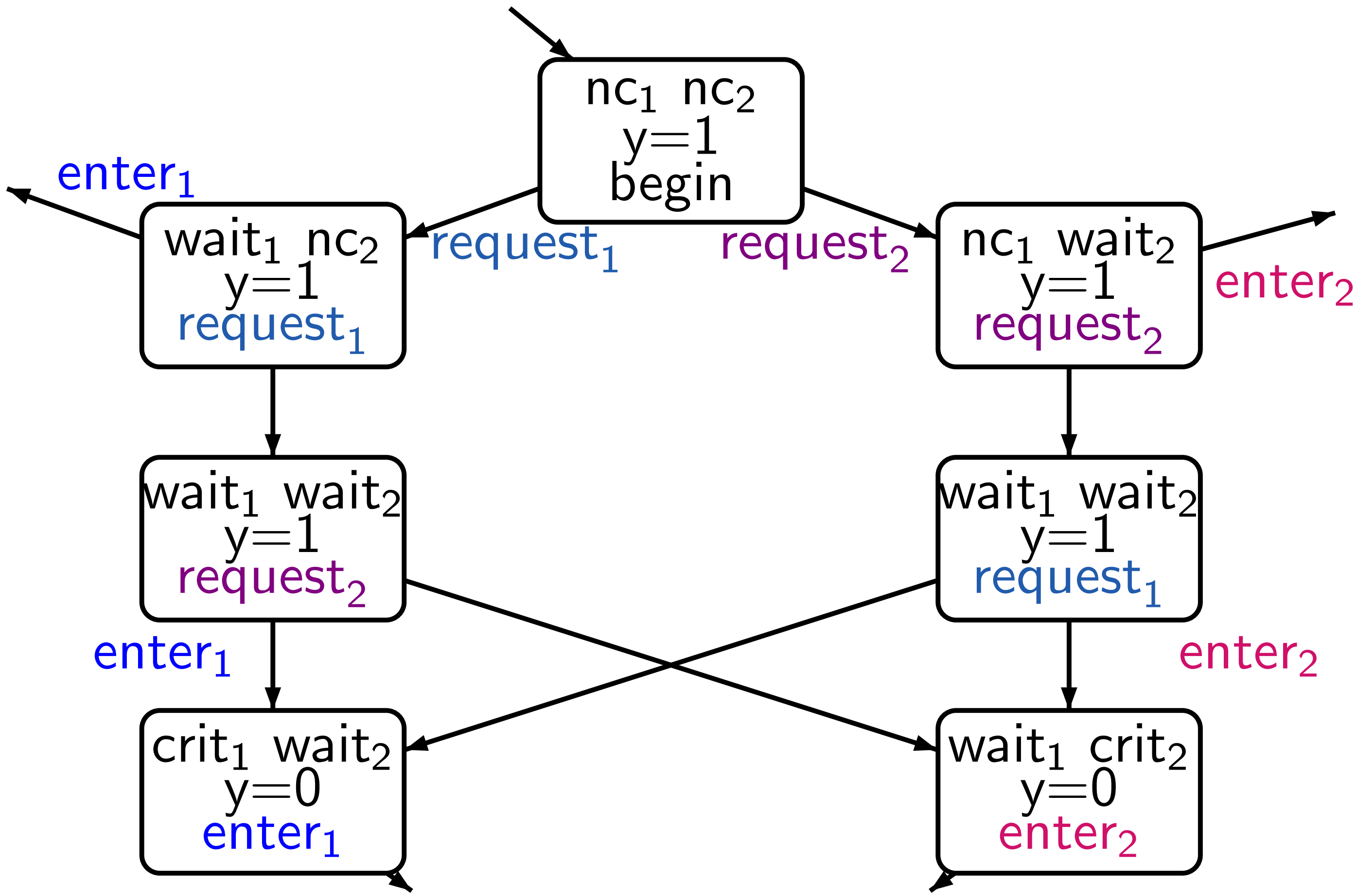
Example: mutual exclusion with semaphore

add additional variable last_action with domain Act $\cup$ {begin}



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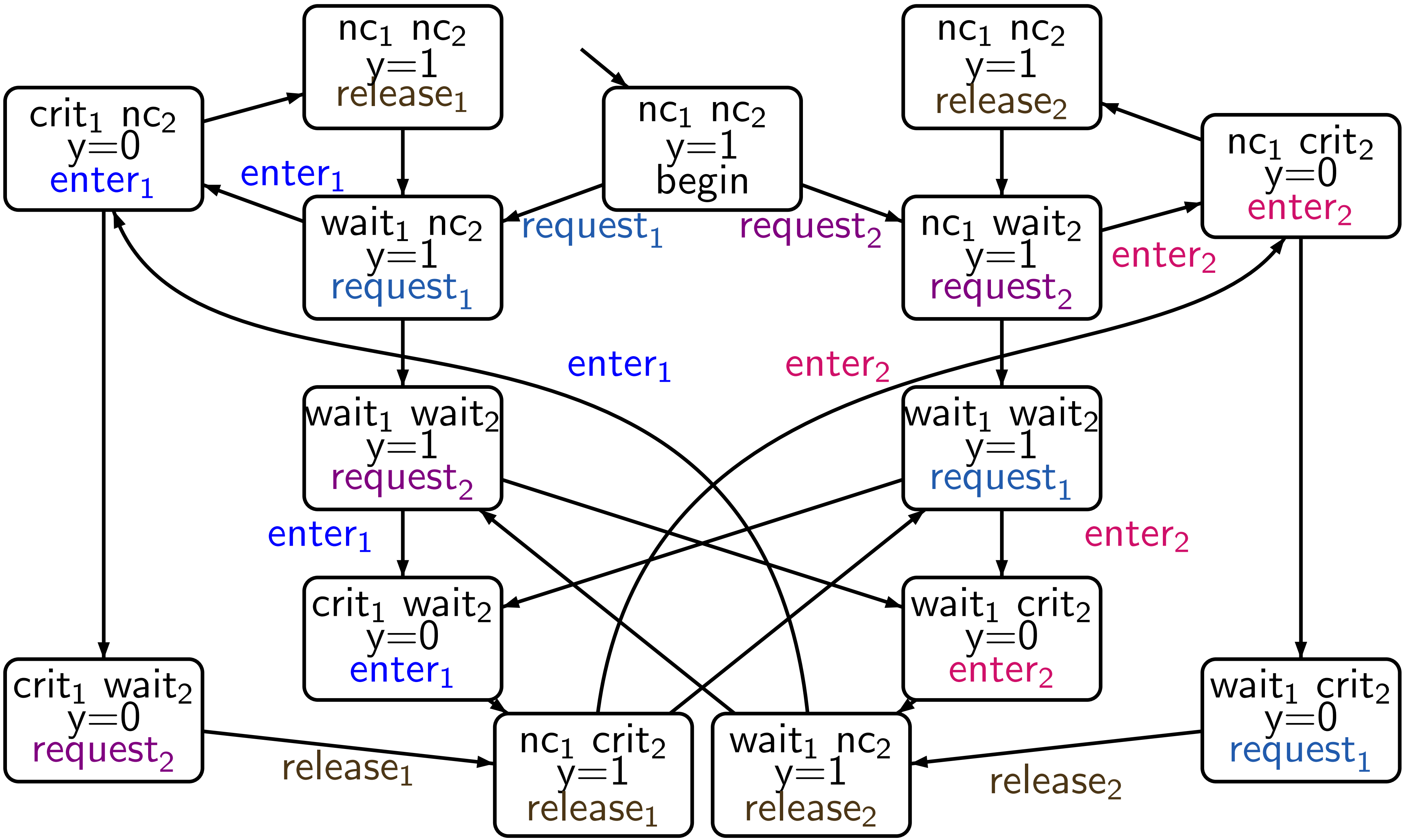

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