Master 1 Informatique **2017-2018**
Université de Rennes 1

Modélisation et Vérification Formelle par Automates

Examination December 19, 2017
Subject leaders: Sophie Pinchinat and Victor Roussanaly

*You may **not** use written notes, published materials, testing aids, or any unauthorized material during the examination. Any of your answer should be justified. You are free to answer in French or in English. The scoring scale is indicative.*

Exercise 1 (Safety) 4pts

Consider the set AP of atomic propositions defined by $AP = \{(x = 0), (x > 1)\}$ and consider a nonterminating sequential computer program P that manipulates the variable x.

1. Formulate the following informally stated properties as LT properties:

1. "initially x differs from zero".
2. "initially x is equal to zero, but at some point x exceeds one".
3. "x exceeds one only finitely many times"
4. "x exceeds one infinitely often".

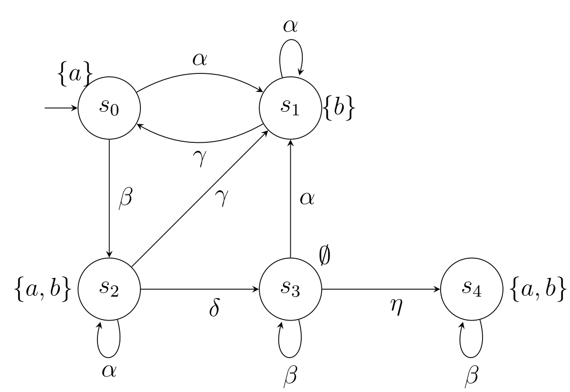
2. Determine which of the provided LT properties are safety properties? Which are liveness properties? Justify your answers by a formal argument.

Exercise 2 (Fairness) 4pts

Let P denote the set of traces of the form $\sigma = A_0A_1A_2 \ldots \in (2^{AP})^\omega$ such that

$$\overset{\infty}{\exists}\, k.A_k = \{a, b\} \wedge \exists n \geq 0.\forall k \geq n.(a \in A_k \Rightarrow b \in A_{k+1})$$

Consider the following fairness assumptions with respect to the transition system TS outlined on the right:



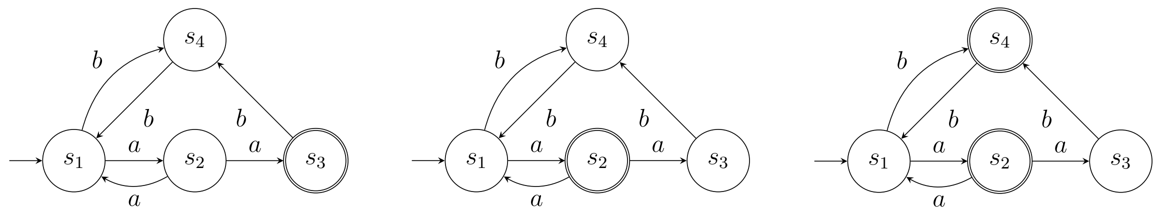
1. $\mathcal{F}_1 = (\{\{\alpha\}\}, \{\{\beta\}, \{\delta, \gamma\}, \{\eta\}\}, \emptyset)$.
Decide whether $TS \models_{\mathcal{F}_1} P$.

2. $\mathcal{F}_2 = (\{\{\alpha\}\}, \{\{\beta\}, \{\gamma\}\}, \{\{\eta\}\})$.
Decide whether $TS \models_{\mathcal{F}_2} P$.

Justify your answers.

Exercise 3 (NBA) 6pts

1. Give the language for the following three NBA (make your ω-regular expression for these languages as simple as possible):



2. Give an NBA over alphabet $\Sigma = \{a, b, c\}$ for:

- "initially a occurs, and at some point b occurs"
- "if a occurs somewhere, then afterwards (b occurs infinitely often iff c occurs infinitely often).

Exercise 4 (LTL2GNBA) 4pts

We consider the LTL formula $\psi = \Box(a \leftrightarrow \bigcirc\neg a)$ over the set of atomic propositions $AP = \{a\}$.

1. Convert $\neg\psi$ into an equivalent LTL-formula φ which is constructed according to the following grammar: $\varphi, \varphi' ::= \texttt{true} \mid \texttt{false} \mid a \mid \varphi \wedge \varphi' \mid \neg\varphi \mid \bigcirc\varphi \mid \varphi U \varphi'$.

2. Construct the set $cl(\varphi)$, which is the closure of φ.

3. Compute all elementary sets with respect to $cl(\varphi)$.
(Hint: There are six elementary sets.)

4. Construct the GNBA $\mathcal{G}_\varphi$ with $\mathcal{L}_\omega(\mathcal{G}_\varphi) = \text{Words}(\varphi)$. To that end,
(i) define its set of initial states and its acceptance components.
(ii) for each elementary set B, define $\delta(B, B \cap AP)$.

Exercise 5 (CTL Model Checking) 2pts

Consider the following CTL-formulas: $\Phi_1 = \exists\Diamond\forall\Box c$ and $\Phi_2 = \forall(aU\forall\Diamond c)$.
and the transition system outlined on the right. Decide whether $TS \models \Phi_i$ for $i = 1, 2$ using the CTL model checking algorithm from the lecture. *(Hint: Do not forget to translate to existential normal form and compute the satisfaction sets for subformulas.)*