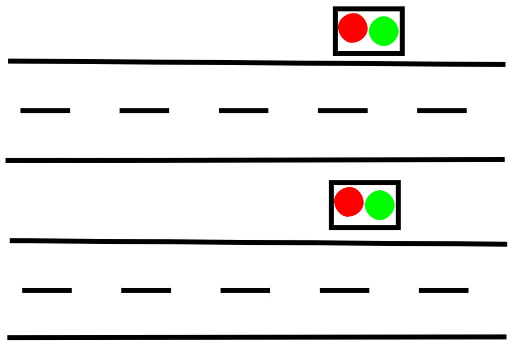
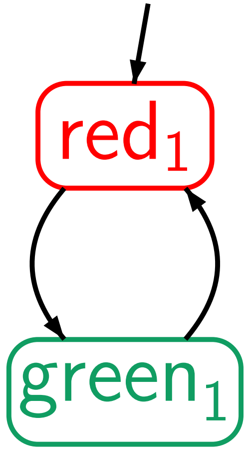
Observation

liveness properties are often violated
although we expect them to hold

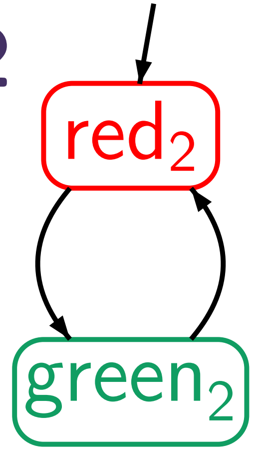
Two independent traffic lights



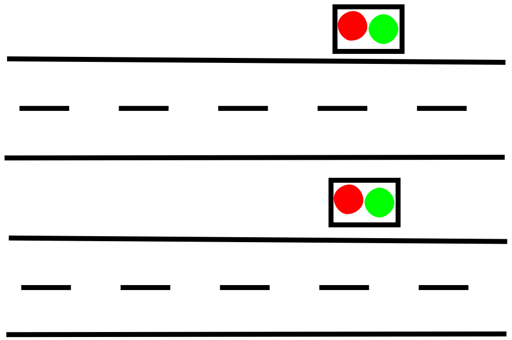
light 1



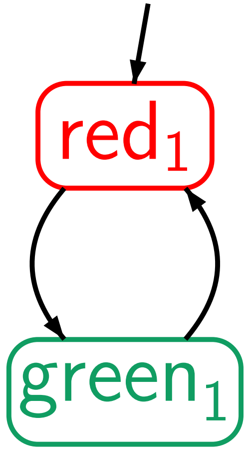
light 2



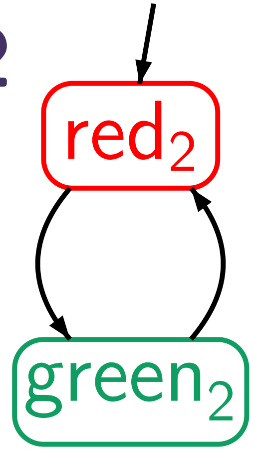
Two independent traffic lights



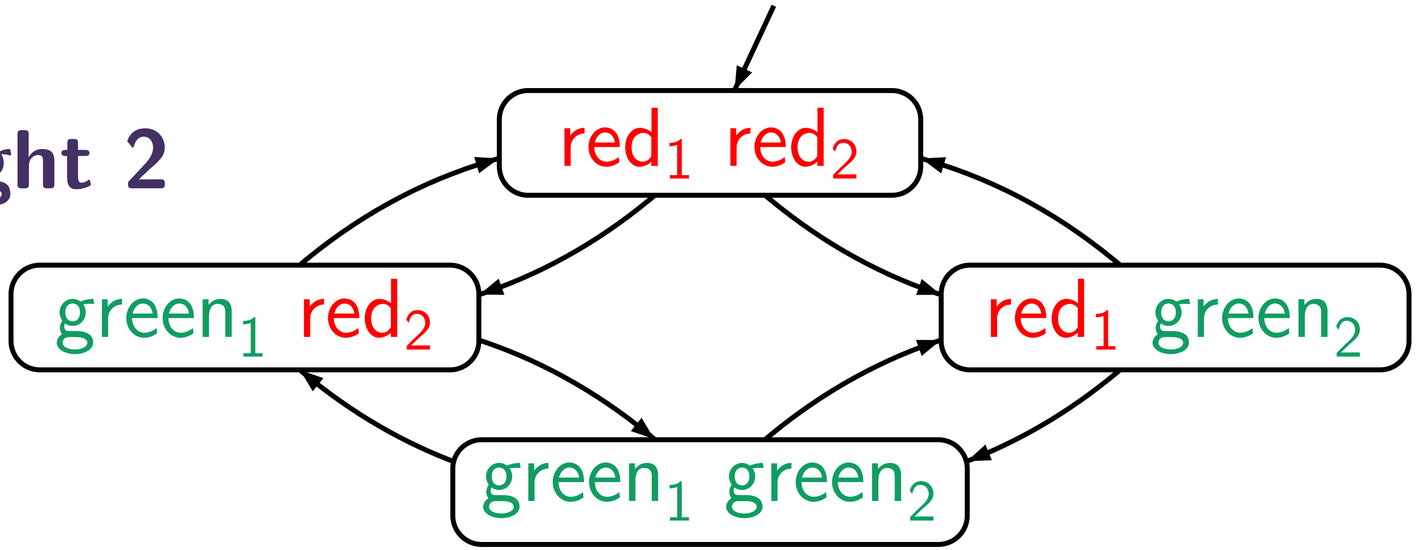
light 1



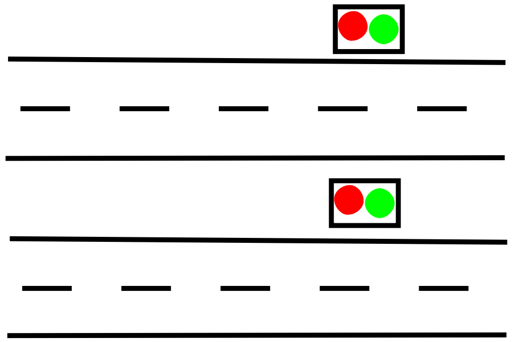
light 2



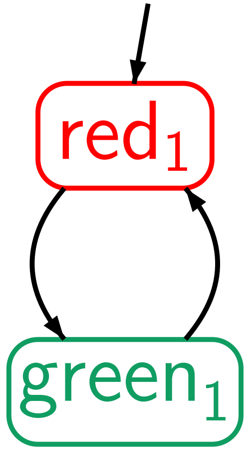
light 1 ||| light 2



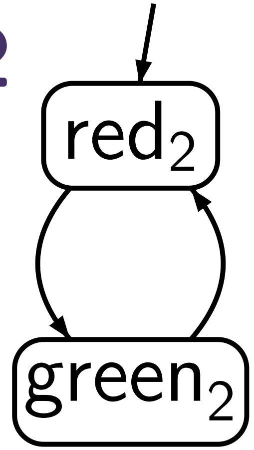
Two independent traffic lights



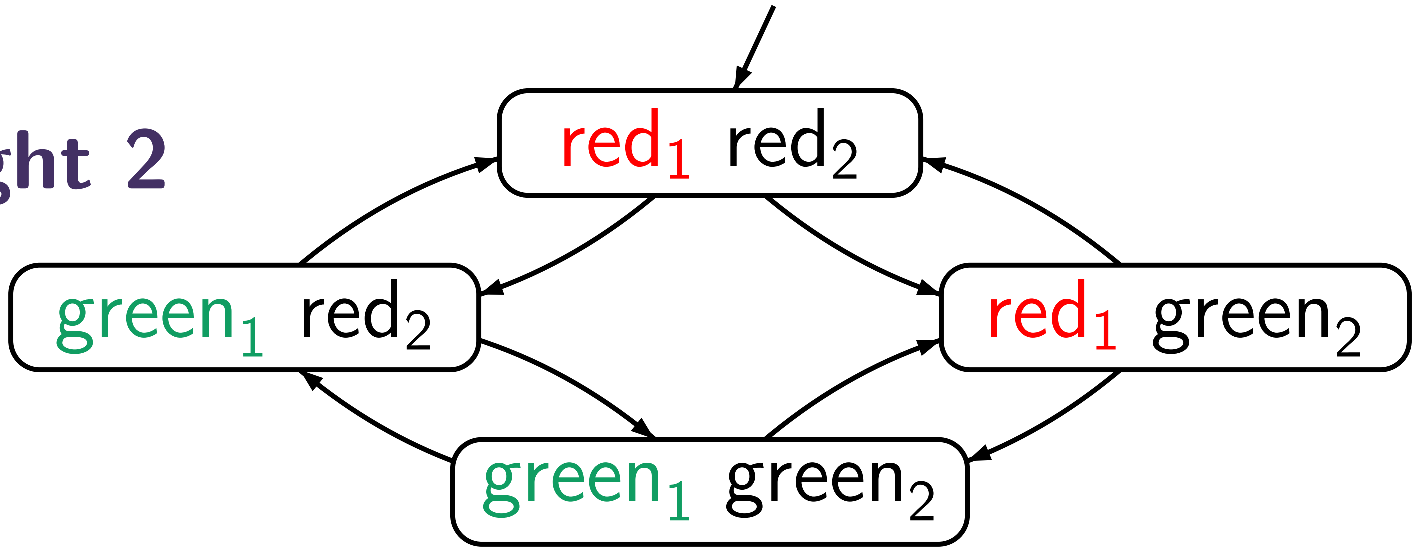
light 1



light 2

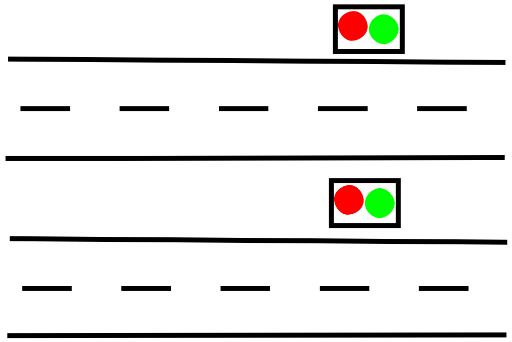


light 1 ||| light 2

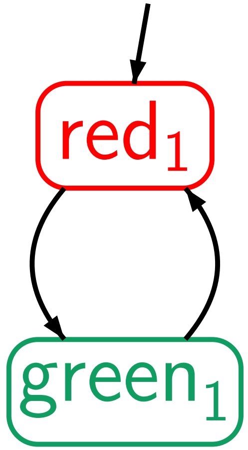


light 1 ||| light 2 $\not\models$ "infinitely often ***$green_1$***"

Two independent traffic lights



light 1



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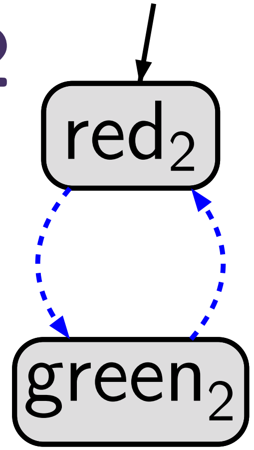
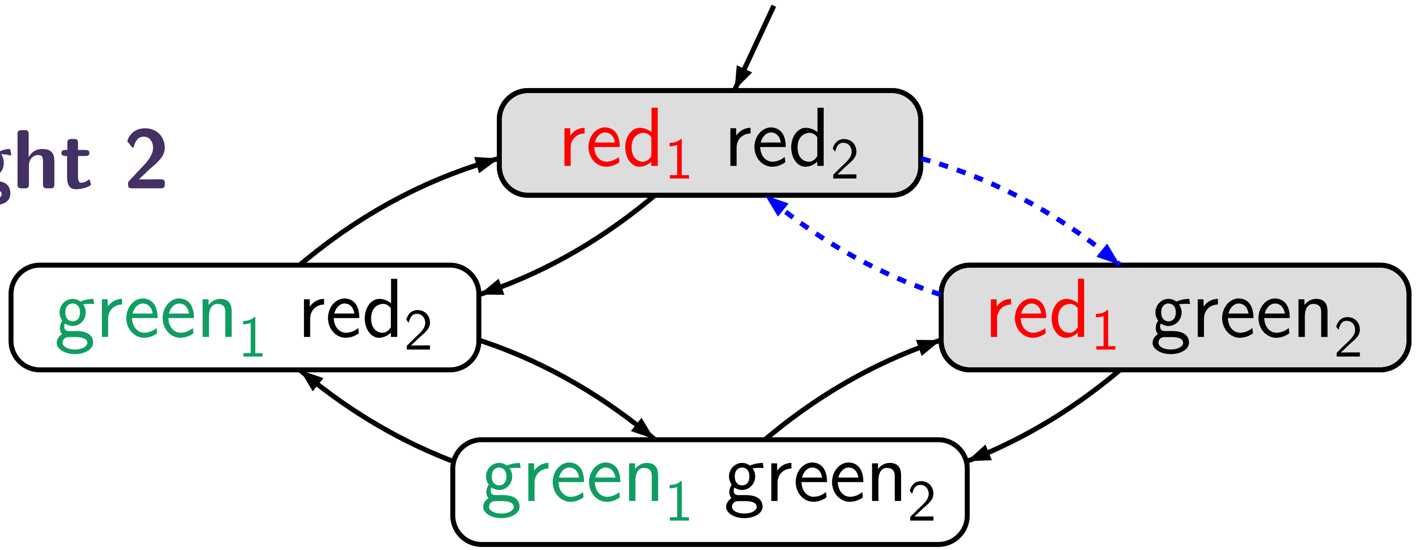
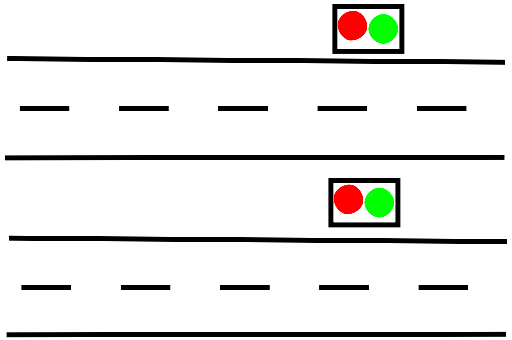


light 1 ||| light 2

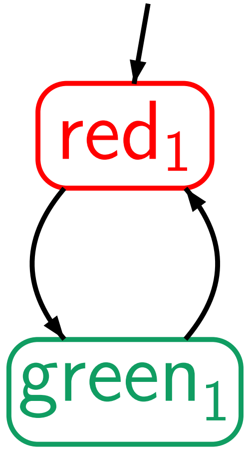


light 1 ||| light 2 $\not\models$ "infinitely often ***green***$_1$"

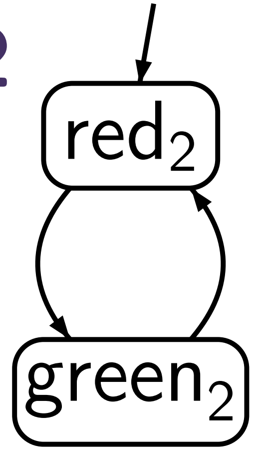
Two independent traffic lights



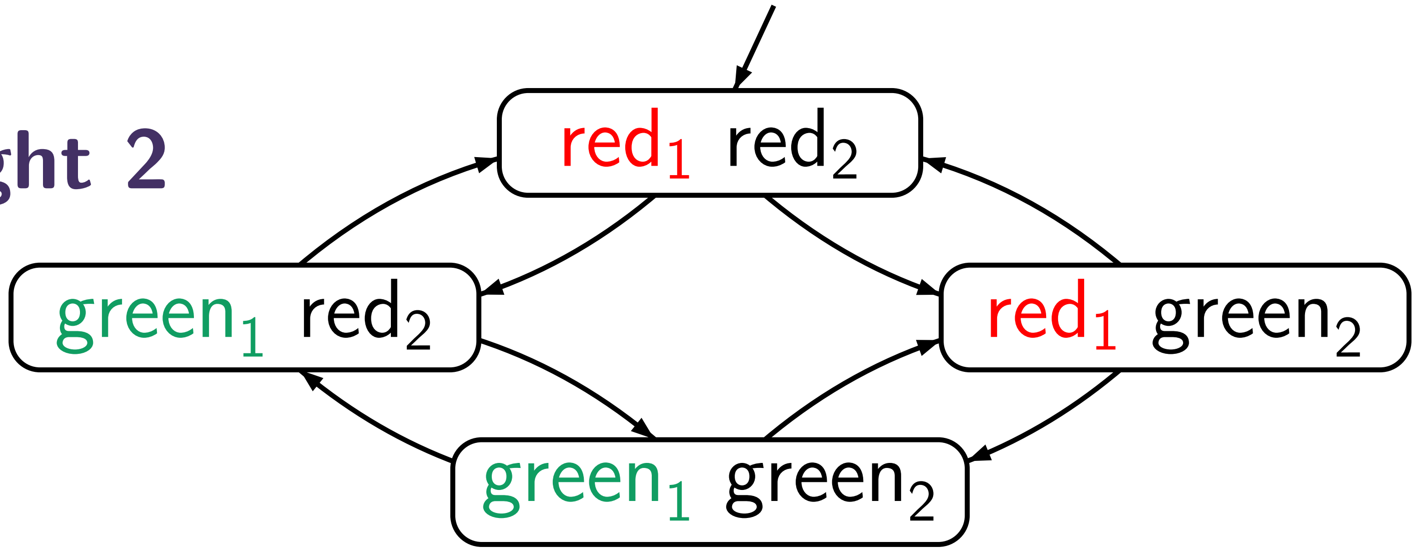
light 1



light 2



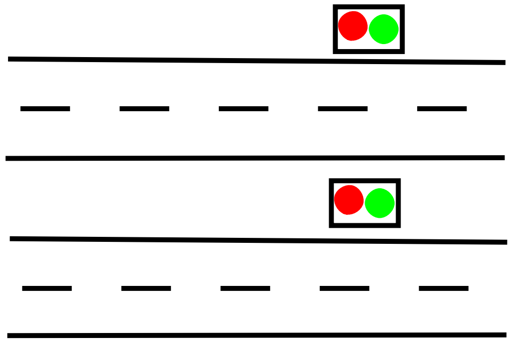
light 1 ||| light 2



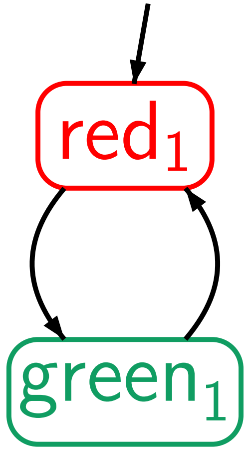
light 1 ||| light 2 $\not\models$ "infinitely often ***green***$_1$"

although **light 1** $\models$ "infinitely often ***green***$_1$"

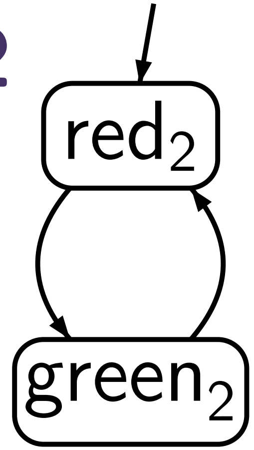
Two independent traffic lights



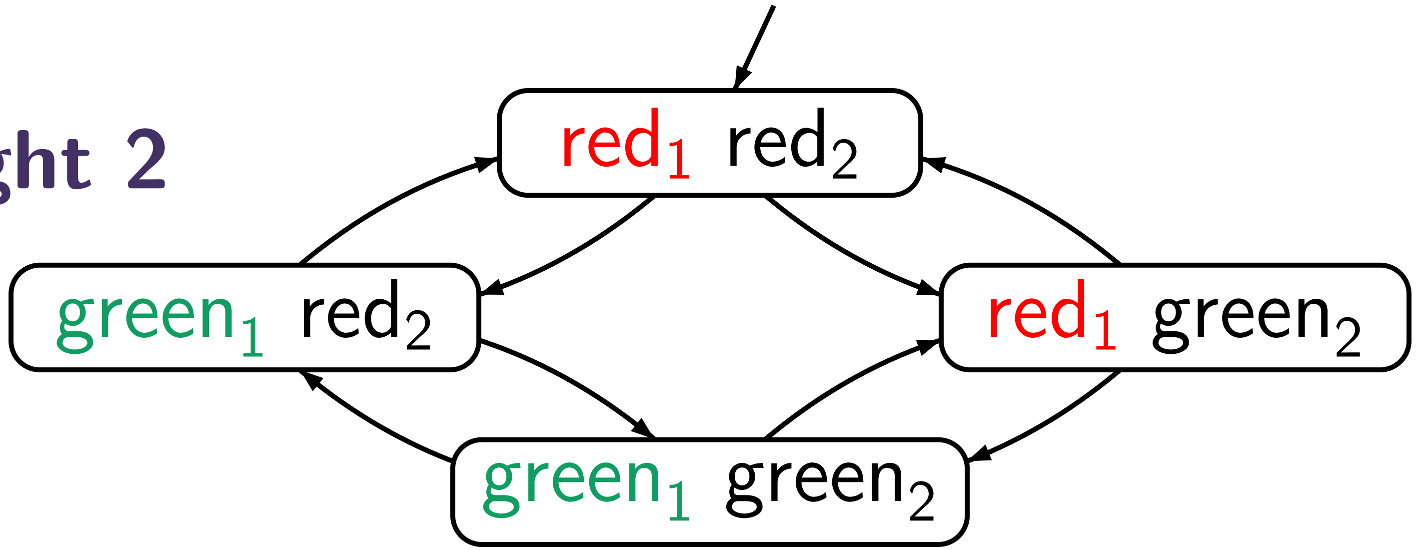
light 1



light 2



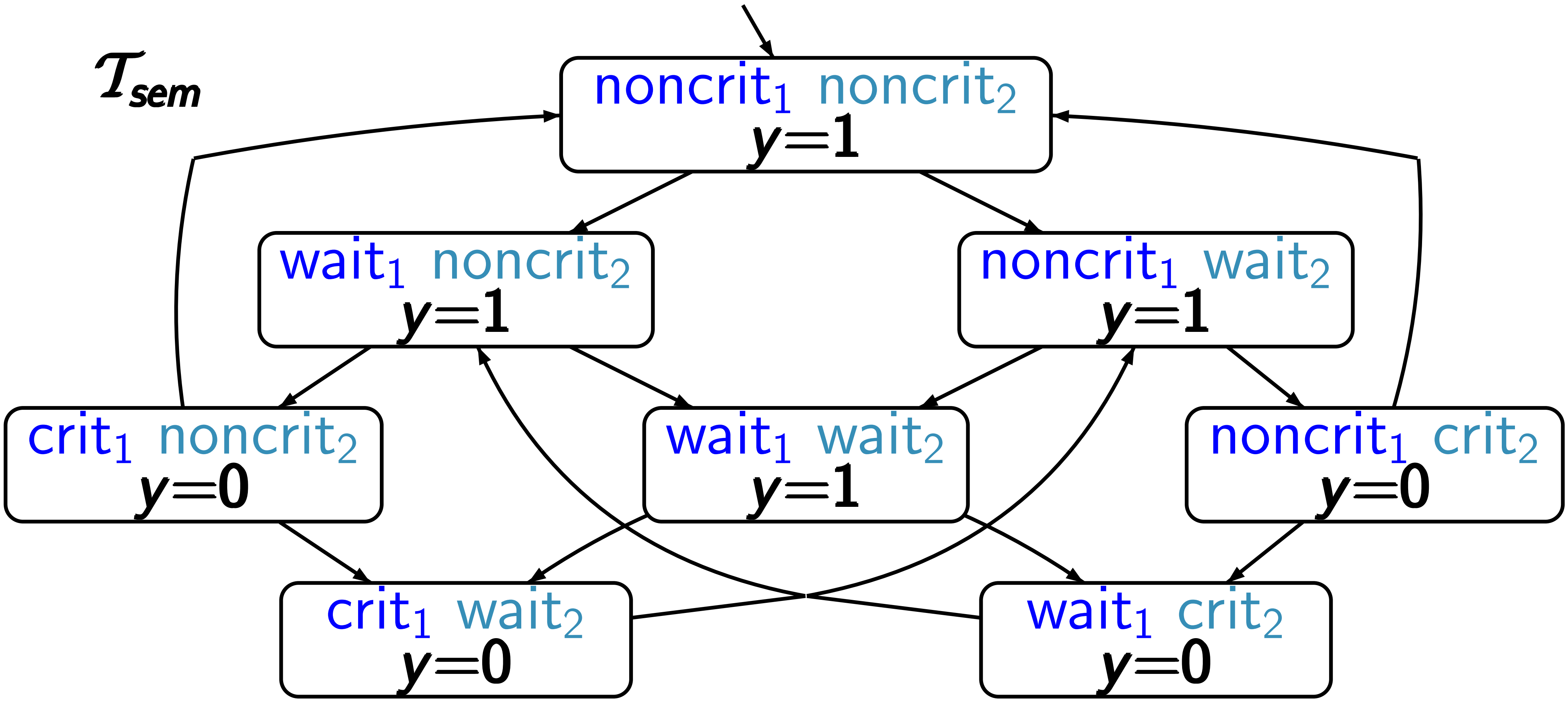
light 1 ||| light 2



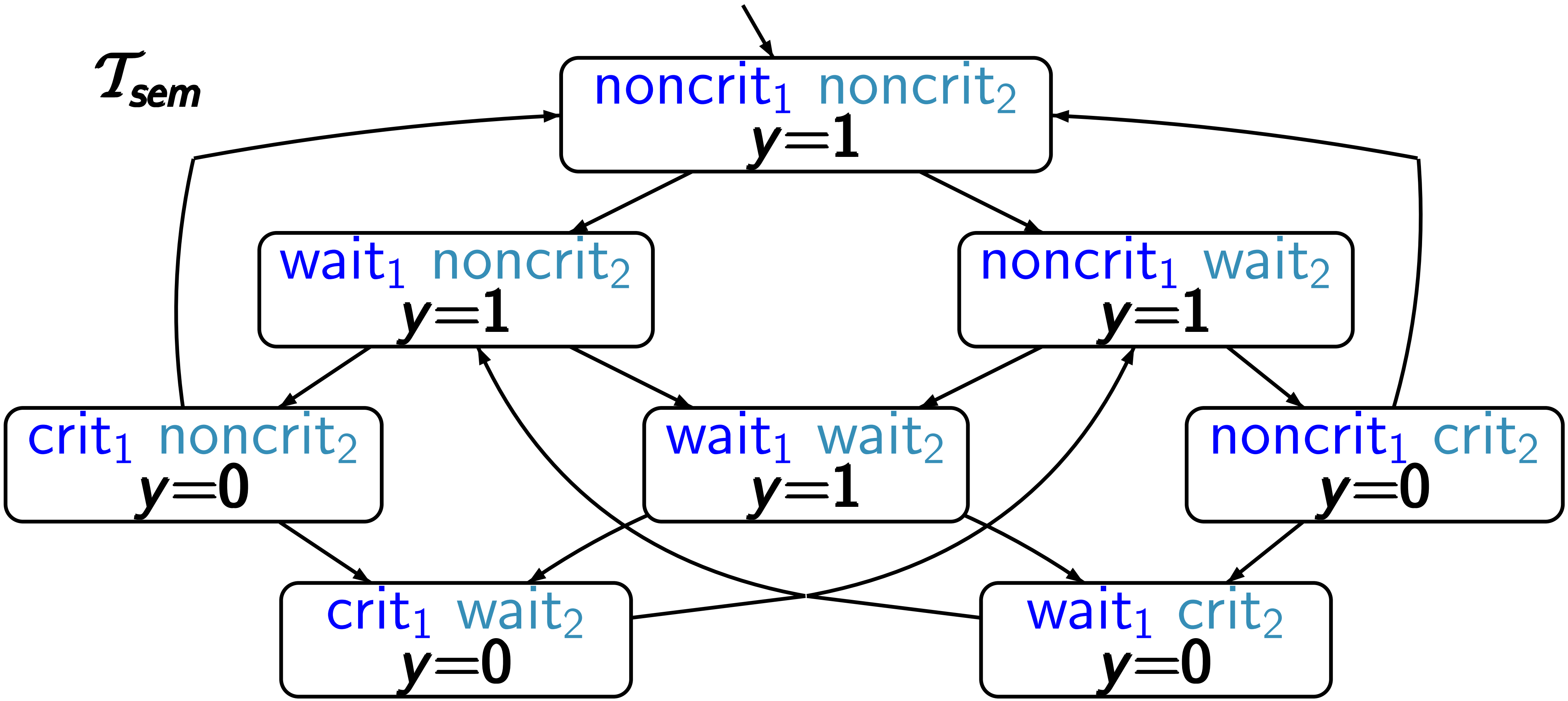
light 1 ||| light 2 $\not\models$ “infinitely often ***green***$_1$”

interleaving is completely time abstract **!**

Mutual exclusion (semaphore)

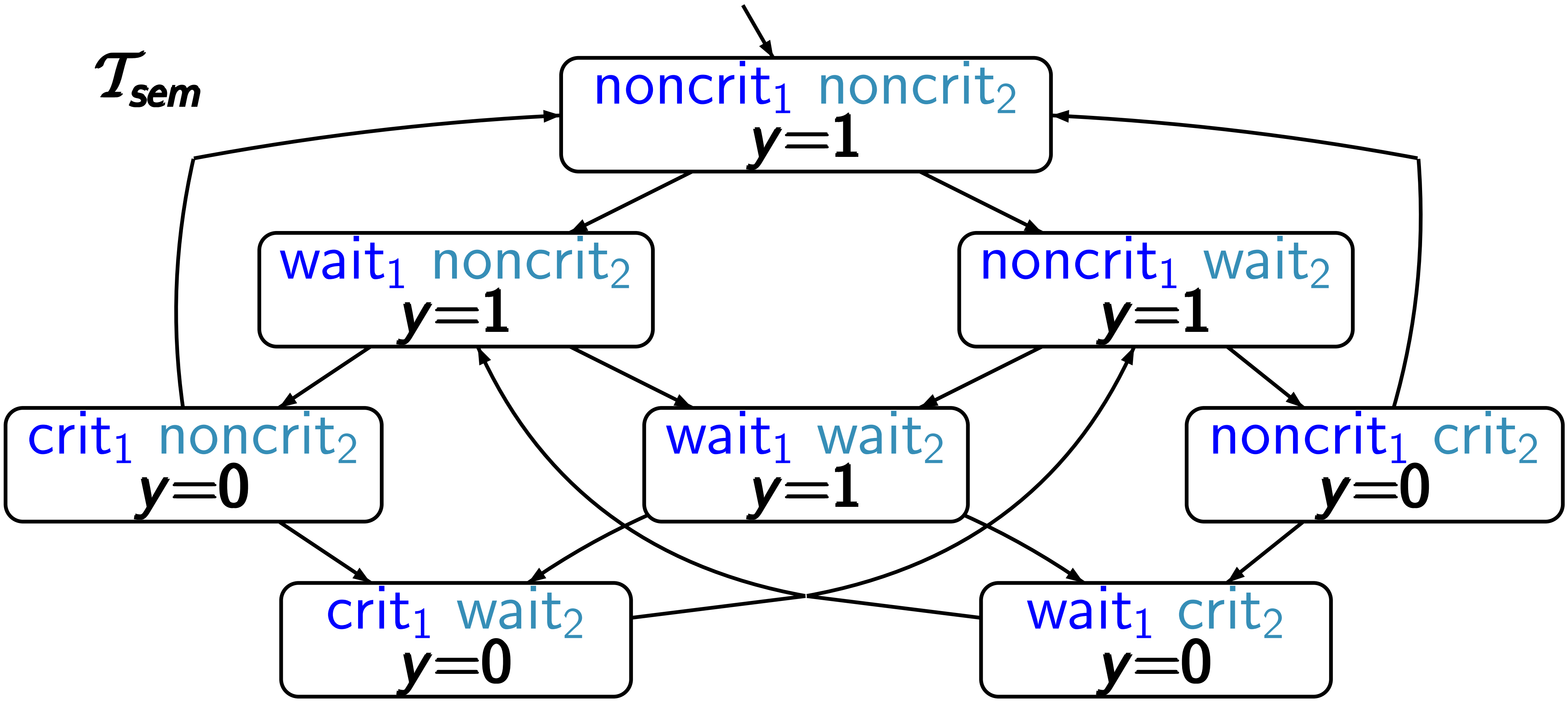


Mutual exclusion (semaphore)



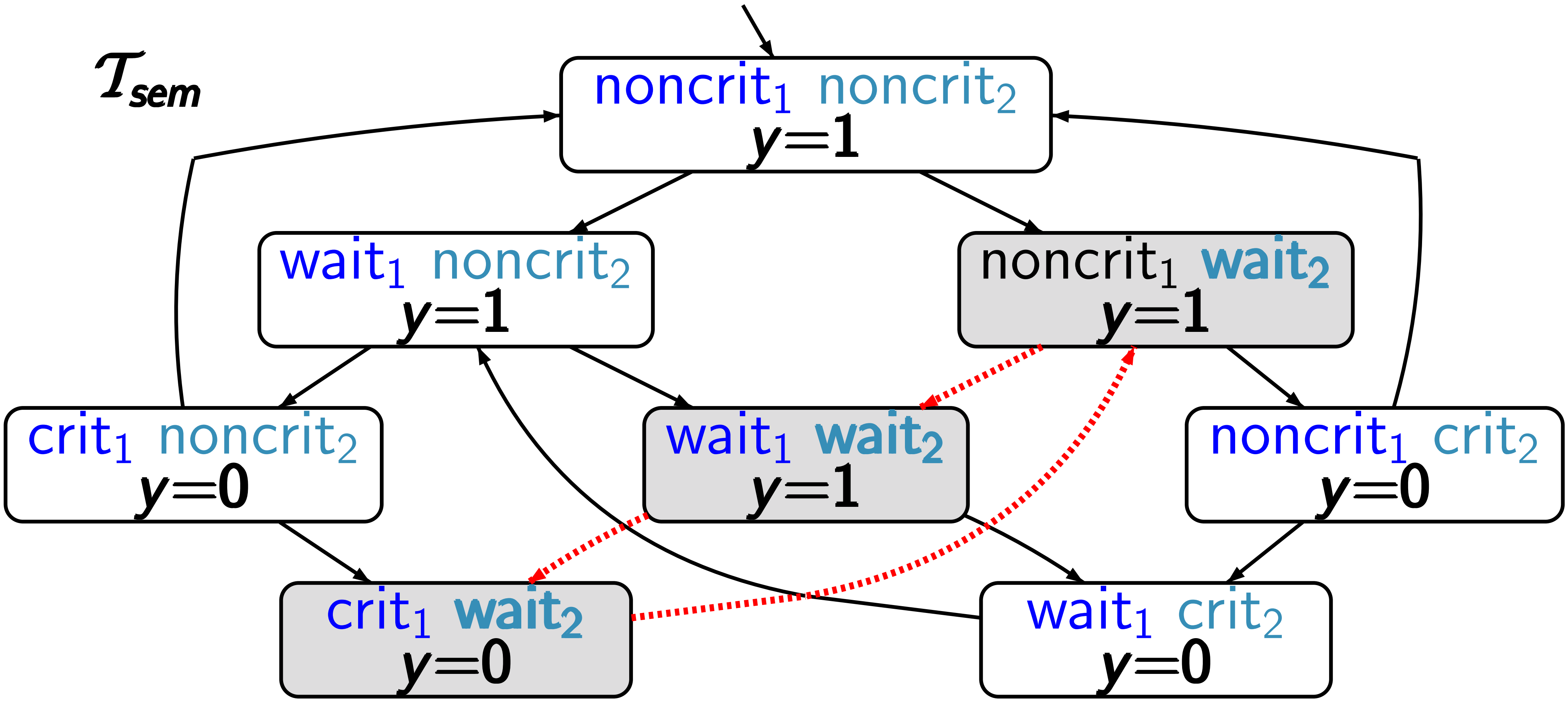
liveness property $\widehat{=}$ "each waiting process will eventually enter its critical section"

Mutual exclusion (semaphore)



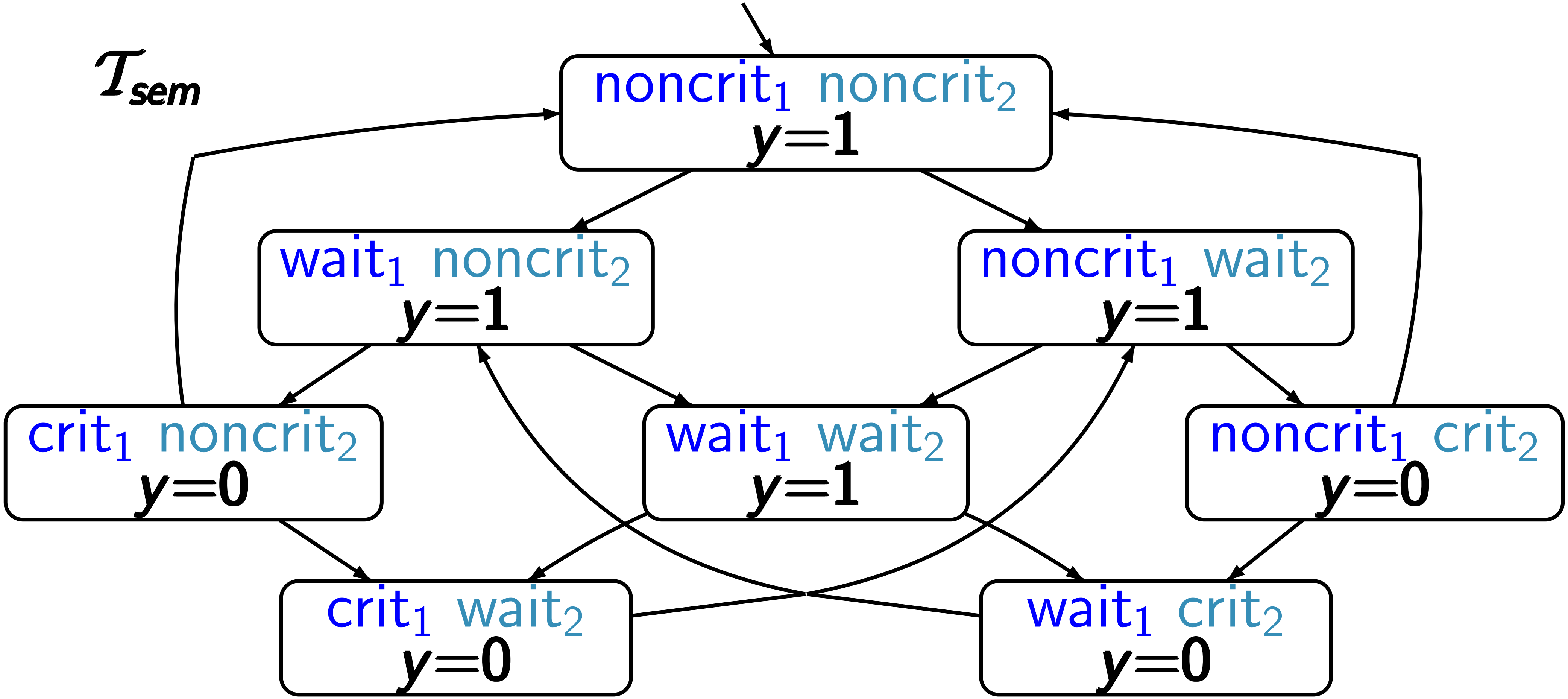
$\mathcal{T}_{sem} \not\models$ "each waiting process will eventually enter its critical section"

Mutual exclusion (semaphore)



$\mathcal{T}_{sem} \not\models$ "each waiting process will eventually enter its critical section"

Mutual exclusion (semaphore)



$\mathcal{T}_{sem} \not\models$ "each waiting process will eventually enter its critical section"

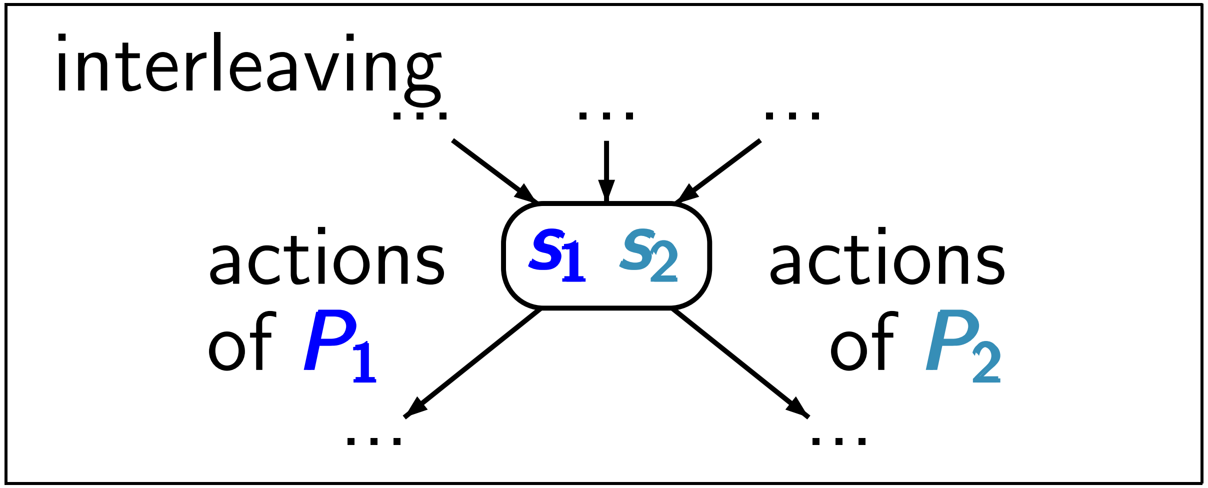
level of abstraction is too coarse !

Process fairness

LF2.6-5

Process fairness

two independent non-communicating processes $P_1 \mid\mid\mid P_2$

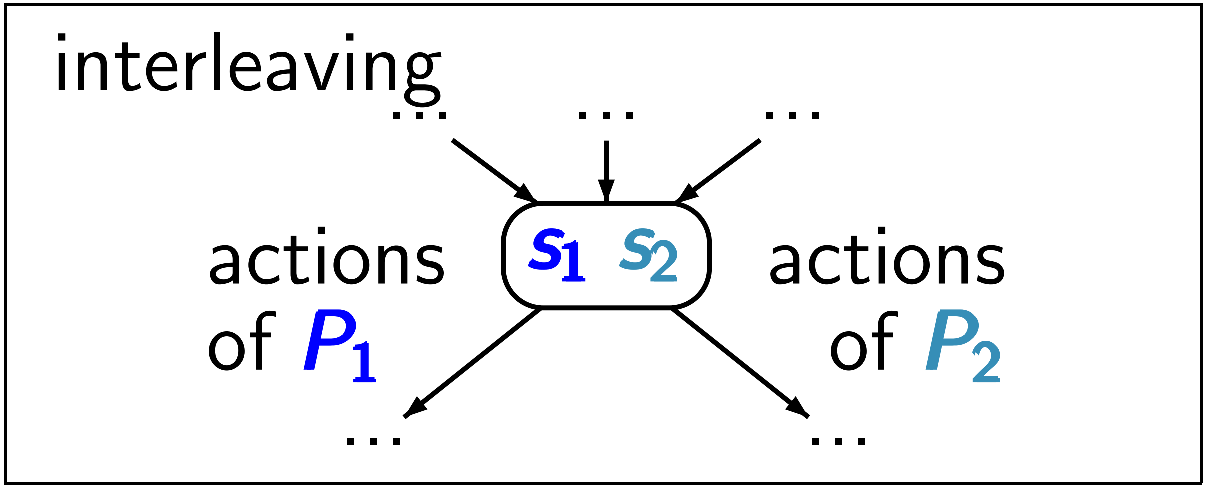


possible interleavings:

P_1 P_2 P_2 P_1 P_1 P_1 P_2 P_1 P_2 P_2 P_2 P_1 P_1 ...
P_1 P_1 P_2 P_1 P_1 P_2 P_1 P_1 P_2 P_1 P_1 P_2 P_1 ...

Process fairness

two independent non-communicating processes $P_1 \mid\mid\mid P_2$



possible interleavings:

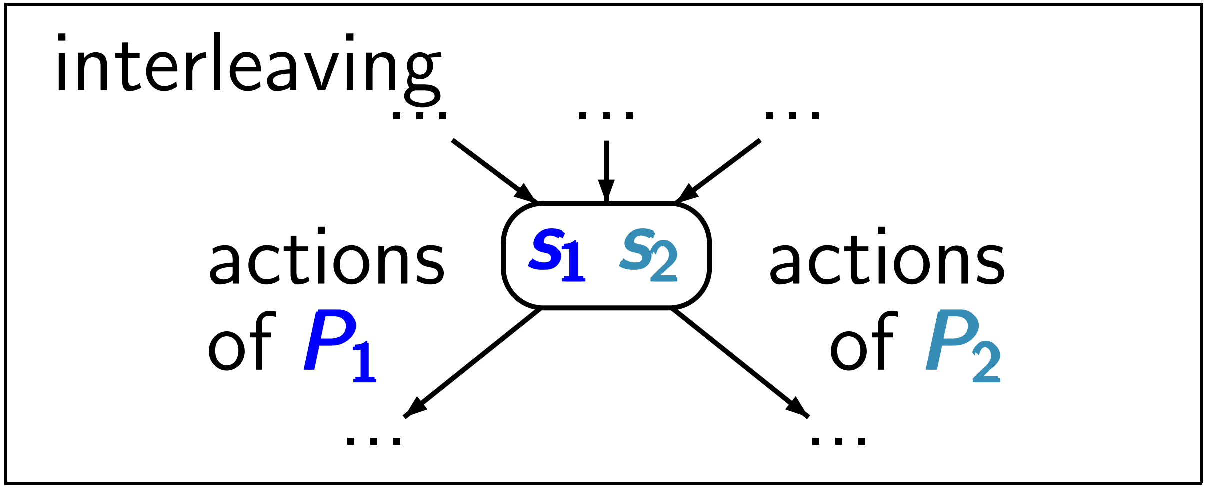
$P_1\ P_2\ P_2\ P_1\ P_1\ P_1\ P_2\ P_1\ P_2\ P_2\ P_2\ P_1\ P_1\ \ldots$

$P_1\ P_1\ P_2\ P_1\ P_1\ P_2\ P_1\ P_1\ P_2\ P_1\ P_1\ P_2\ P_1\ \ldots$

$P_1\ P_1\ P_1\ P_1\ P_1\ P_1\ P_1\ P_1\ P_1\ P_1\ P_1\ P_1\ P_1\ \ldots$

Process fairness

two independent non-communicating processes $P_1 \,|||\, P_2$



possible interleavings:

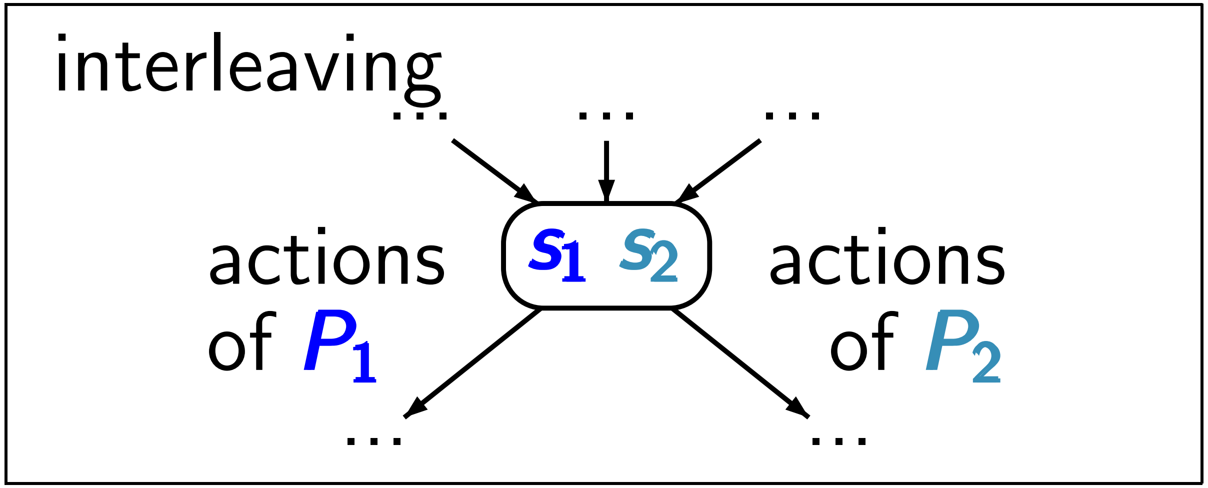
$P_1\ P_2\ P_2\ P_1\ P_1\ P_1\ P_2\ P_1\ P_2\ P_2\ P_2\ P_1\ P_1\ \ldots$ fair

$P_1\ P_1\ P_2\ P_1\ P_1\ P_2\ P_1\ P_1\ P_2\ P_1\ P_1\ P_2\ P_1\ \ldots$ fair

$P_1\ P_1\ P_1\ P_1\ P_1\ P_1\ P_1\ P_1\ P_1\ P_1\ P_1\ P_1\ P_1\ \ldots$ unfair

Process fairness

two independent non-communicating processes $P_1 \mid\mid\mid P_2$



possible interleavings:

P_1 P_2 P_2 P_1 P_1 P_1 P_2 P_1 P_2 P_2 P_2 P_1 P_1 ... fair

P_1 P_1 P_2 P_1 P_1 P_2 P_1 P_1 P_2 P_1 P_1 P_2 P_1 ... fair

P_1 P_1 P_1 P_1 P_1 P_1 P_1 P_1 P_1 P_1 P_1 P_1 P_1 ... unfair

process fairness assumes an appropriate resolution of the nondeterminism resulting from interleaving and competitions

Nuances of fairness

- unconditional fairness

- strong fairness

- weak fairness

Nuances of fairness

- unconditional fairness, e.g.,

  every process enters gets its turn infinitely often.

- strong fairness

- weak fairness

Nuances of fairness

LF2.6-6

- unconditional fairness, e.g.,

  every process enters gets its turn infinitely often.

- strong fairness, e.g.,

  every process that is enabled infinitely often
  gets its turn infinitely often.

- weak fairness

Nuances of fairness

- unconditional fairness, e.g.,

  every process enters gets its turn infinitely often.

- strong fairness, e.g.,

  every process that is enabled infinitely often
  gets its turn infinitely often.

- weak fairness, e.g.,

  every process that is continuously enabled
  from a certain time instance on,
  gets its turn infinitely often.

Fairness for action-set LF2.6-7

Fairness for action-set

Let $\mathcal{T}$ be a TS with action-set Act, $A \subseteq Act$ and

$\rho = s_0 \xrightarrow{\alpha_0} s_1 \xrightarrow{\alpha_1} s_2 \xrightarrow{\alpha_2} \ldots$ infinite execution fragment

Fairness for action-set

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we will provide conditions for

- unconditional A-fairness of ρ
- strong A-fairness of ρ
- weak A-fairness of ρ

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we will provide conditions for

- unconditional A-fairness of ρ
- strong A-fairness of ρ
- weak A-fairness of ρ

using the following notations:

$$Act(s_i) \;=\; \{\beta \in Act : \exists s' \text{ s.t. } s_i \xrightarrow{\beta} s'\}$$

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$$\overset{\infty}{\exists} \;\widehat{=}\; \text{“there exists infinitely many ...”}$$

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- weak A-fairness of ρ

using the following notations:

$$Act(s_i) = \{\beta \in Act : \exists s' \text{ s.t. } s_i \xrightarrow{\beta} s'\}$$

$$\overset{\infty}{\exists} \quad \widehat{=} \quad \text{“there exists infinitely many ...”}$$

$$\overset{\infty}{\forall} \quad \widehat{=} \quad \text{“for all, but finitely many ...”}$$

Fairness for action-set

Let $\mathcal{T}$ be a TS with action-set Act, $A \subseteq Act$ and

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- ρ is unconditionally A-fair, if

Fairness for action-set

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"actions in A will be taken infinitely many times"

Fairness for action-set

Let $\mathcal{T}$ be a TS with action-set Act, $A \subseteq Act$ and

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Fairness for action-set

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$$\overset{\infty}{\exists}\, i \geq 0.\, A \cap Act(s_i) \neq \varnothing \quad \Longrightarrow \quad \overset{\infty}{\exists}\, i \geq 0.\, \alpha_i \in A$$

"If infinitely many times some action in A is enabled, then actions in A will be taken infinitely many times."

Fairness for action-set

Let $\mathcal{T}$ be a TS with action-set Act, $A \subseteq Act$ and

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- ρ is weakly A-fair, if

Fairness for action-set

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- ρ is weakly A-fair, if

$$\overset{\infty}{\forall}\, i \geq 0.\, A \cap Act(s_i) \neq \varnothing \quad \Longrightarrow \quad \overset{\infty}{\exists}\, i \geq 0.\, \alpha_i \in A$$

"If from some moment, actions in A are enabled, then actions in A will be taken infinitely many times."

Fairness for action-set

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unconditionally A-fair $\Longrightarrow$ strongly A-fair
$\Longrightarrow$ weakly A-fair

Fairness for action-set

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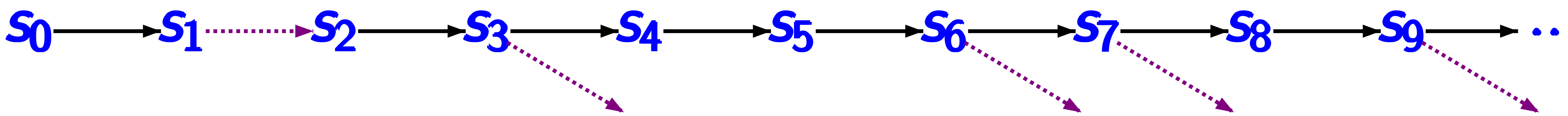
- ρ is weakly A-fair, if

$$\stackrel{\infty}{\forall} i \geq 0. \, A \cap Act(s_i) \neq \varnothing \quad \Longrightarrow \quad \stackrel{\infty}{\exists} i \geq 0. \, \alpha_i \in A$$

unconditionally A-fair	$\Longrightarrow$	strongly A-fair
	$\Longrightarrow$	weakly A-fair

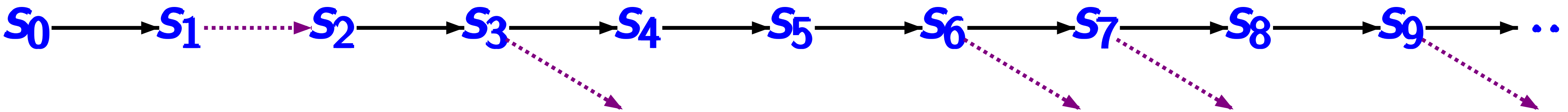
strong A-fairness is *violated* if



- no A-actions are executed from a certain moment
- A-actions are enabled infinitely many times

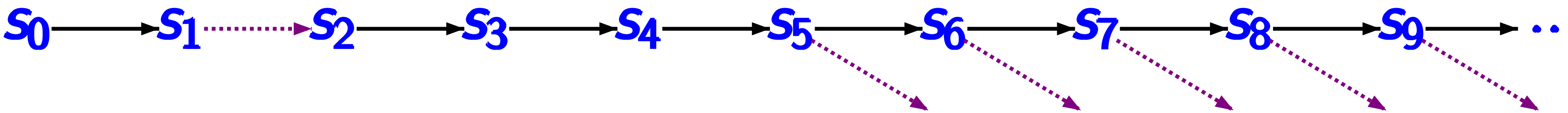
Strong and weak action fairness

strong A-fairness is *violated* if



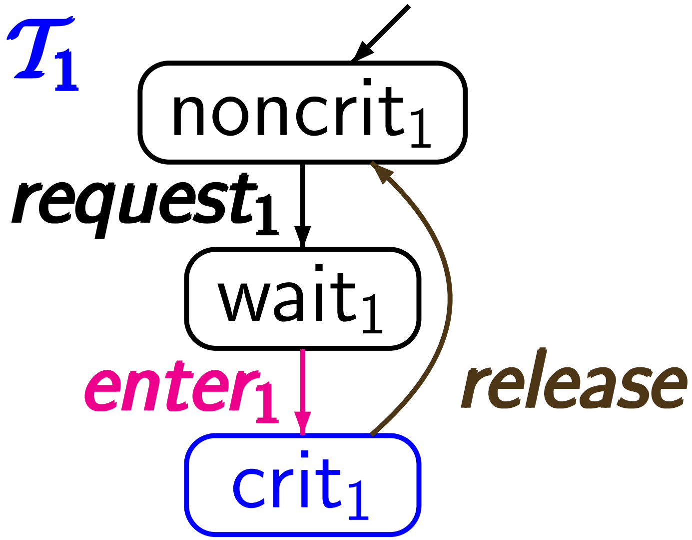
- no A-actions are executed from a certain moment
- A-actions are enabled infinitely many times

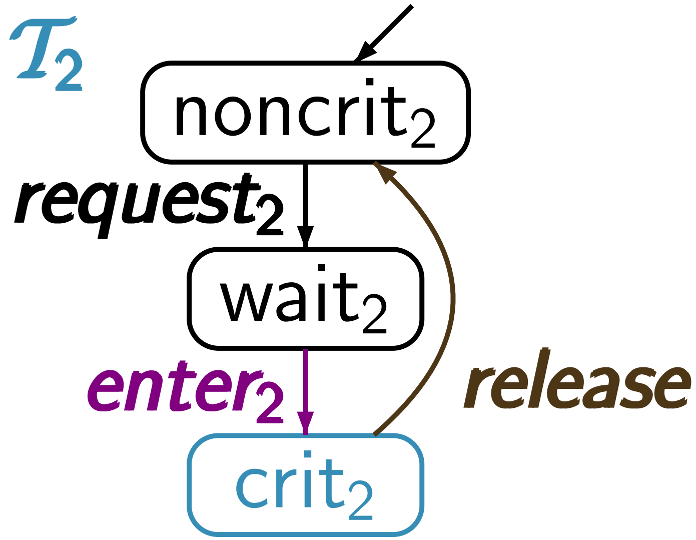
weak A-fairness is *violated* if



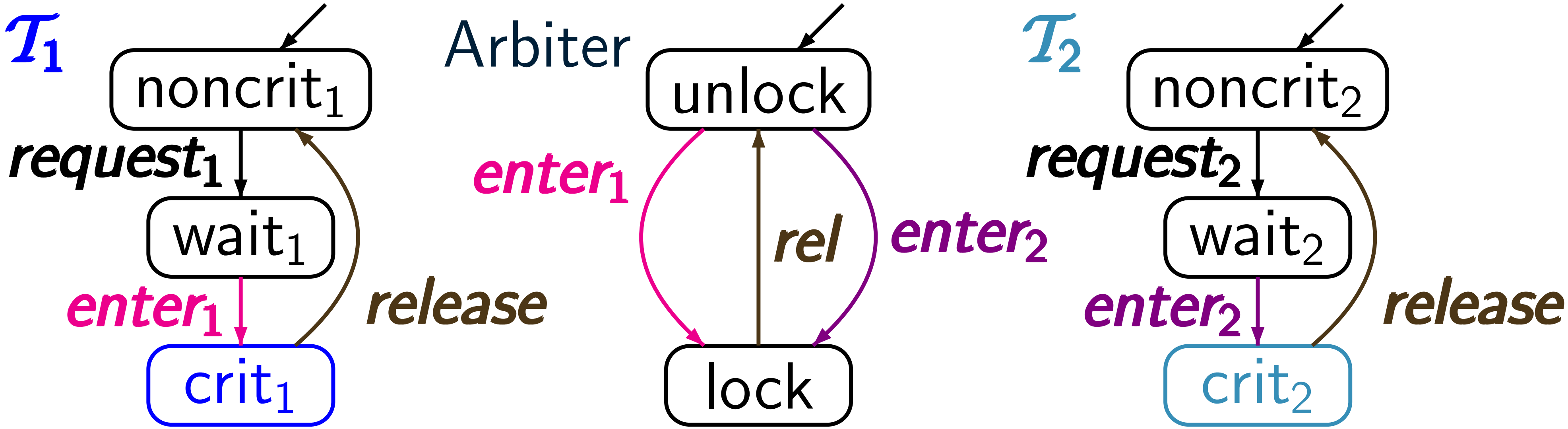
- no A-actions are executed from a certain moment
- A-actions are continuously enabled from some moment on

Mutual exclusion with arbiter

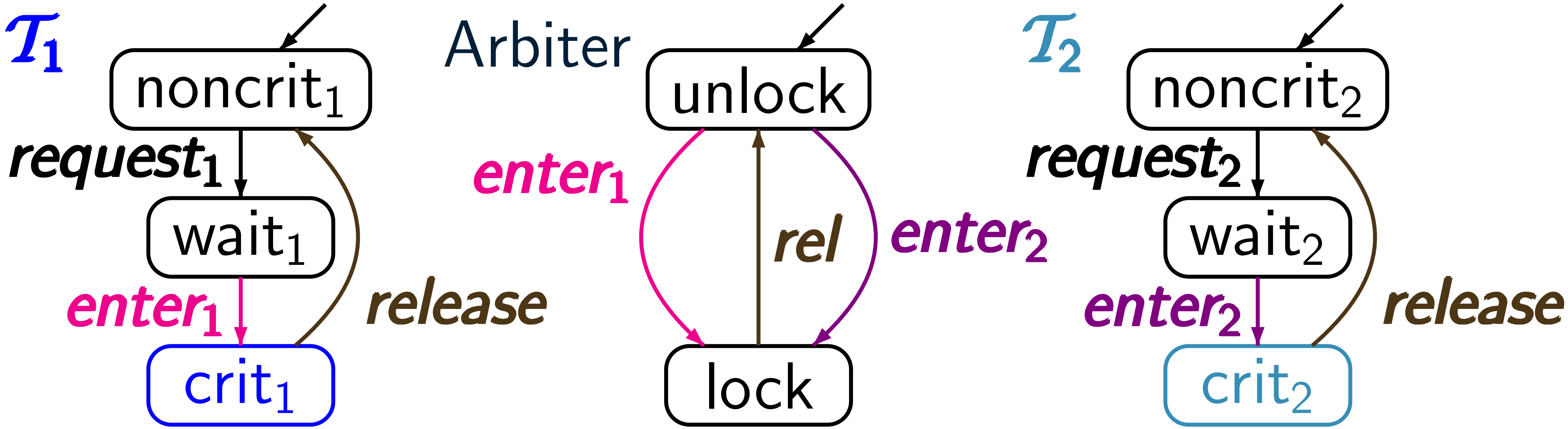


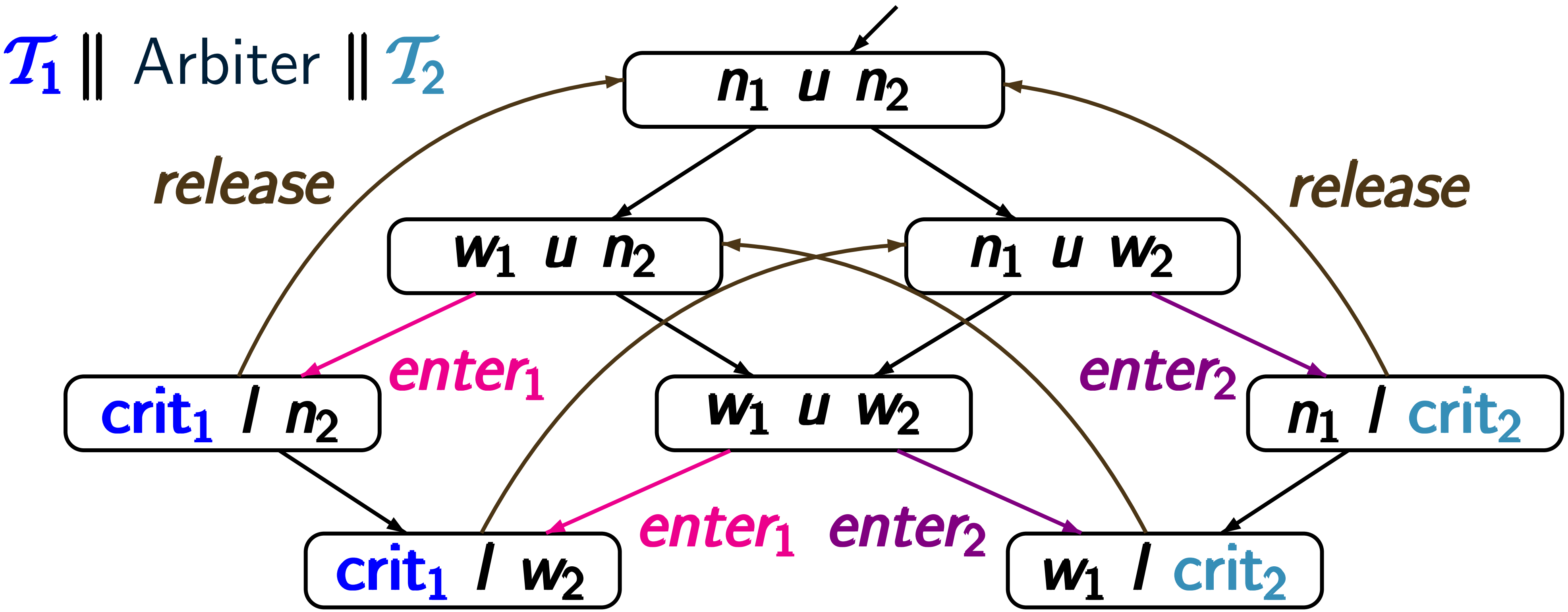


Mutual exclusion with arbiter



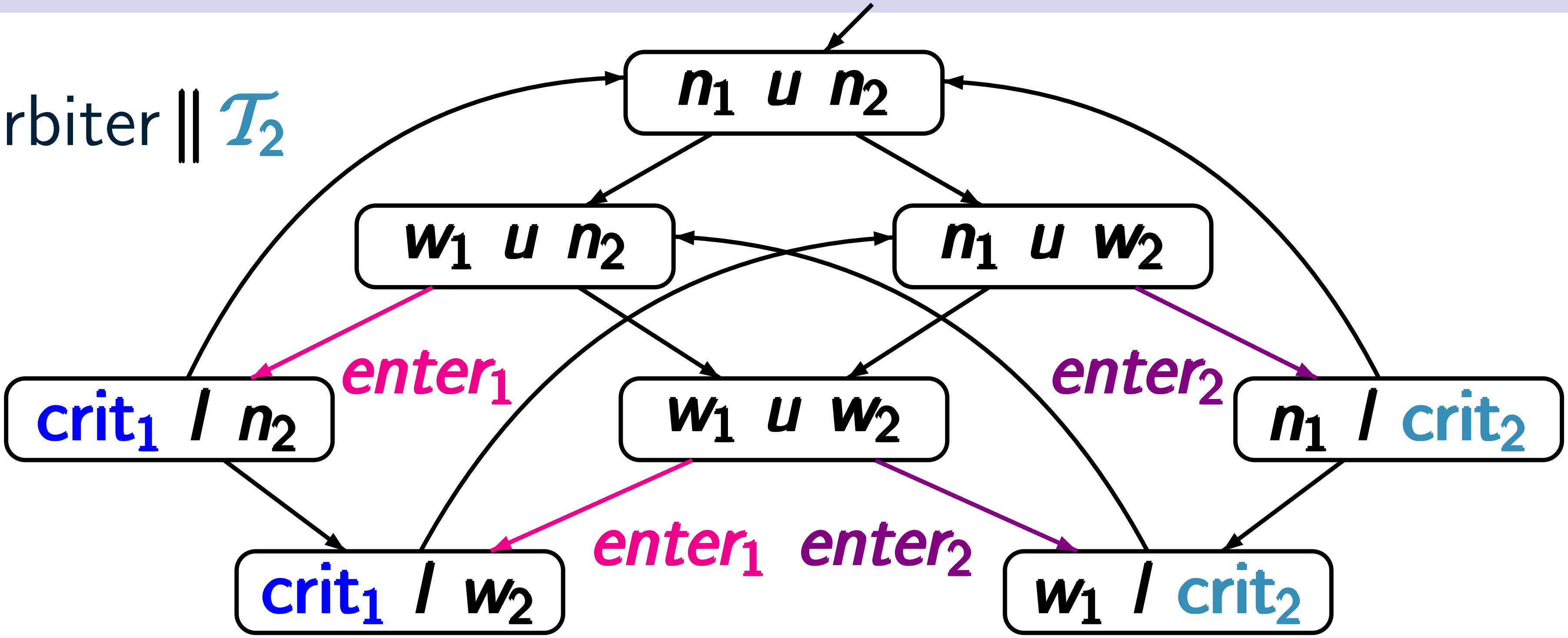
Mutual exclusion with arbiter





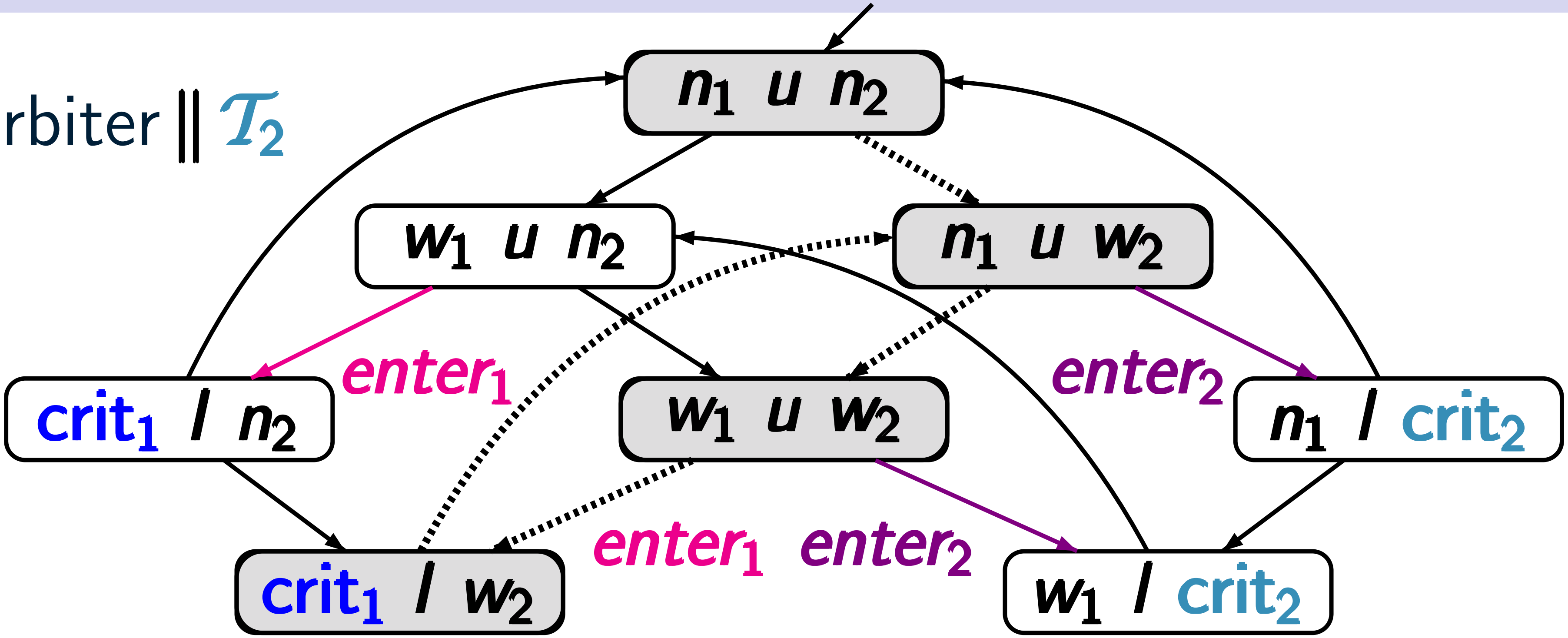
Unconditional, strongly or weakly fair?

$\mathcal{T}_1 \parallel \text{Arbiter} \parallel \mathcal{T}_2$



Unconditional, strongly or weakly fair?

$\mathcal{T}_1 \parallel \text{Arbiter} \parallel \mathcal{T}_2$

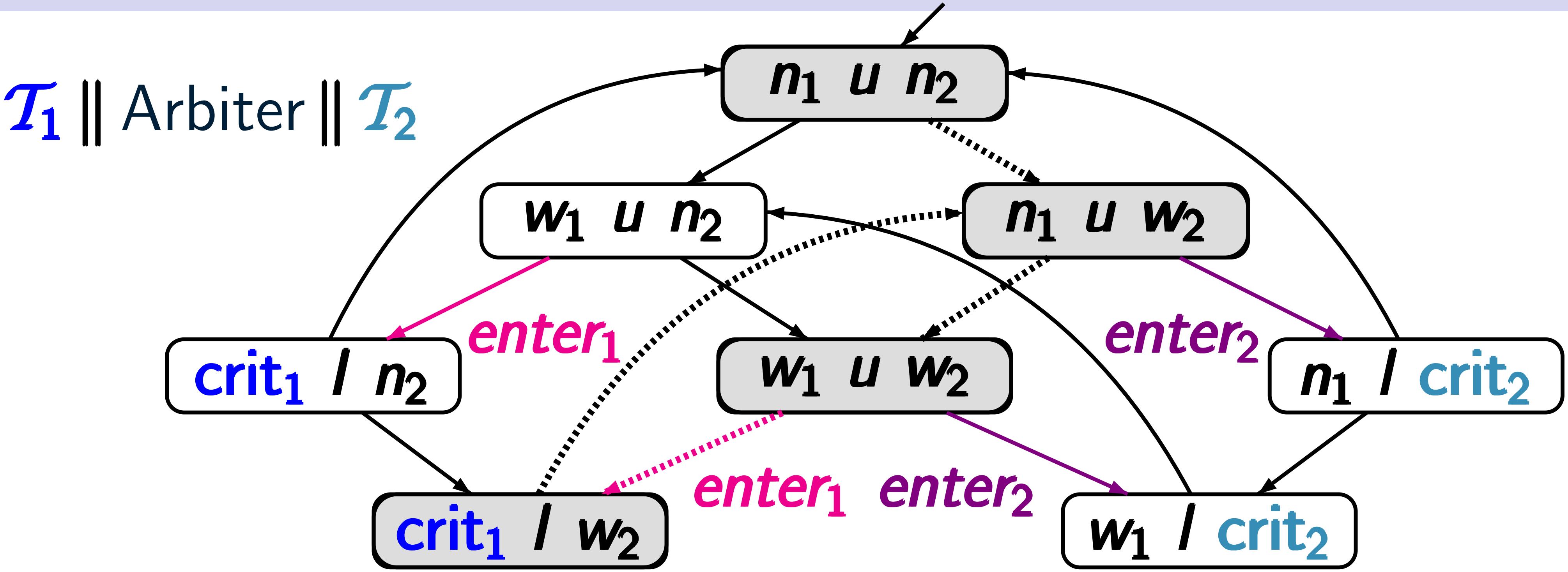


fairness for action set $A = \{enter_1\}$:

$$\langle n_1, u, n_2 \rangle \rightarrow \Big(\langle n_1, u, w_2 \rangle \rightarrow \langle w_1, u, w_2 \rangle \rightarrow \langle crit_1, l, w_2 \rangle \Big)^{\omega}$$

- unconditional A-fairness:
- strong A-fairness:
- weak A-fairness:

Unconditional, strongly or weakly fair?



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- unconditional A-fairness: **yes**
- strong A-fairness: **yes** $\longleftarrow$ unconditionally fair
- weak A-fairness: **yes** $\longleftarrow$ unconditionally fair

Unconditional, strongly or weakly fair?

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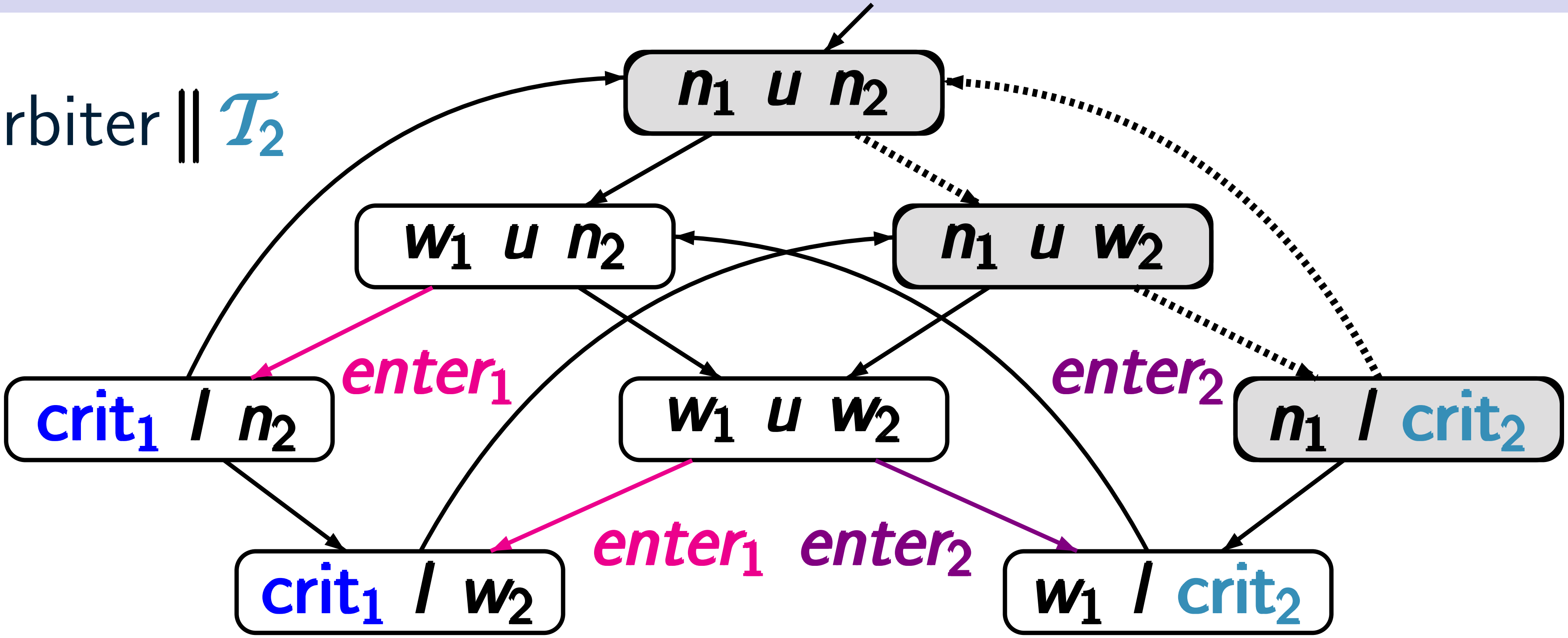

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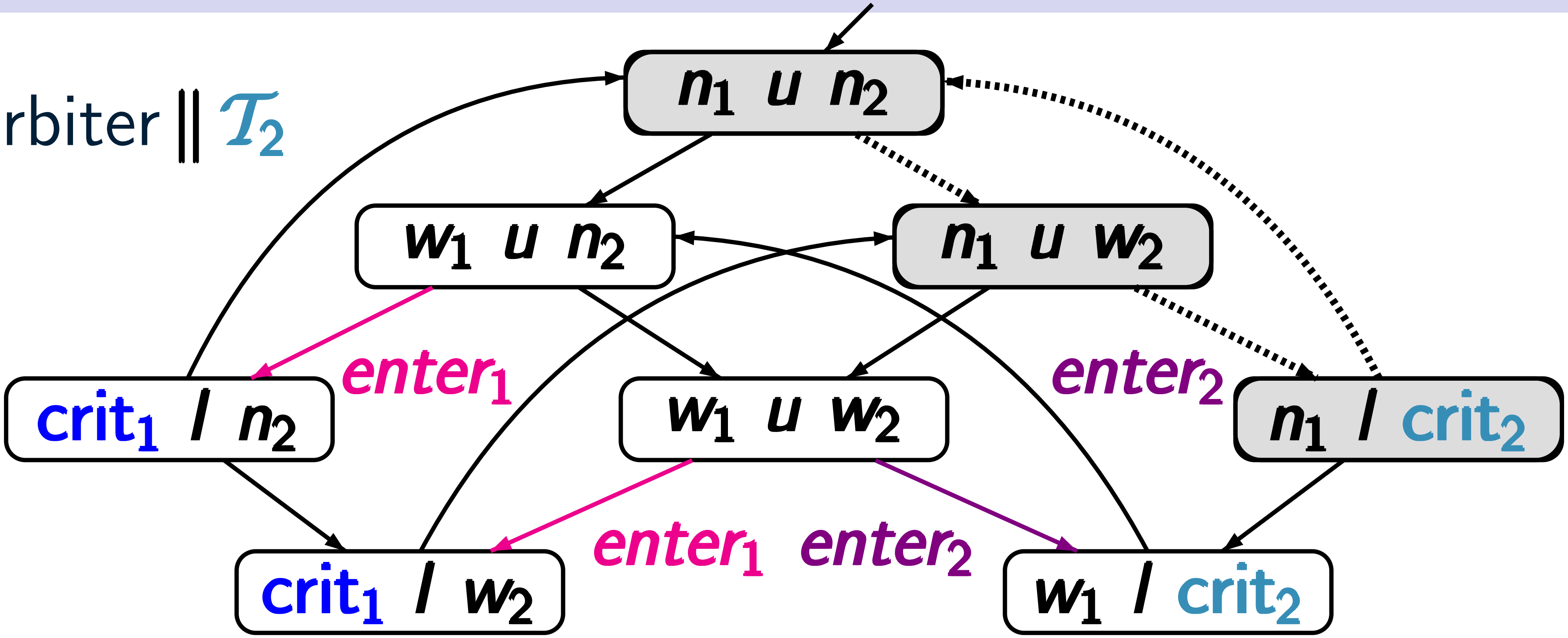

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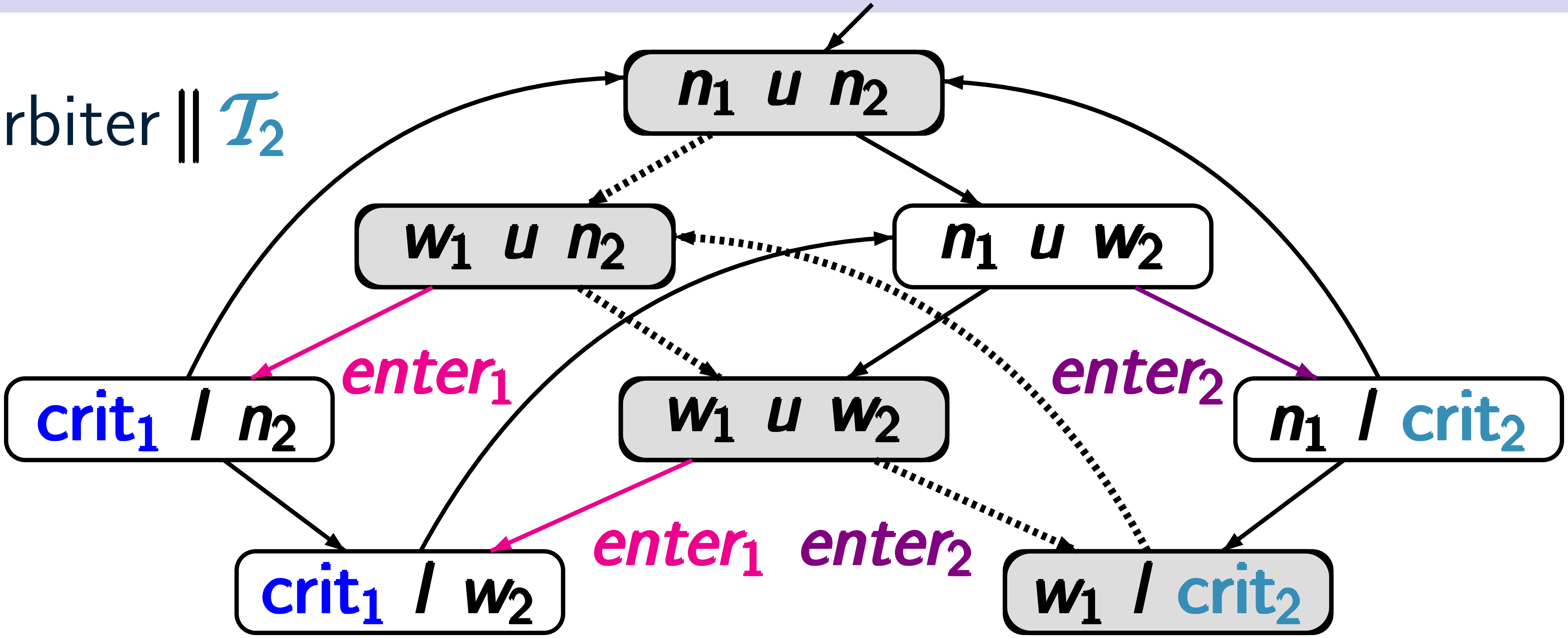
- unconditional A-fairness: **no**
- strong A-fairness: **yes** $\leftarrow$ A never enabled
- weak A-fairness: **yes** $\leftarrow$ strongly A-fair

Unconditional, strongly or weakly fair?

$\mathcal{T}_1 \,\|\, \text{Arbiter} \,\|\, \mathcal{T}_2$



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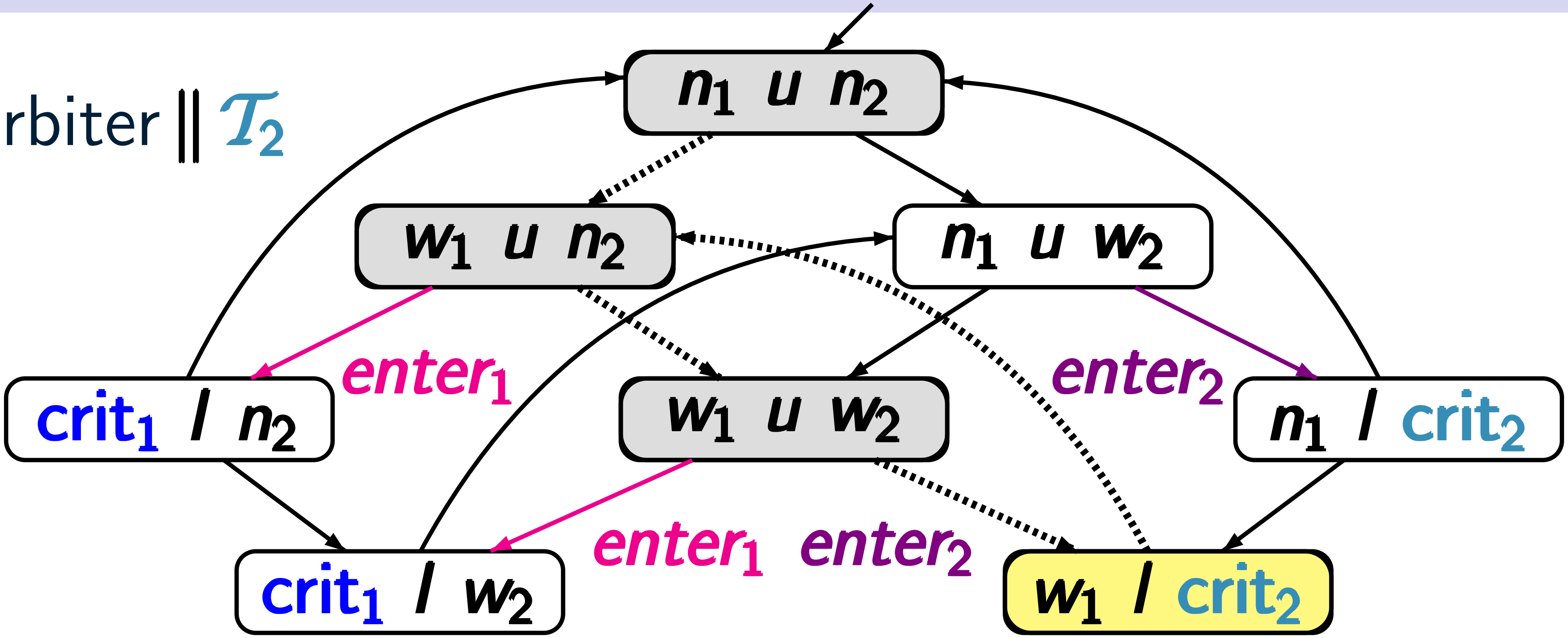

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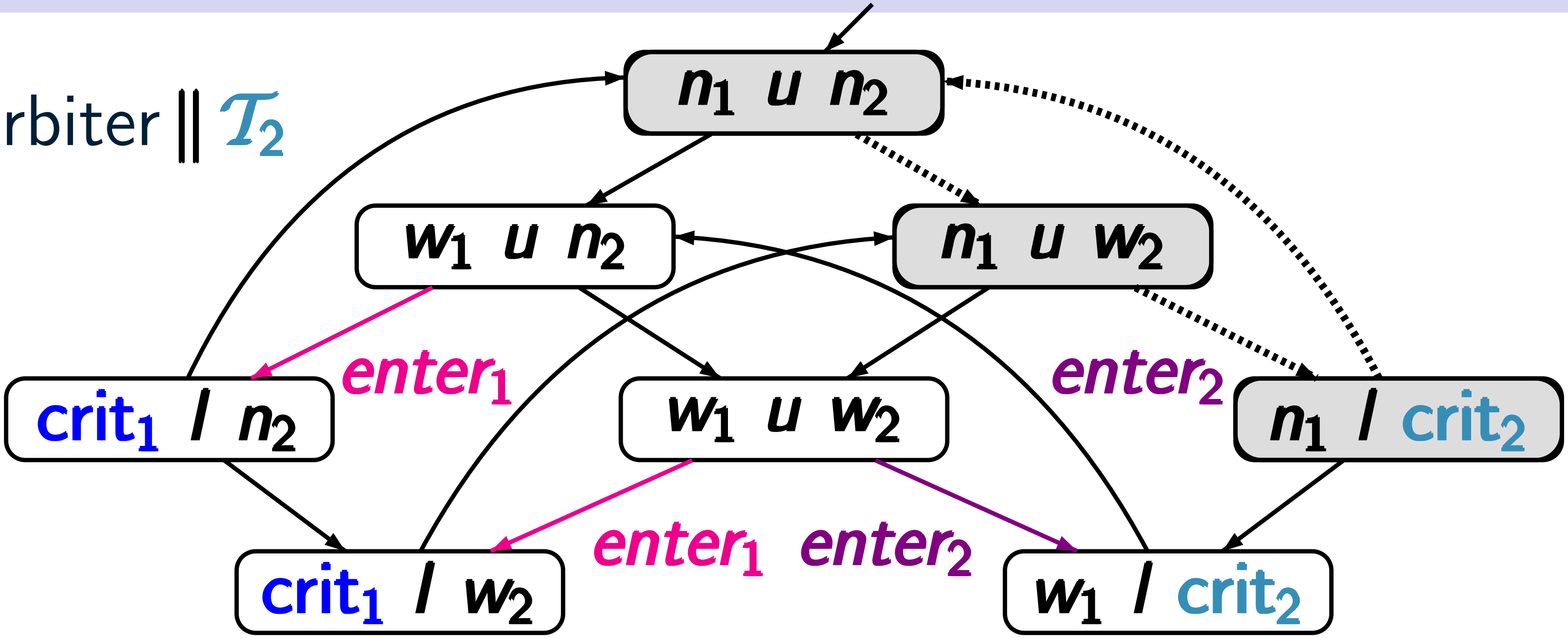

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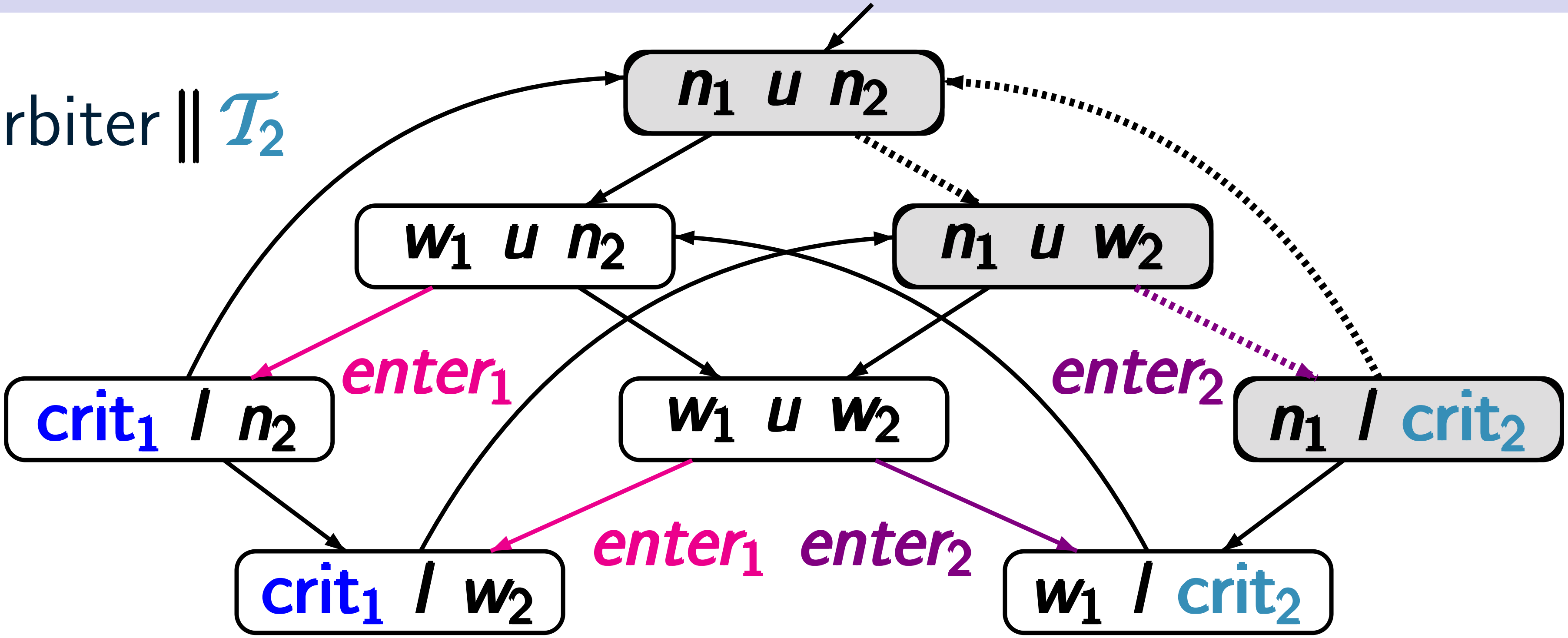

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Action-based fairness assumptions

Action-based fairness assumptions

Let $\mathcal{T}$ be a transition system with action-set Act.
A fairness assumption for $\mathcal{T}$ is a triple

$$\mathcal{F} = (\mathcal{F}_{ucond}, \mathcal{F}_{strong}, \mathcal{F}_{weak})$$

where $\mathcal{F}_{ucond}$, $\mathcal{F}_{strong}$, $\mathcal{F}_{weak} \subseteq 2^{Act}$.

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An execution ρ is called $\mathcal{F}$-fair iff

- ρ is unconditionally A-fair for all $A \in \mathcal{F}_{ucond}$
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$$FairTraces_{\mathcal{F}}(\mathcal{T}) \stackrel{\text{def}}{=} \{trace(\rho) : \rho \text{ is a } \mathcal{F}\text{-fair execution of } \mathcal{T}\}$$

Fair satisfaction relation

Fair satisfaction relation

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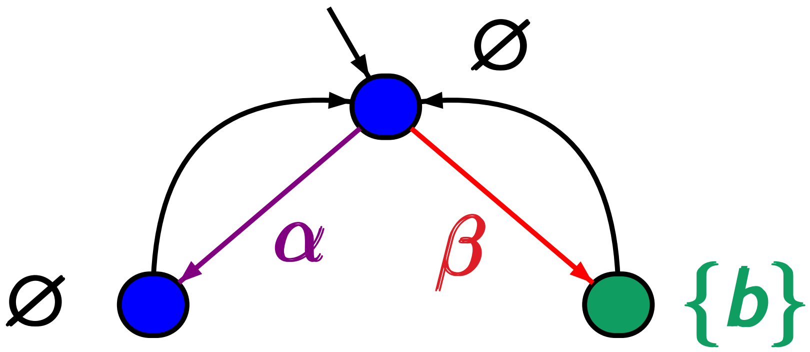
If $\mathcal{T}$ is a TS and E a LT property over AP then:

$$\mathcal{T} \models_{\mathcal{F}} E \quad \overset{\text{def}}{\Longleftrightarrow} \quad FairTraces_{\mathcal{F}}(\mathcal{T}) \subseteq E$$



fairness assumption $\mathcal{F}$

- no unconditional fairness condition
- strong fairness for $\{\alpha, \beta\}$
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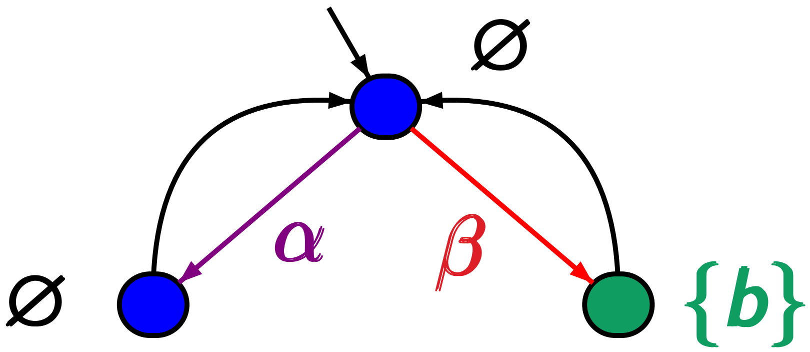
Example: fair satisfaction relation



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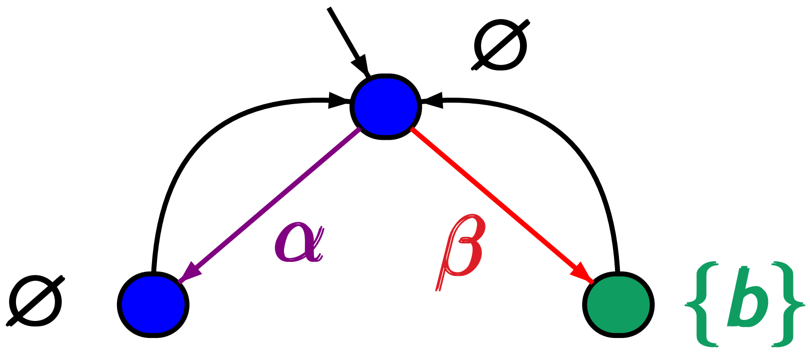

$\mathcal{T} \models_{\mathcal{F}}$ "infinitely often b" ?

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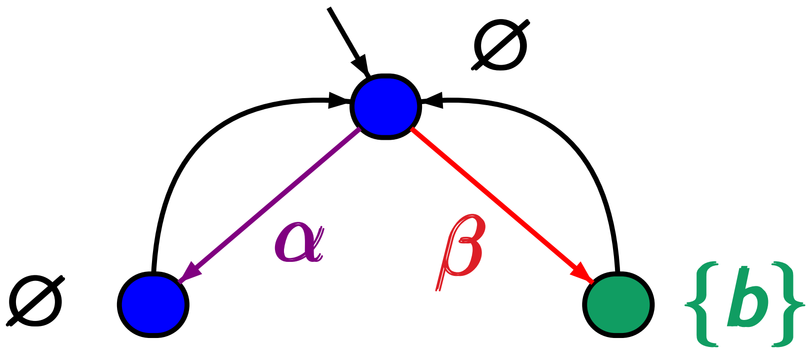

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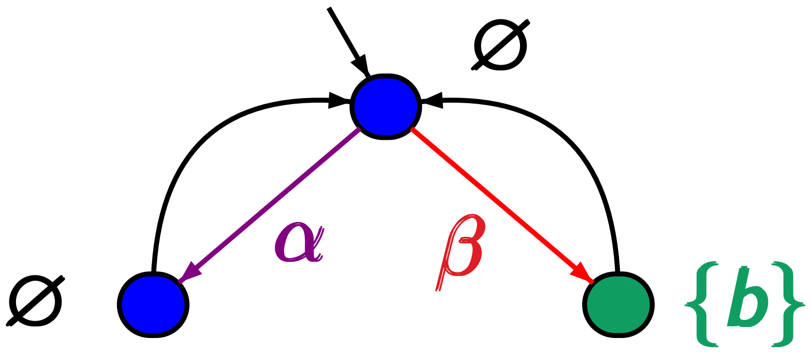

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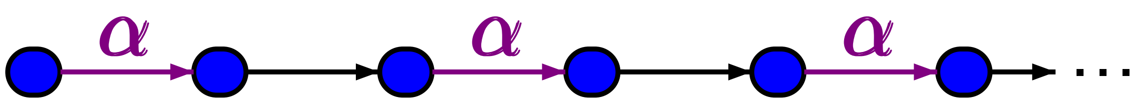
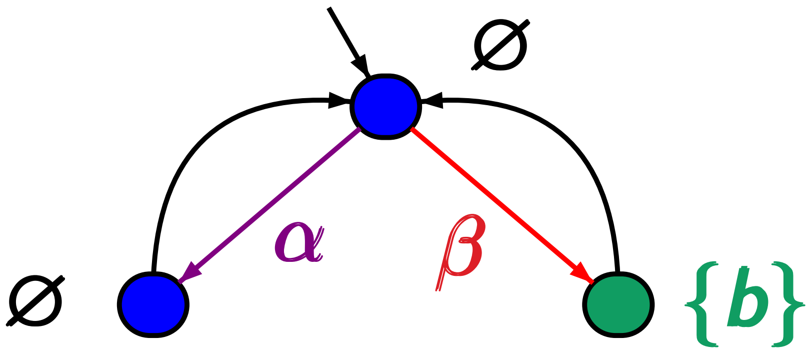


$\mathcal{F}$-fair

actions in $\{\alpha, \beta\}$ are executed infinitely many times

Example: fair satisfaction relation



fairness assumption $\mathcal{F}$

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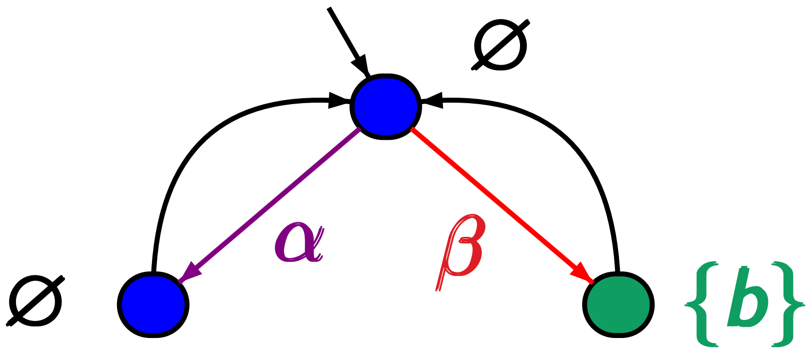
Example: fair satisfaction relation



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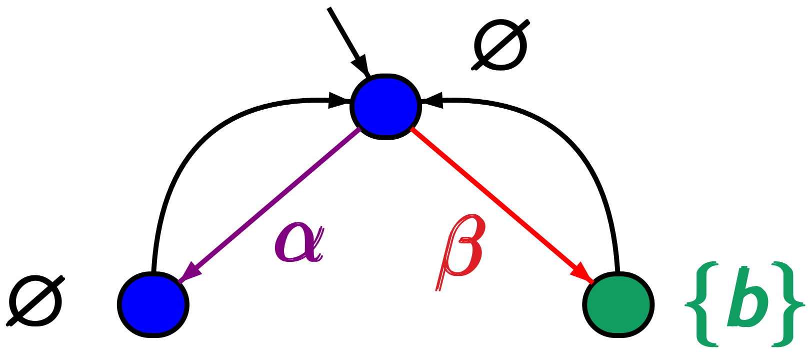

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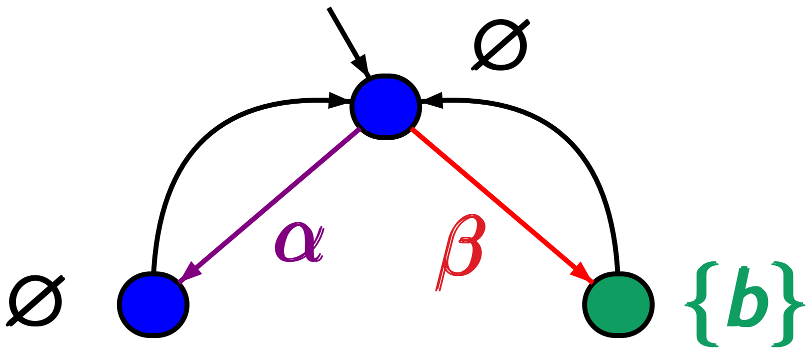

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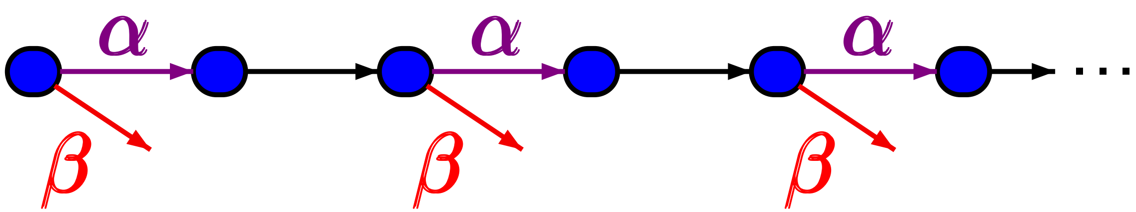
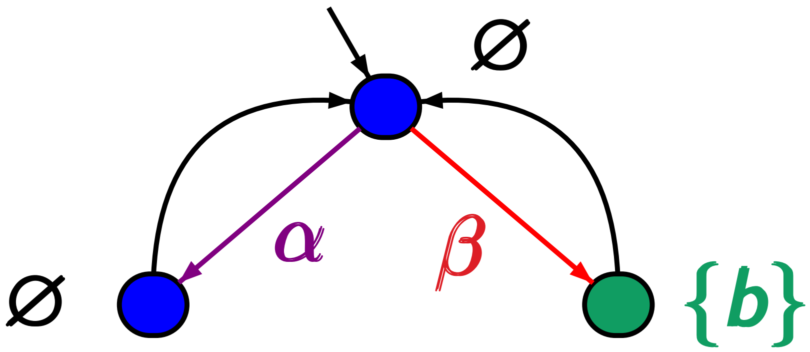


$\mathcal{F}$-fair

Example: fair satisfaction relation



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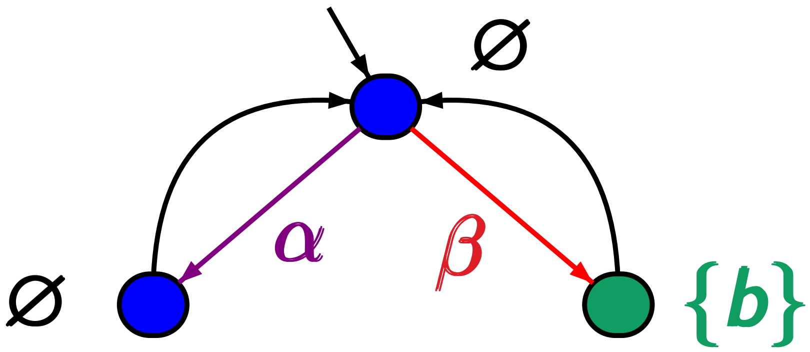

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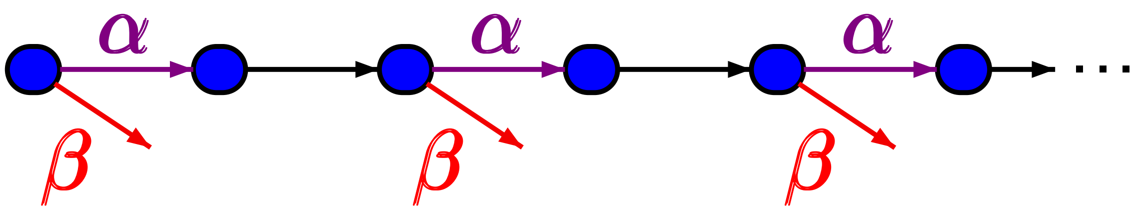


is not $\mathcal{F}$-fair

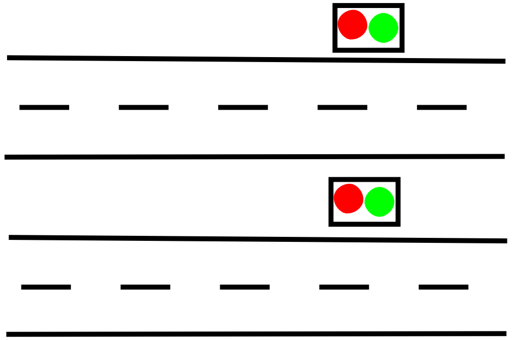
Which type of fairness?

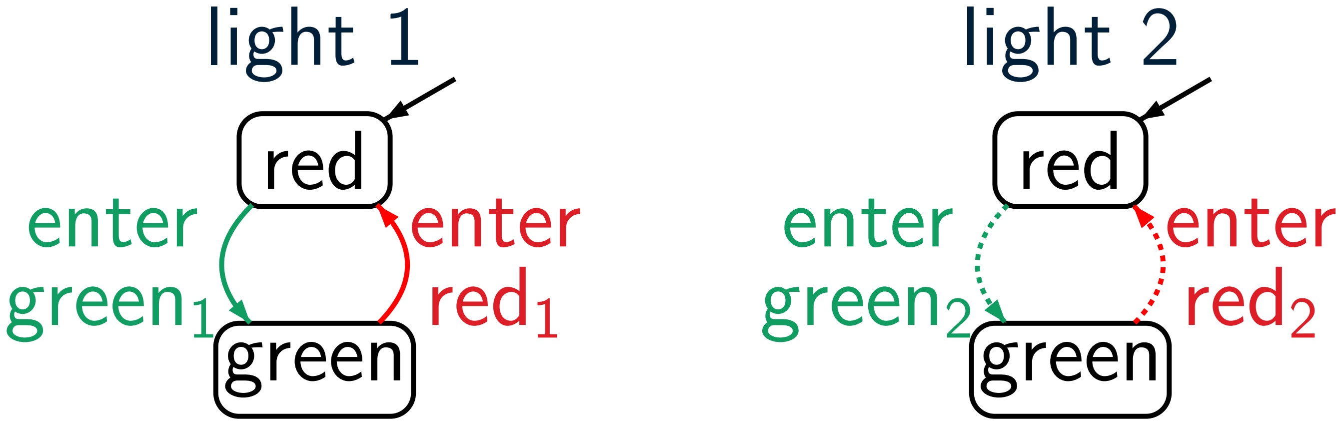
Which type of fairness?

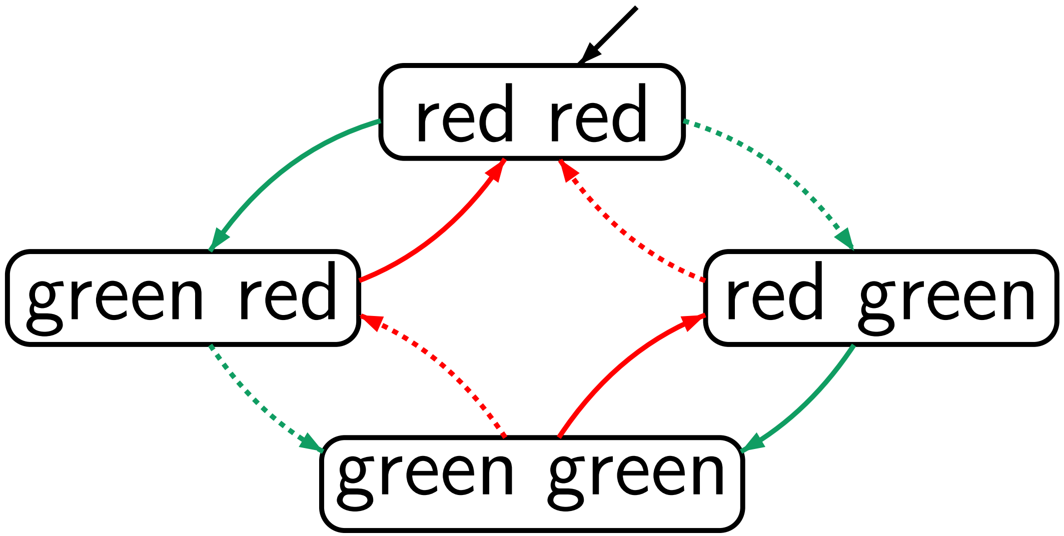
LF2.6-13A

fairness assumptions should be
as weak as possible

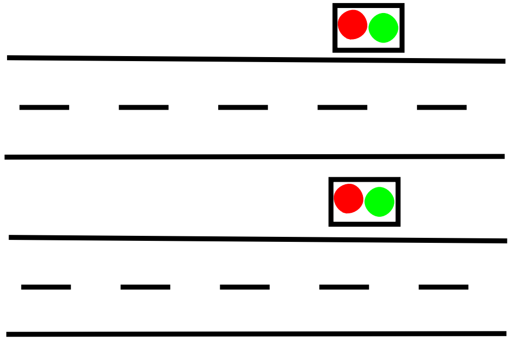
Two independent traffic lights

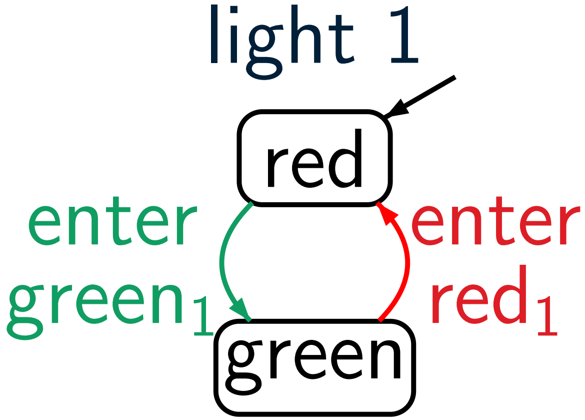


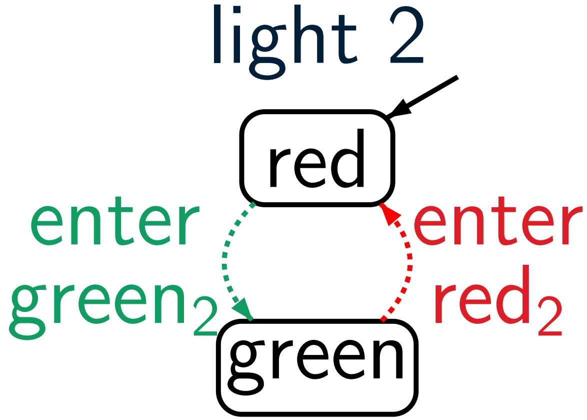


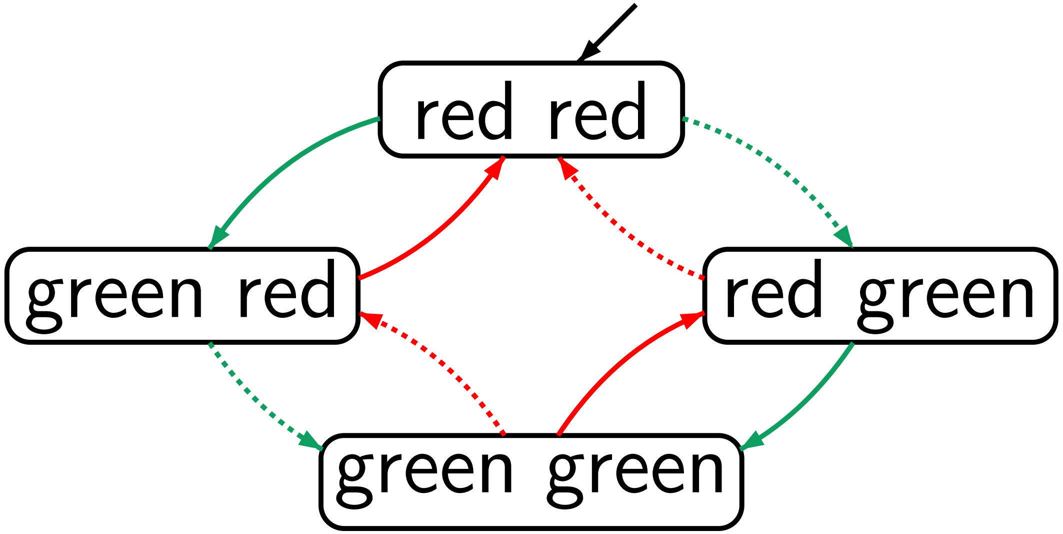


Two independent traffic lights









fairness assumption $\mathcal{F}$:

$\mathcal{F}_{ucond} = ?$

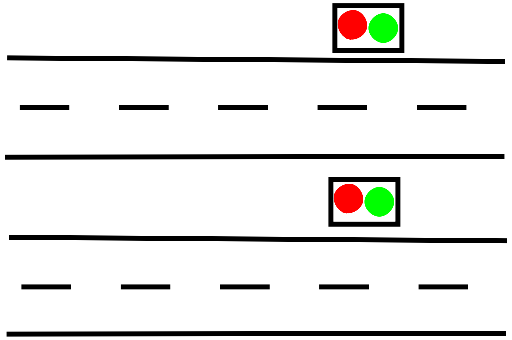
$\mathcal{F}_{strong} = ?$

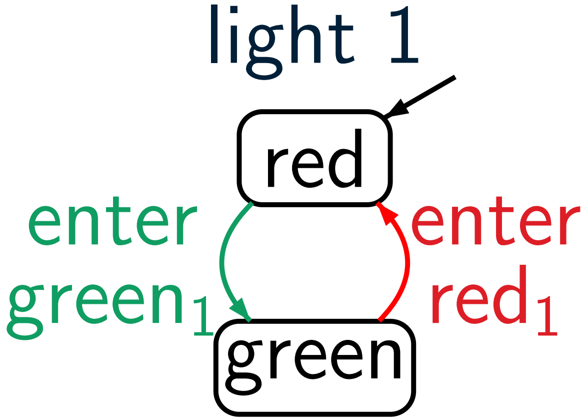
$\mathcal{F}_{weak} = ?$

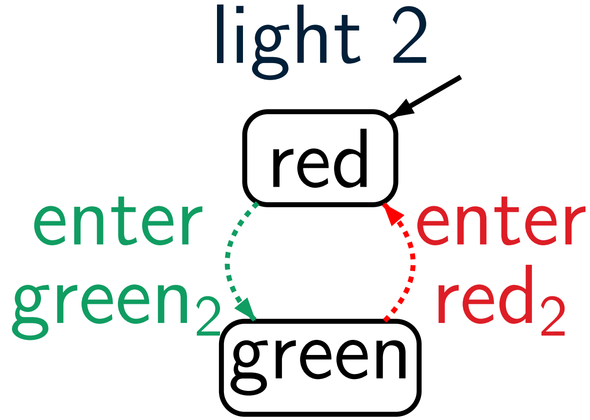
light 1 $|||$ light 2 $\models_{\mathcal{F}}$ E

$E \mathrel{\widehat{=}}$ "both lights are infinitely often green"

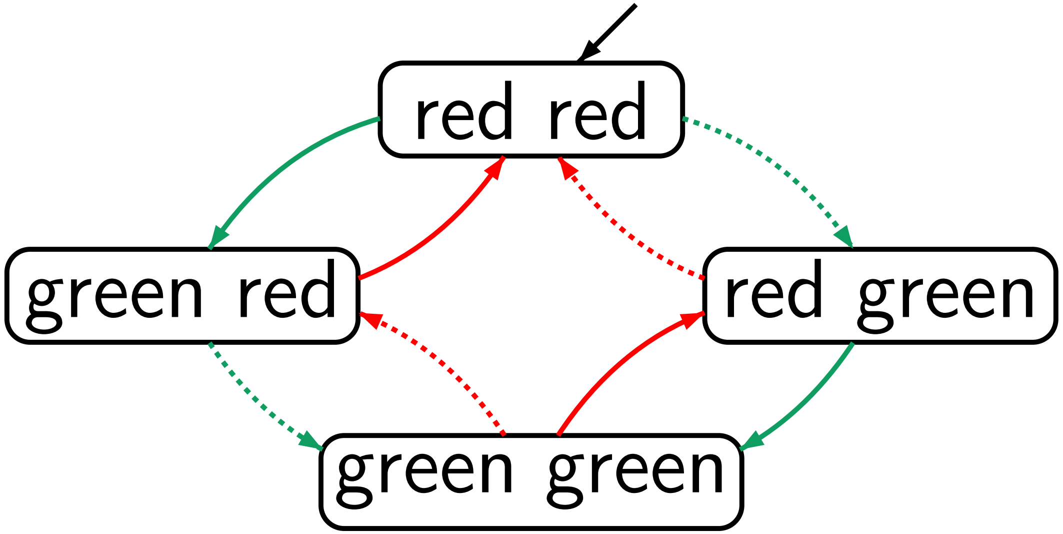
Two independent traffic lights







A_1 = actions of light 1
A_2 = actions of light 2



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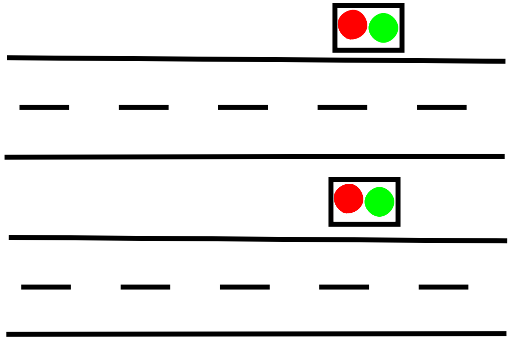

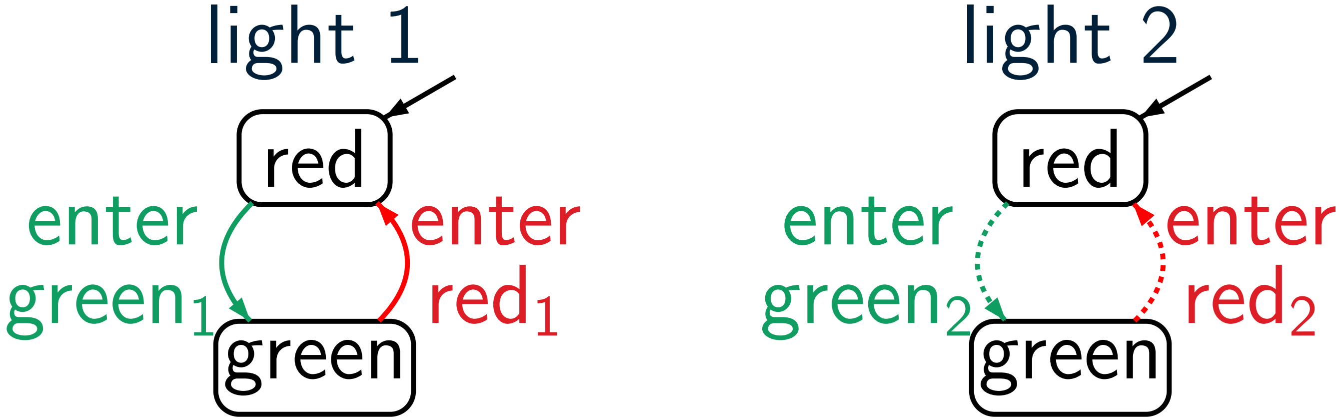

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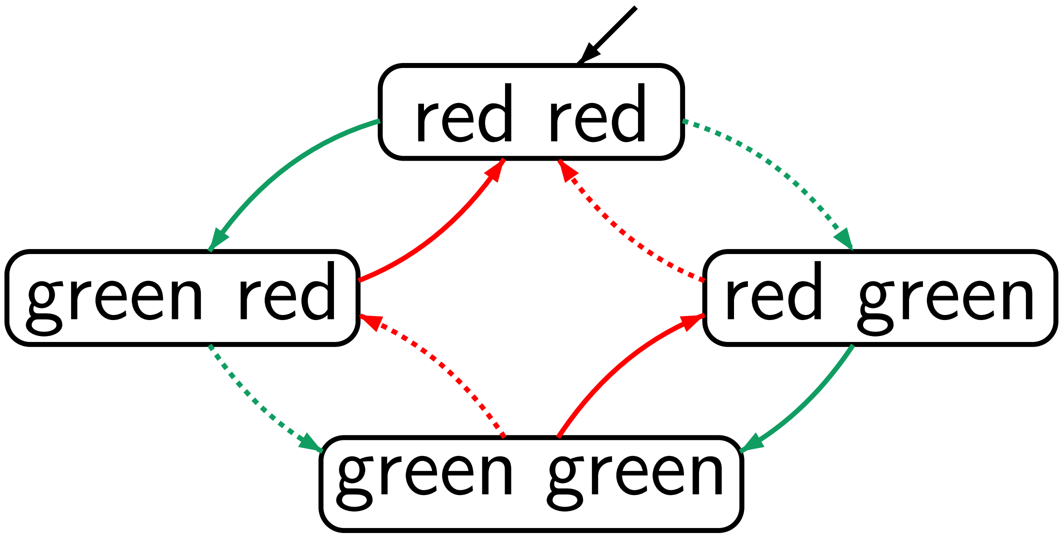

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$\mathcal{F}_{weak} = \{A_1, A_2\}$

light 1 $|||$ light 2 $\models_{\mathcal{F}} E$

$E \mathrel{\widehat{=}}$ "both lights are infinitely often green"

Example: MUTEX with fair arbiter

$$\mathcal{T} = \mathcal{T}_1 \parallel \text{Arbiter} \parallel \mathcal{T}_2$$

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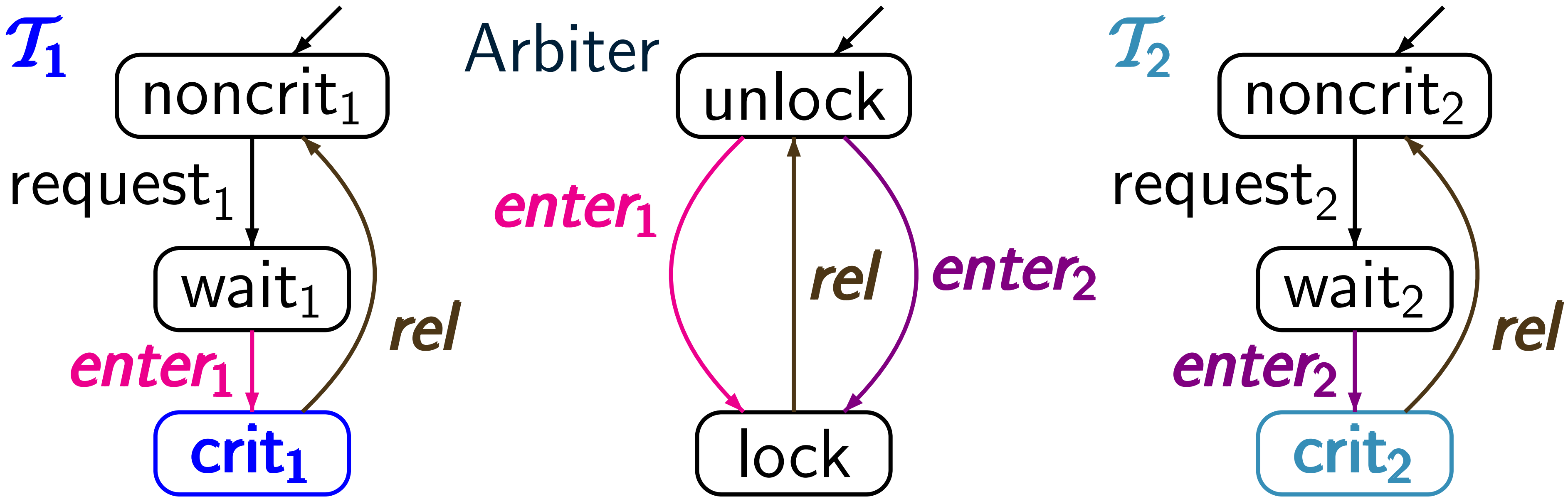

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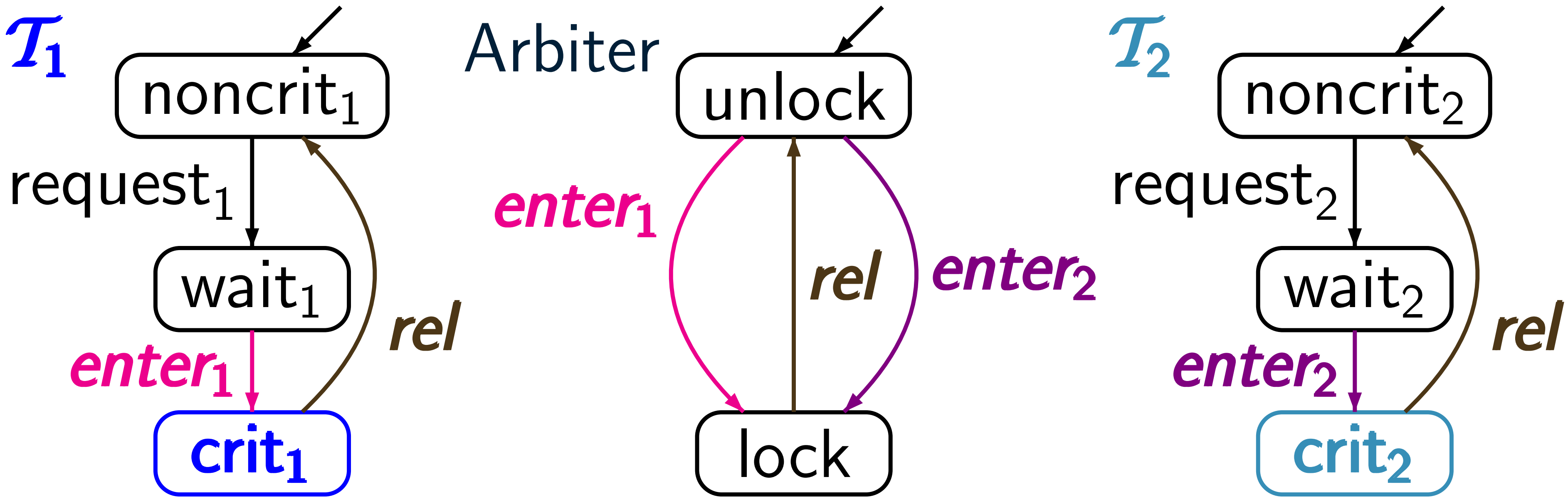
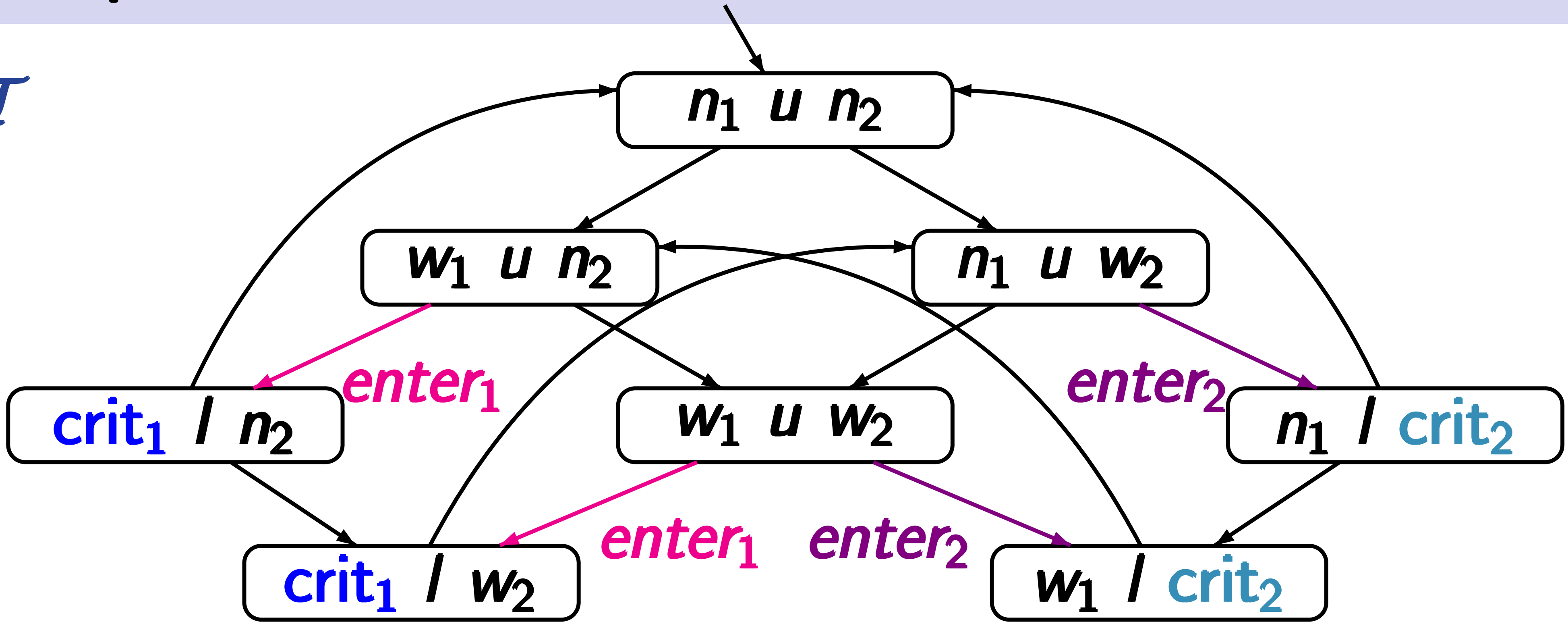


$\mathcal{T}_1$ and $\mathcal{T}_2$ compete to communicate with the arbiter by means of the actions *enter*$_1$ and *enter*$_2$, respectively

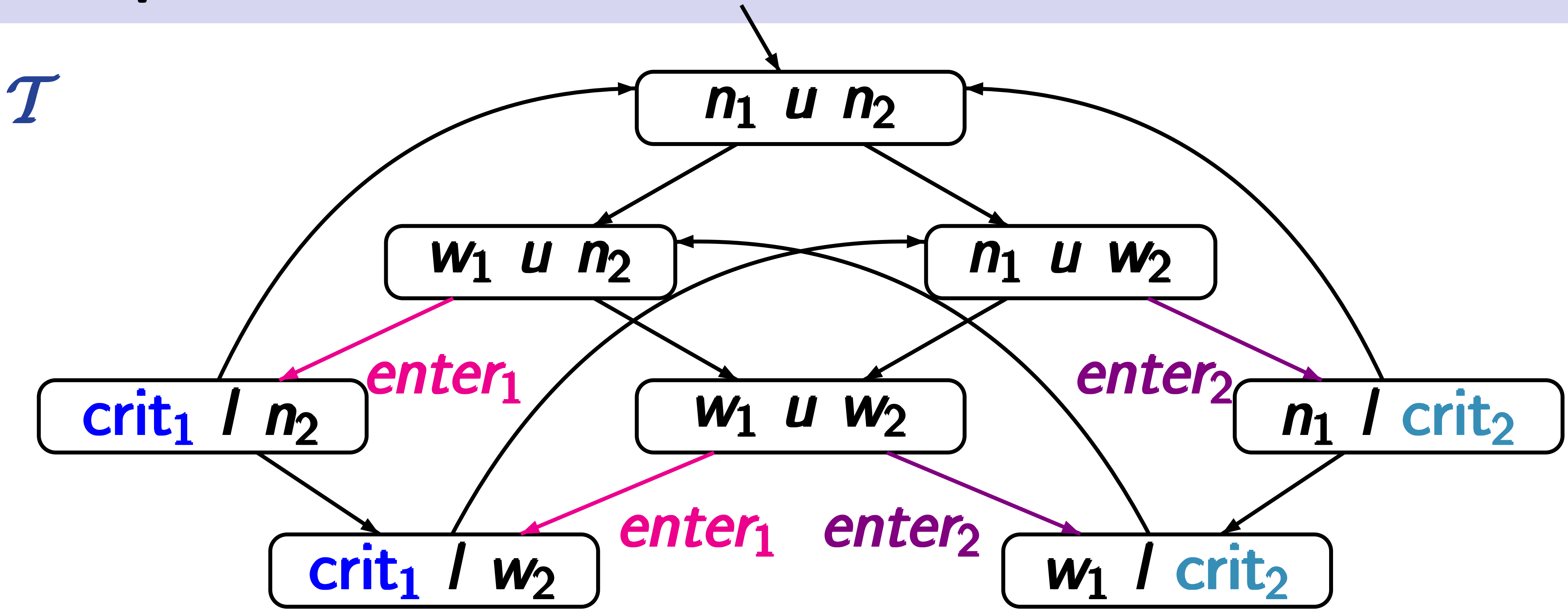
$\mathcal{T}$



LT property E: each waiting process eventually enters its critical section

$\mathcal{T} \not\models E$

Example: MUTEX with fair arbiter



LT property E: each waiting process eventually enters its critical section

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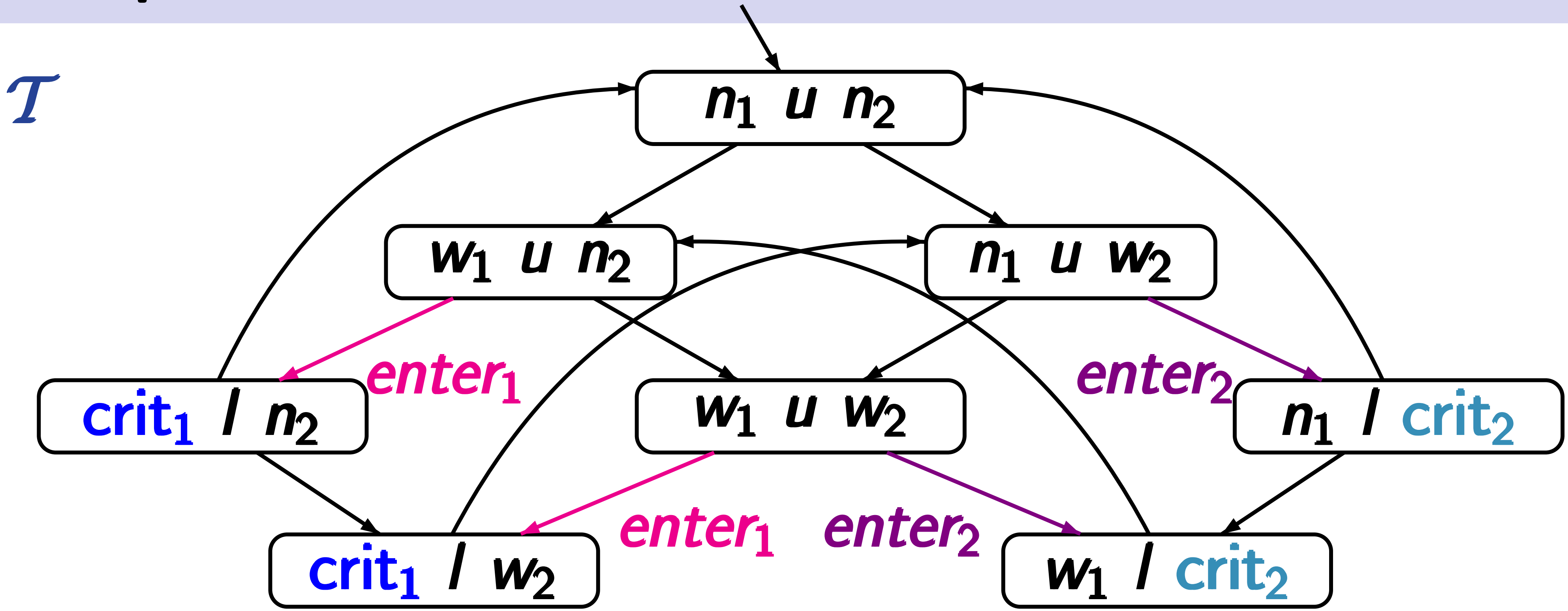
$$\mathcal{F}_{weak} = \{\{enter_1\}, \{enter_2\}\}$$

does $\mathcal{T} \models_{\mathcal{F}} E$ hold ?

Example: MUTEX with fair arbiter



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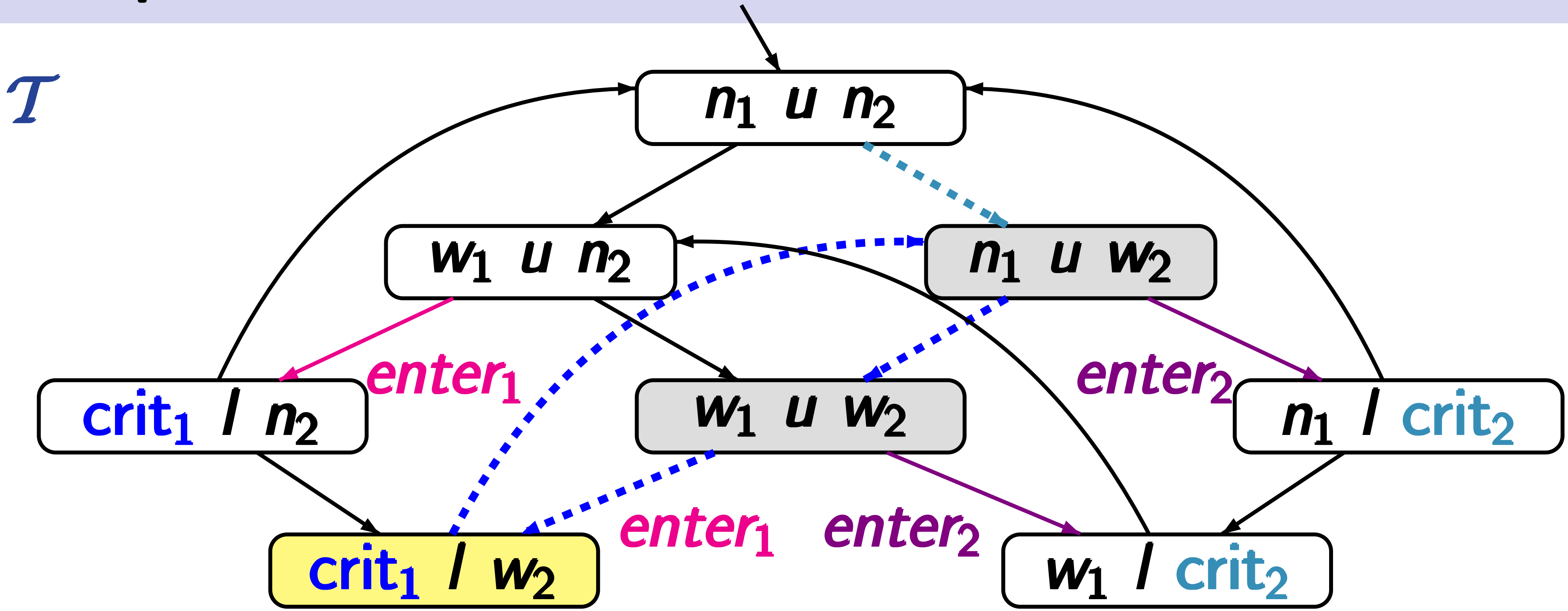
answer: **no**

Example: MUTEX with fair arbiter

$\mathcal{T}$



LT property E: each waiting process eventually enters its critical section

fairness assumption $\mathcal{F}$

$\mathcal{F}_{ucond} = \mathcal{F}_{strong} = \varnothing$

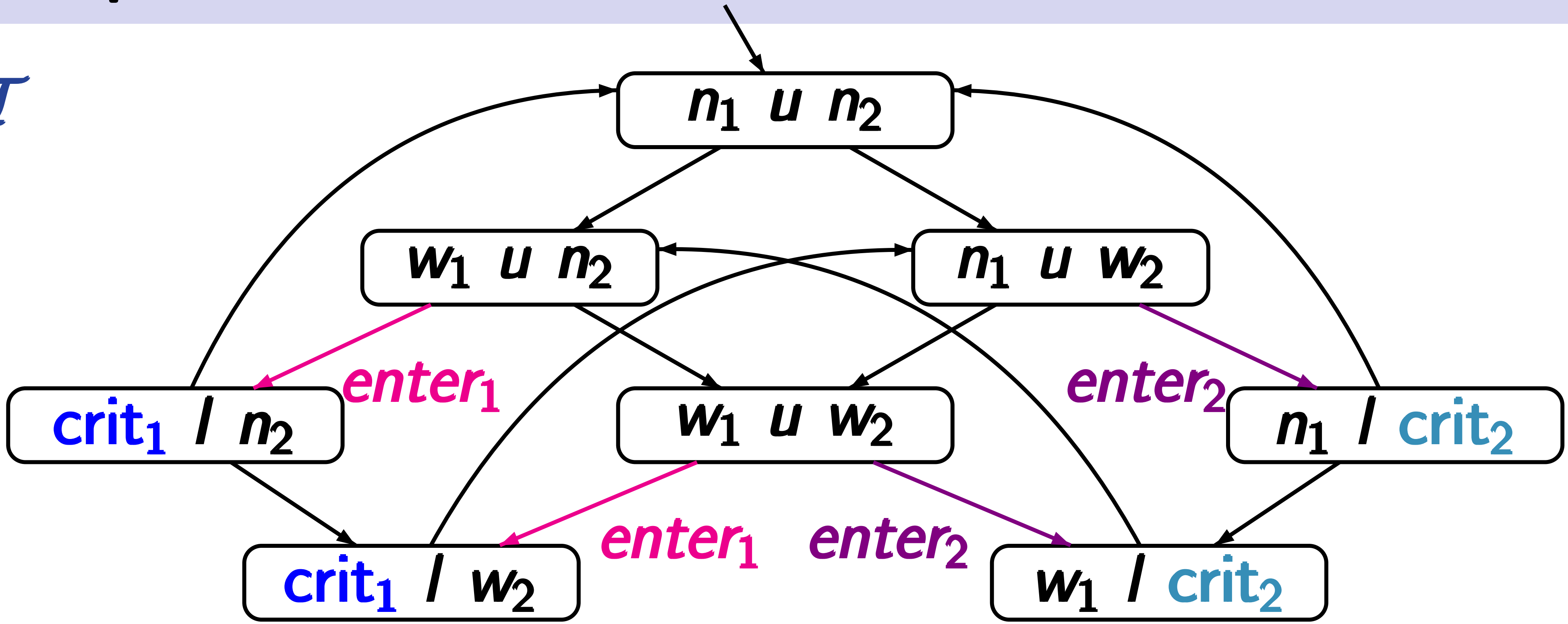
$\mathcal{F}_{weak} = \{\{enter_1\}, \{enter_2\}\}$

$\mathcal{T} \not\models_{\mathcal{F}} E$

as $enter_2$ is not enabled in $\langle crit_1, l, w_2 \rangle$

Example: MUTEX with fair arbiter

$\mathcal{T}$



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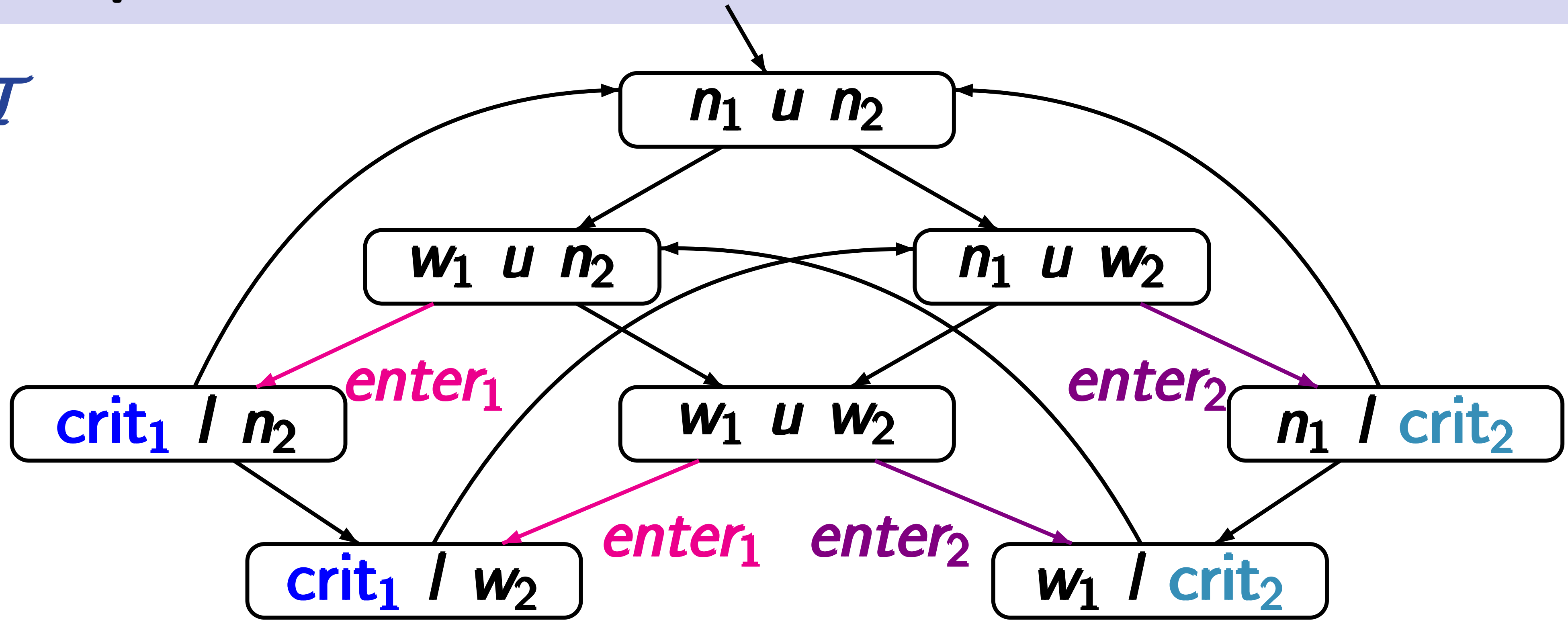

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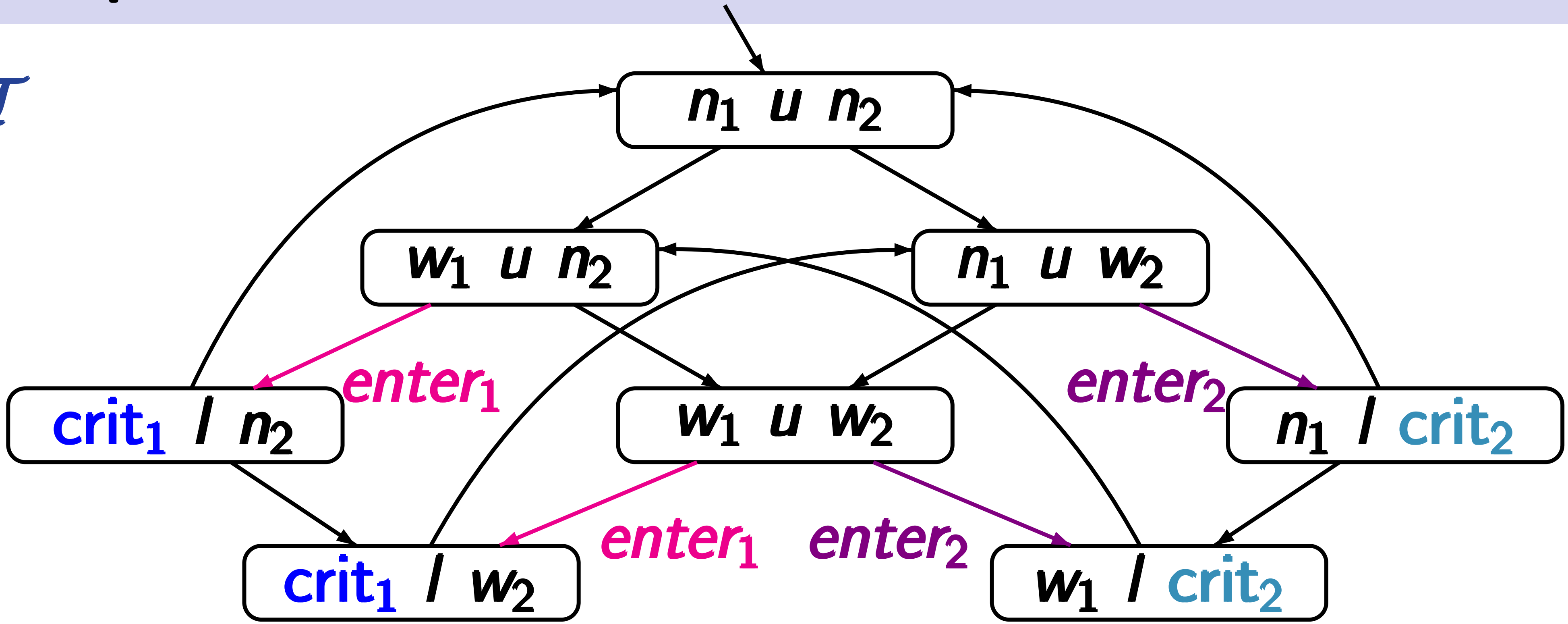

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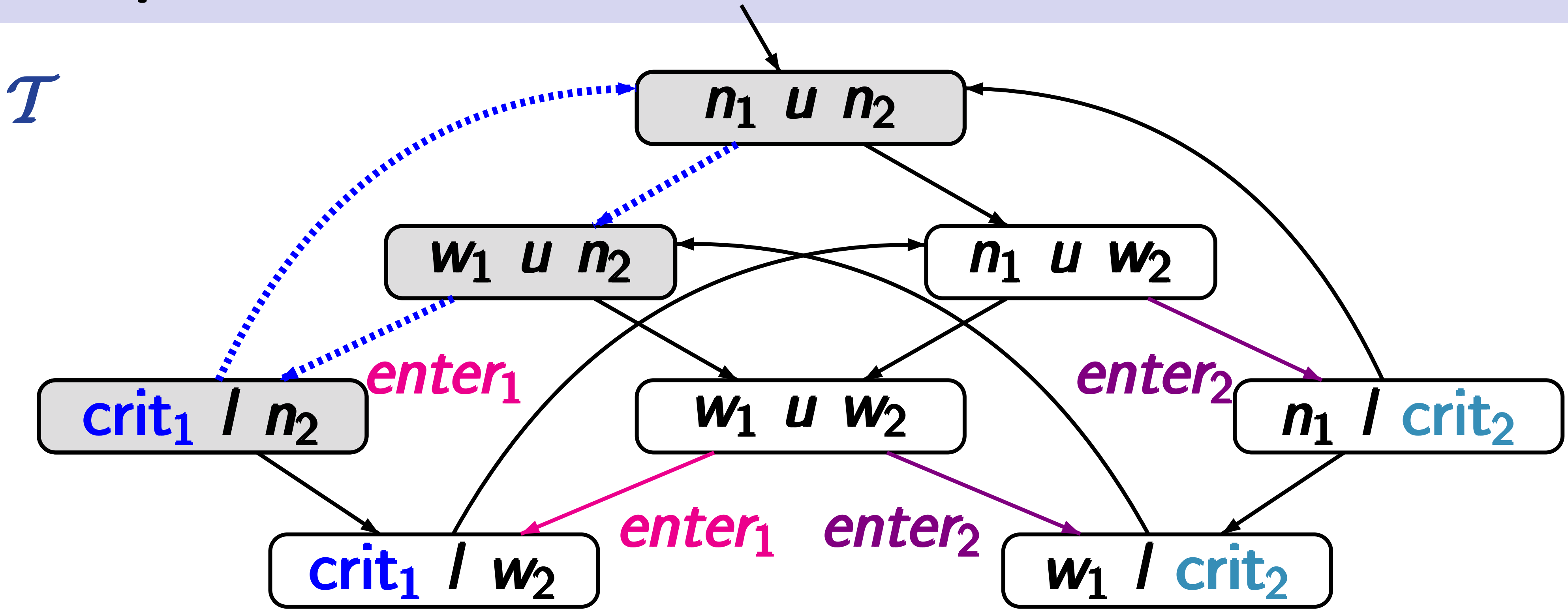

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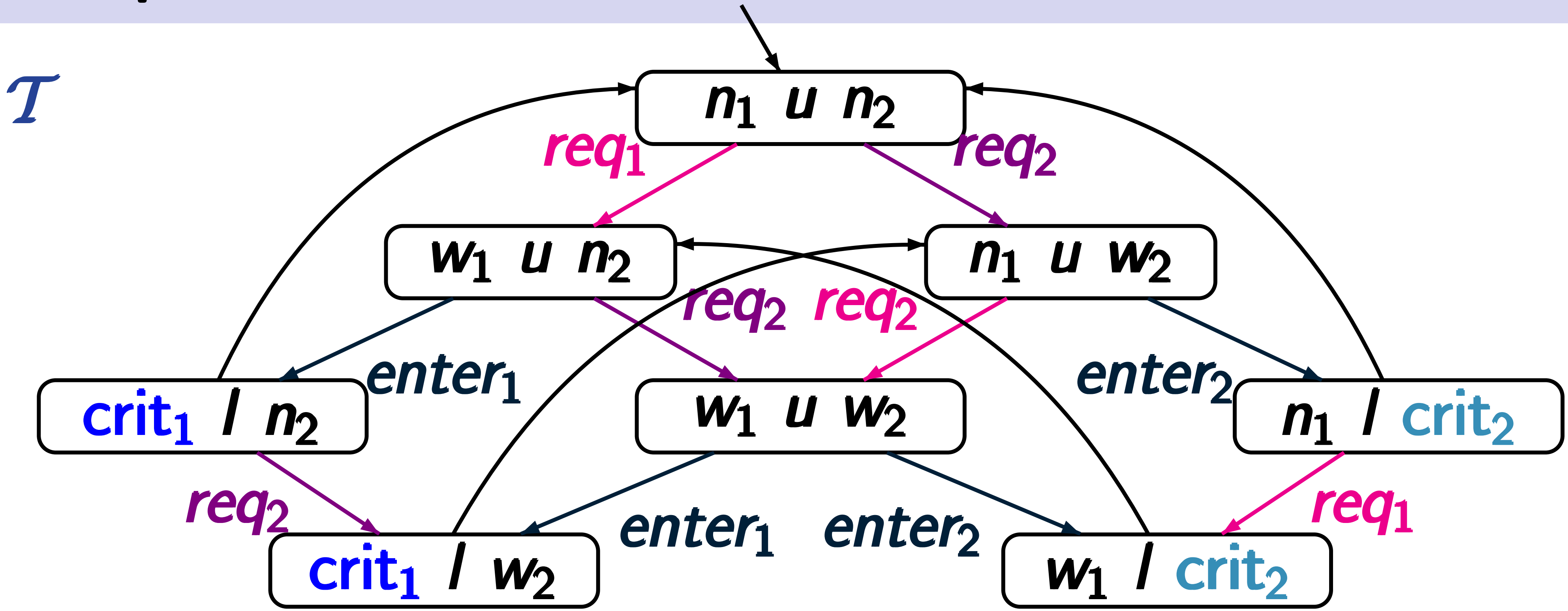

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Process fairness

LF2.6-19

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For asynchronous systems:

$$\text{parallelism} \;=\; \text{interleaving} + \text{fairness}$$

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- weak fairness for the nondetermism obtained from the interleaving of independent actions
- unconditional fairness: only of theoretical interest

Purpose of fairness conditions

parallelism = interleaving + fairness

Process fairness and other fairness conditions

- can compensate information loss due to interleaving
  or rule out other unrealistic pathological cases
- can be requirements for a scheduler
  or requirements for environment
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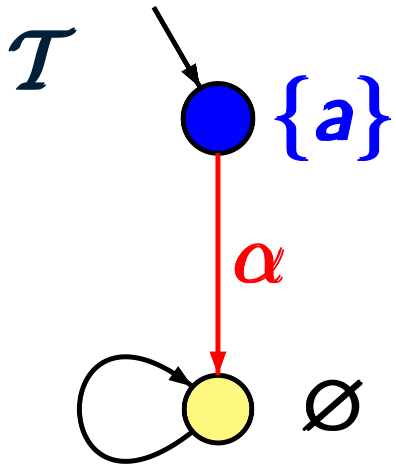
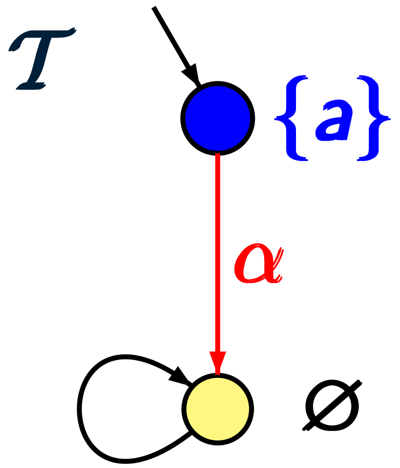
liveness properties:	fairness can be essential
safety properties:	fairness is irrelevant

Fairness



fairness assumption $\mathcal{F}$:
unconditional fairness for action set $\{\alpha\}$

does $\mathcal{T} \models_{\mathcal{F}}$ "infinitely often a" hold ?



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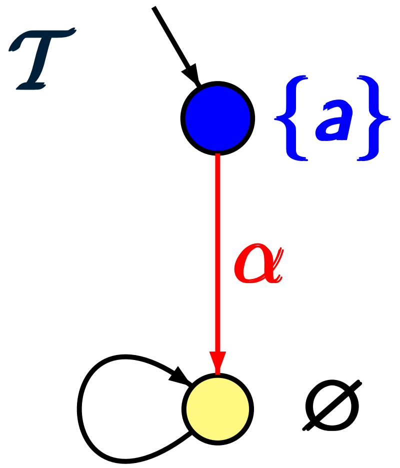

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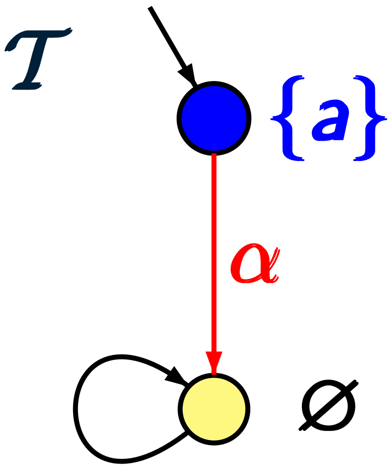

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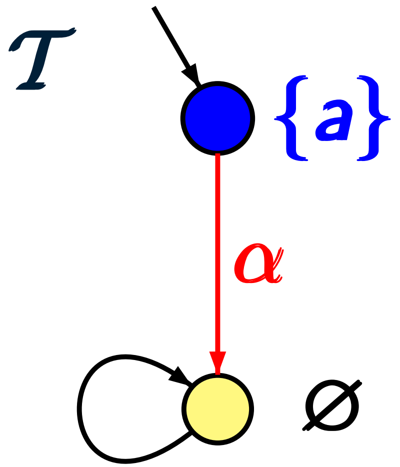
Realizability requires that each initial finite path fragment can be extended to a $\mathcal{F}$-fair path

Realizability of fairness assumptions



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Fairness assumption $\mathcal{F}$ is said to be realizable for a transition system $\mathcal{T}$ if for each reachable state s in $\mathcal{T}$ there exists a $\mathcal{F}$-fair path starting in s

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$\uparrow$

can always be guaranteed by a scheduler, i.e.,
an instance that resolves the nondeterminism in $\mathcal{T}$

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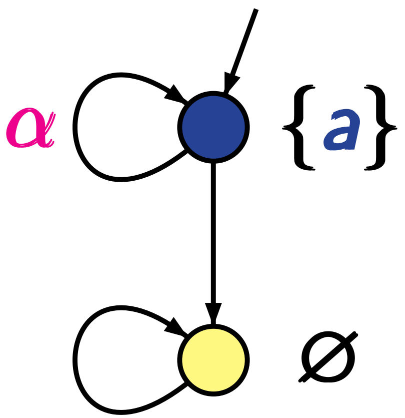

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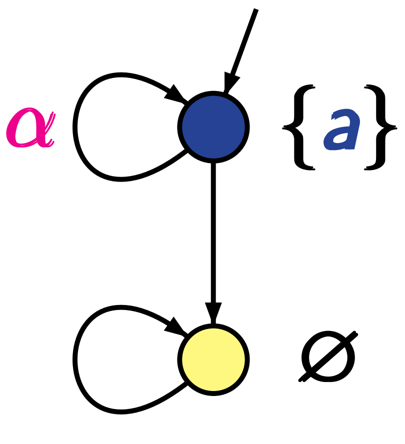

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E = invariant "always a"

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