

Introduction

Modelling parallel systems

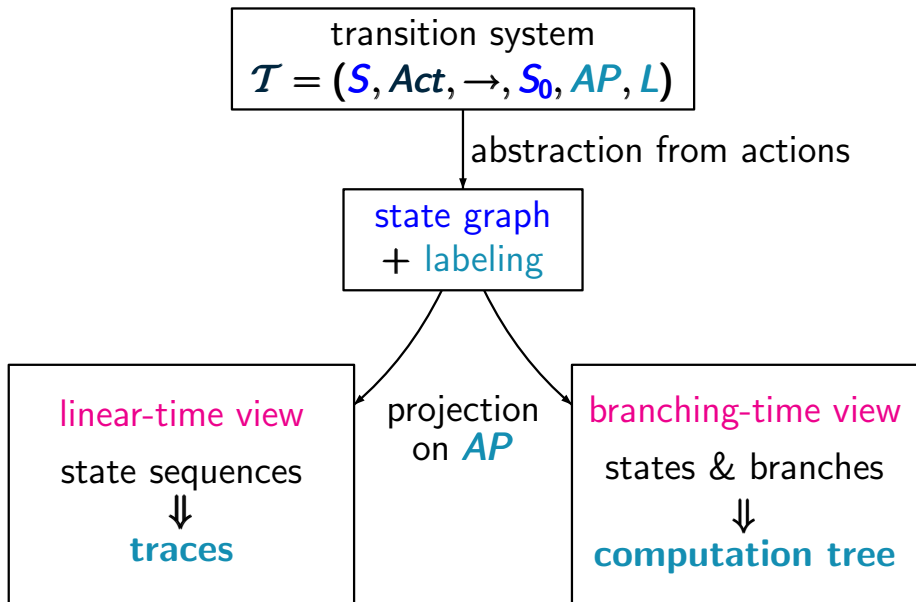
Linear Time Properties

Regular Properties

Linear Temporal Logic (LTL)

Computation Tree Logic

Equivalences and Abstraction



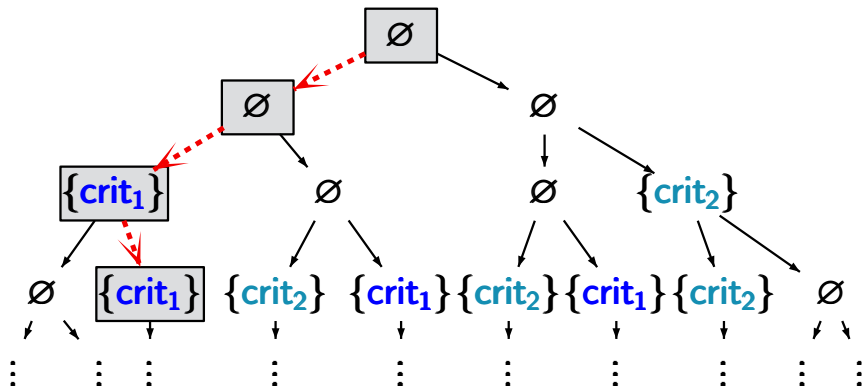
The computation tree of state s_0 in a transition system $\mathcal{T} = (\mathcal{S}, \text{Act}, \rightarrow, S_0, AP, L)$ arises by:

- unfolding $\mathcal{T}_{s_0} = (\mathcal{S}, \text{Act}, \rightarrow, s_0, AP, L)$ into a tree
- abstraction from the actions
- projection of the states s to their labels $L(s) \subseteq AP$

Example: computation tree

CTLSS4.1-1A

mutual exclusion with semaphore and $AP = \{\text{crit}_1, \text{crit}_2\}$:



path	$\langle nc_1, nc_2 \rangle$	$\langle wait_1, nc_2 \rangle$	$\langle \text{crit}_1, nc_2 \rangle$	$\langle \text{crit}_1, wait_2 \rangle$...
trace	\emptyset	\emptyset	$\{\text{crit}_1\}$	$\{\text{crit}_1\}$...

Linear vs. branching time

CTLSS4.1-2

	linear time	branching time
behavior	path based traces	state based computation tree
temporal logic	LTL path formulas	CTL state formulas
model checking	PSPACE-complete $\mathcal{O}(\text{size}(\mathcal{T}) \cdot \exp(\varphi))$	PTIME $\mathcal{O}(\text{size}(\mathcal{T}) \cdot \Phi)$
impl. relation	trace inclusion trace equivalence PSPACE-complete	simulation bisimulation PTIME
fairness	can be encoded	requires special treatment

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 syntax and semantics of CTL



 expressiveness of CTL and LTL

 CTL model checking

 fairness, counterexamples/witnesses

 CTL⁺ and CTL*

Equivalences and Abstraction

CTL (state) formulas:

$$\Phi ::= \text{true} \mid a \mid \Phi_1 \wedge \Phi_2 \mid \neg\Phi \mid \exists\varphi \mid \forall\varphi$$

CTL path formulas:

$$\varphi ::= \bigcirc\Phi \mid \Phi_1 \mathbf{U} \Phi_2$$

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eventually:

$$\exists\Diamond\Phi \stackrel{\text{def}}{=} \exists(\mathit{true} \mathbf{U} \Phi)$$

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$\diamond \hat{=}$ eventually

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unconditional process fairness $\forall\square\forall\diamond\mathit{crit}_1 \wedge \forall\square\forall\diamond\mathit{crit}_2$

Example: 15-puzzle

CTLSS4.1-5

6	8	2	12
4	1	13	5
	9	10	14
7	11	15	3



1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

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states:	game configurations
transitions:	legal moves

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- representation as parallel system:

left || *up* || *down* || *right*

with shared variables *field*[*i*] for $i = 1, \dots, 16$

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CTL specification:

$$\exists \diamond \bigwedge_{1 \leq i \leq 15} \text{“piece } i \text{ on field}[i]\text{”}$$

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CTL specification: seeking for a **witness** for

$$\exists \diamond \bigwedge_{1 \leq i \leq 15} \text{“piece } i \text{ on field}[i]\text{”}$$

define a satisfaction relation \models for CTL formulas over AP and a given TS $\mathcal{T} = (\mathcal{S}, Act, \rightarrow, \mathcal{S}_0, AP, L)$ without terminal states

- interpretation of **state formulas** over the **states**
- interpretation of **path formulas** over the **paths** (infinite path fragments)

for infinite path fragment $\pi = s_0 s_1 s_2 \dots$:

$\pi \models \text{true}$

$\pi \models a$ iff $s_0 \models a$, i.e., $a \in L(s_0)$

$\pi \models \varphi_1 \wedge \varphi_2$ iff $\pi \models \varphi_1$ and $\pi \models \varphi_2$

$\pi \models \neg \varphi$ iff $\pi \not\models \varphi$

$\pi \models \bigcirc \varphi$ iff $\text{suffix}(\pi, 1) = s_1 s_2 s_3 \dots \models \varphi$

$\pi \models \varphi_1 \mathbf{U} \varphi_2$ iff there exists $j \geq 0$ such that

$\text{suffix}(\pi, j) = s_j s_{j+1} s_{j+2} \dots \models \varphi_2$ and

$\text{suffix}(\pi, k) = s_k s_{k+1} s_{k+2} \dots \models \varphi_1$ for $0 \leq k < j$

Let $\pi = s_0 s_1 s_2 \dots$ be an infinite path fragment.

$$\pi \models \bigcirc \Phi \quad \text{iff} \quad s_1 \models \Phi$$

$$\pi \models \Phi_1 \mathbf{U} \Phi_2 \quad \text{iff} \quad \text{there exists } j \geq 0 \text{ such that}$$

$$s_j \models \Phi_2$$

$$s_k \models \Phi_1 \text{ for } 0 \leq k < j$$

semantics of derived operators:

$$\pi \models \diamond \Phi \quad \text{iff} \quad \text{there exists } j \geq 0 \text{ with } s_j \models \Phi$$

$$\pi \models \square \Phi \quad \text{iff} \quad \text{for all } j \geq 0 \text{ we have: } s_j \models \Phi$$

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s.t. $\pi \models \varphi$

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$s \models \forall\varphi$ iff for each path $\pi \in \text{Paths}(s)$:
 $\pi \models \varphi$

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satisfaction set for state formula Φ :

$$\text{Sat}(\Phi) \stackrel{\text{def}}{=} \{s \in S : s \models \Phi\}$$

satisfaction of state formulas over a TS \mathcal{T} :

$$\begin{aligned} \mathcal{T} \models \Phi & \text{ iff } S_0 \subseteq \text{Sat}(\Phi) \\ & \text{ iff } s_0 \models \Phi \text{ for all initial states } s_0 \text{ of } \mathcal{T} \end{aligned}$$

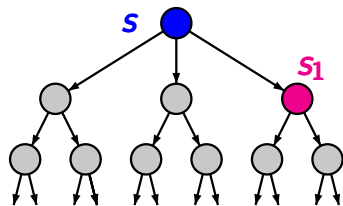
where S_0 is the set of initial states

$$\text{recall: } \text{Sat}(\Phi) = \{s \in S : s \models \Phi\}$$

$s \models \exists \bigcirc \Phi$ iff there exists $\pi = s s_1 s_2 \dots \in \text{Paths}(s)$
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$\exists \bigcirc \Phi$

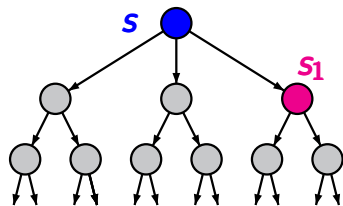


$\text{Post}(s) \cap \text{Sat}(\Phi) \neq \emptyset$

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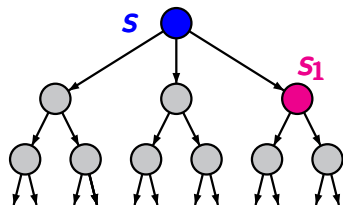
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Semantics of the next operator

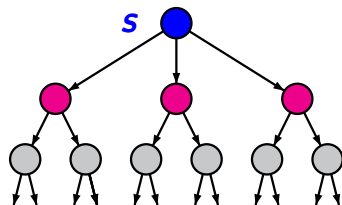
CTLSS4.1-8

$s \models \exists \bigcirc \Phi$ iff there exists $\pi = s s_1 s_2 \dots \in Paths(s)$
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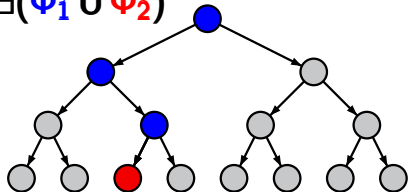
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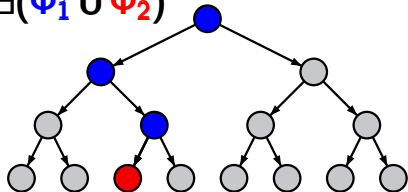
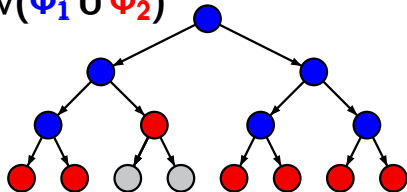
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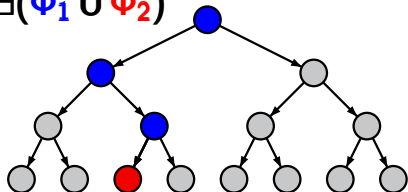
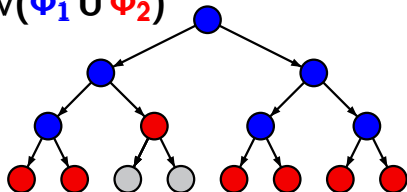
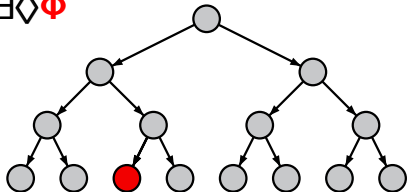
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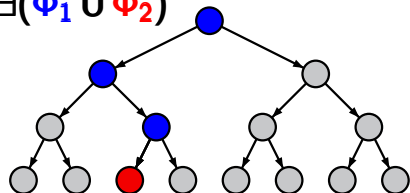
$Post(s) \subseteq Sat(\Phi)$

$\exists(\phi_1 \text{ U } \phi_2)$ 

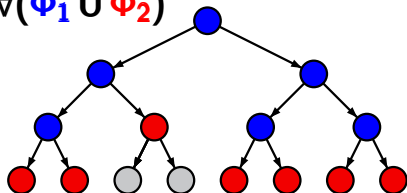
$\exists(\phi_1 \text{ U } \phi_2)$  $\forall(\phi_1 \text{ U } \phi_2)$ 

$\exists(\phi_1 U \phi_2)$

 $\forall(\phi_1 U \phi_2)$

 $\exists \diamond \phi$


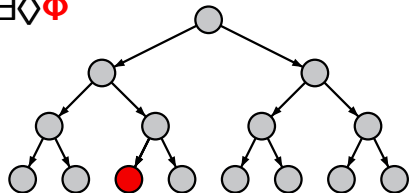
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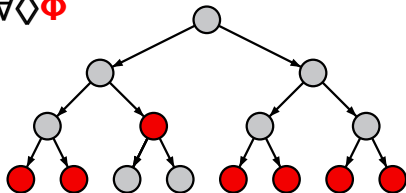
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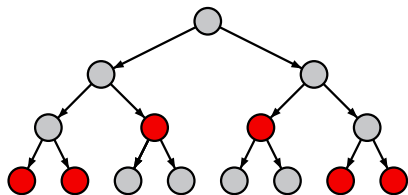
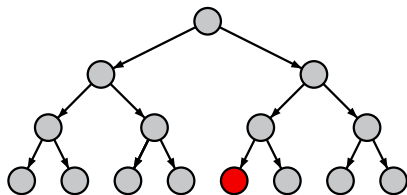


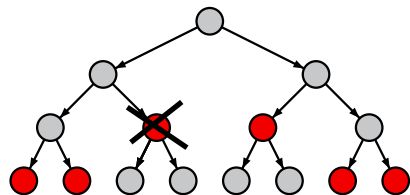
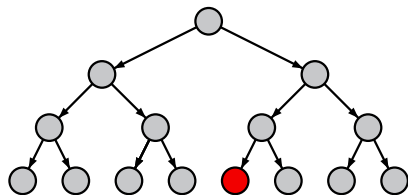
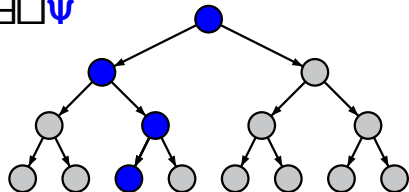
$$\exists \diamond \phi$$

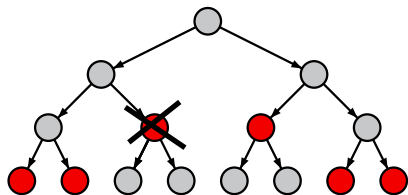
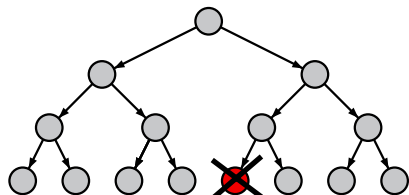
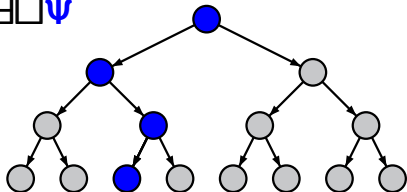
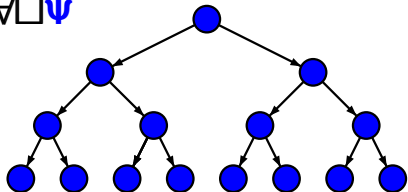


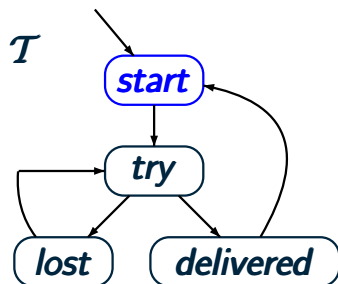
$$\forall \diamond \phi$$

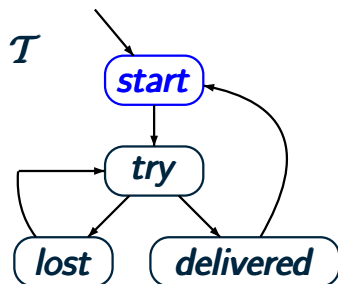


$\forall \diamond \phi$  $\exists \diamond \phi$ 

$\neg \forall \Diamond \Phi$

 $\exists \Diamond \Phi$

 $\exists \Box \Psi$


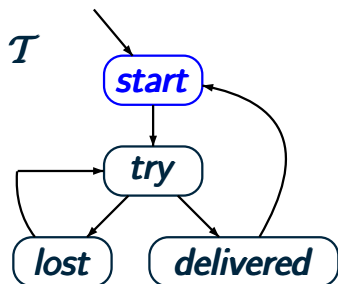
$\neg \forall \Diamond \phi$

 $\neg \exists \Diamond \phi$

 $\exists \Box \psi$

 $\forall \Box \psi$






CTL formula

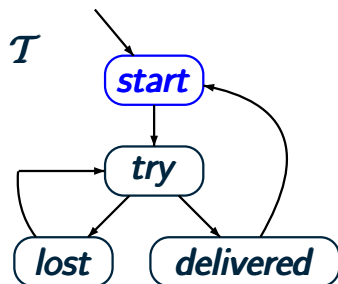
$$\phi = \forall \square \neg \forall \diamond \text{start}$$



CTL formula

$$\phi = \forall \square \forall \diamond \textit{start}$$

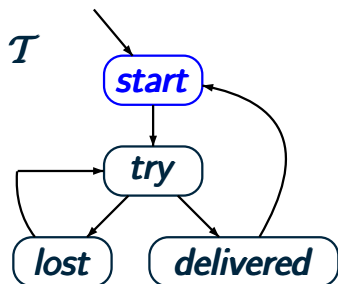
$$\textit{Sat}(\forall \diamond \textit{start}) = ?$$



CTL formula

$$\phi = \forall \square \forall \diamond \text{start}$$

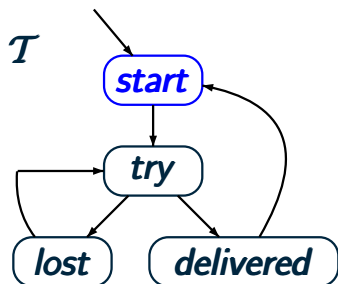
$$\text{Sat}(\forall \diamond \text{start}) = \{\text{start}, \text{delivered}\}$$



CTL formula

$$\phi = \forall \square \forall \diamond \text{start} \quad \cong \quad \forall \square (\text{start} \vee \text{delivered})$$

$$\text{Sat}(\forall \diamond \text{start}) = \{ \text{start}, \text{delivered} \}$$

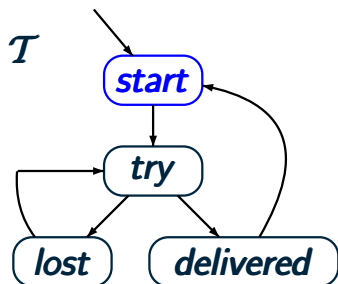


CTL formula

$$\Phi = \forall \square \forall \diamond \text{start} \quad \cong \quad \forall \square (\text{start} \vee \text{delivered})$$

$$\text{Sat}(\forall \diamond \text{start}) = \{\text{start}, \text{delivered}\}$$

$$\text{Sat}(\Phi) = \emptyset$$



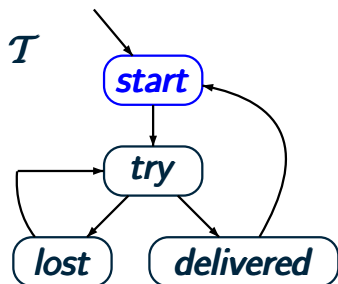
$$\mathcal{T} \not\models \forall \square \forall \diamond \text{start}$$

CTL formula

$$\Phi = \forall \square \forall \diamond \text{start} \quad \cong \quad \forall \square (\text{start} \vee \text{delivered})$$

$$\text{Sat}(\forall \diamond \text{start}) = \{ \text{start}, \text{delivered} \}$$

$$\text{Sat}(\Phi) = \emptyset$$



$$\mathcal{T} \not\models \forall \square \forall \diamond \text{start}$$

“infinitely often *start*”

CTL formula

$$\Phi = \forall \square \forall \diamond \text{start} \quad \cong \quad \forall \square (\text{start} \vee \text{delivered})$$

$$\text{Sat}(\forall \diamond \text{start}) = \{ \text{start}, \text{delivered} \}$$

$$\text{Sat}(\Phi) = \emptyset$$

Specifying “infinitely often” in CTL

CTLSS4.1-INF-OFTEN.TEX

If s is a state in a TS and $a \in AP$ then:

$$s \models_{\text{CTL}} \forall \square \forall \diamond a$$

iff for all paths $\pi = s_0 s_1 s_2 \dots \in \text{Paths}(s)$:

$$\exists i \geq 0. \text{ s.t. } s_i \models a$$

If s is a state in a TS and $a \in AP$ then:

$$s \models_{\text{CTL}} \forall \square \forall \diamond a$$

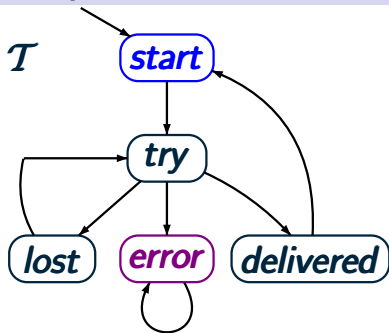
iff for all paths $\pi = s_0 s_1 s_2 \dots \in \text{Paths}(s)$:

$$\exists i \geq 0. \quad \text{s.t.} \quad s_i \models a$$

iff $s \models_{\text{LTL}} \square \diamond a$

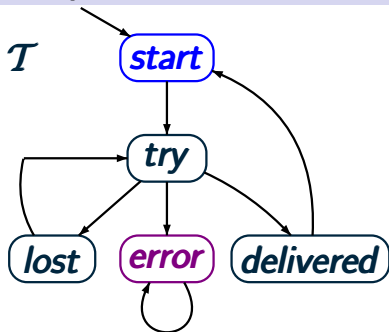
Example: CTL semantics

CTLSS4.1-16



$\mathcal{T} \models \exists \Diamond \Box \neg \text{start}$?

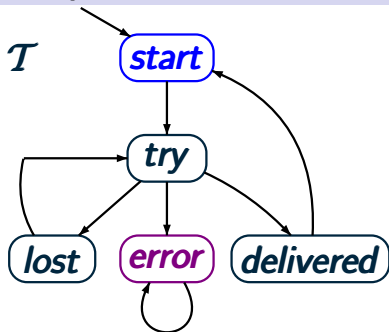
$\Phi_1 = \exists \Diamond \Box \neg \text{start}$



$\mathcal{T} \models \exists \diamond \forall \square \neg \text{start}$?

$$\phi_1 = \exists \diamond \forall \square \neg \text{start}$$

$$\text{Sat}(\forall \square \neg \text{start}) = \{ \text{error} \}$$



$\mathcal{T} \models \exists \diamond \forall \square \neg \text{start}$?

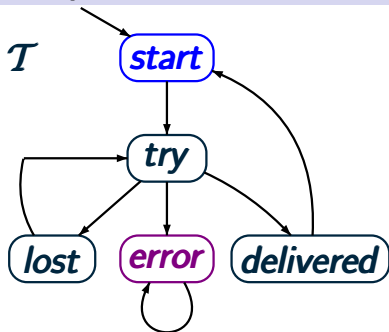
$$\phi_1 = \exists \diamond \forall \square \neg \text{start} \rightsquigarrow \exists \diamond \text{error}$$

$$\text{Sat}(\forall \square \neg \text{start}) = \{ \text{error} \}$$

$$\text{Sat}(\exists \diamond \forall \square \neg \text{start}) = ?$$

Example: CTL semantics

CTLSS4.1-16



$$\mathcal{T} \models \exists \diamond \forall \square \neg \text{start} \quad \checkmark$$

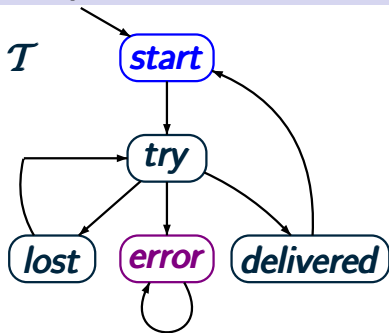
$$\phi_1 = \exists \diamond \forall \square \neg \text{start} \rightsquigarrow \exists \diamond \text{error}$$

$$\text{Sat}(\forall \square \neg \text{start}) = \{\text{error}\}$$

$$\text{Sat}(\exists \diamond \forall \square \neg \text{start}) = \text{Sat}(\exists \diamond \text{error}) = \text{“all states”}$$

Example: CTL semantics

CTLSS4.1-16



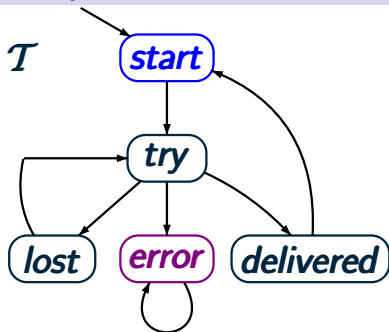
$$\mathcal{T} \models \exists \Diamond \Box \neg \text{start}$$

$$\mathcal{T} \models \forall \bigcirc \bigcirc \bigcirc \Box \neg \text{start} ?$$

$$\Phi_2 = \forall \bigcirc \bigcirc \bigcirc \Box \neg \text{start}$$

Example: CTL semantics

CTLSS4.1-16



$$\mathcal{T} \models \exists \Diamond \Box \neg \text{start}$$

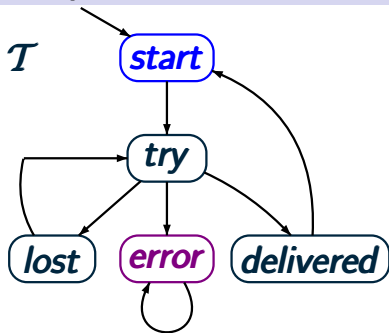
$$\mathcal{T} \models \forall \bigcirc \bigcirc \Box \neg \text{start} ?$$

$$\Phi_2 = \forall \bigcirc \bigcirc \Box \neg \text{start}$$

$$\text{Sat}(\forall \Box \neg \text{start}) = \{\text{error}\}$$

Example: CTL semantics

CTLSS4.1-16



$$\mathcal{T} \models \exists \Diamond \Box \neg \text{start}$$

$$\mathcal{T} \models \forall \Box \Diamond \Box \neg \text{start} ?$$

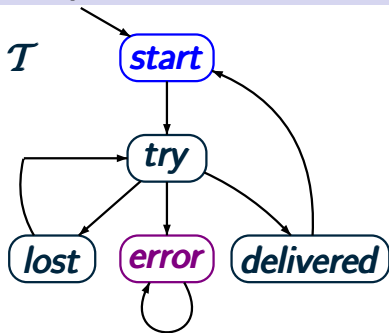
$$\phi_2 = \forall \Box \Diamond \Box \neg \text{start} \rightsquigarrow \forall \Box \Diamond \Box \text{error}$$

$$\text{Sat}(\forall \Box \neg \text{start}) = \{\text{error}\}$$

$$\text{Sat}(\exists \Box \Diamond \Box \neg \text{start}) = \{\text{error}, \text{try}\}$$

Example: CTL semantics

CTLSS4.1-16



$$\mathcal{T} \models \exists \Diamond \Box \neg \text{start}$$

$$\mathcal{T} \models \forall \Box \exists \Box \Box \neg \text{start} ?$$

$$\Phi_2 = \forall \Box \exists \Box \Box \neg \text{start}$$

$$\rightsquigarrow \Box (\text{error} \vee \text{try})$$

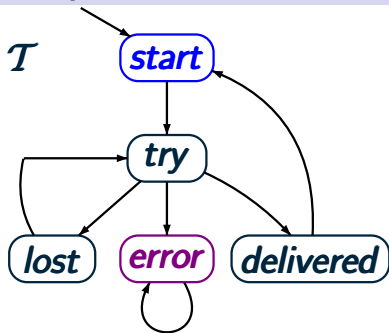
$$\text{Sat}(\forall \Box \neg \text{start}) = \{\text{error}\}$$

$$\text{Sat}(\exists \Box \Box \neg \text{start}) = \{\text{error}, \text{try}\}$$

$$\text{Sat}(\forall \Box \exists \Box \Box \neg \text{start}) = ?$$

Example: CTL semantics

CTLSS4.1-16



$$\mathcal{T} \models \exists \Diamond \Box \neg \text{start}$$

$$\mathcal{T} \models \forall \bigcirc \bigcirc \Box \neg \text{start} \quad \checkmark$$

$$\Phi_2 = \forall \bigcirc \bigcirc \Box \neg \text{start}$$

$$\rightsquigarrow \Box (\text{error} \vee \text{try})$$

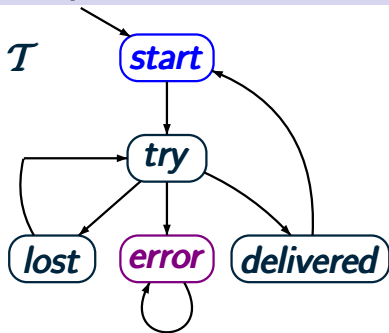
$$\text{Sat}(\forall \Box \neg \text{start}) = \{\text{error}\}$$

$$\text{Sat}(\exists \bigcirc \Box \neg \text{start}) = \{\text{error}, \text{try}\}$$

$$\text{Sat}(\forall \bigcirc \bigcirc \Box \neg \text{start}) = \{\text{error}, \text{lost}, \text{start}\}$$

Example: CTL semantics

CTLSS4.1-16



$$\Phi_3 = \exists \text{O} \text{O} \text{O} \text{O} \square \neg \text{start}$$

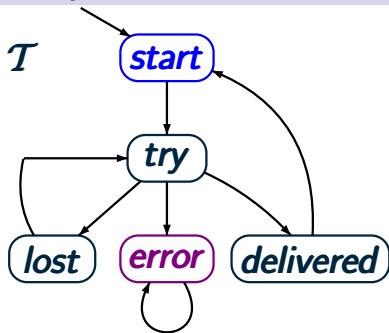
$$\mathcal{T} \models \exists \text{O} \square \neg \text{start}$$

$$\mathcal{T} \models \forall \text{O} \text{E} \text{O} \square \neg \text{start}$$

$$\mathcal{T} \models \exists \text{O} \text{E} \text{O} \square \neg \text{start} ?$$

Example: CTL semantics

CTLSS4.1-16



$$\mathcal{T} \models \exists \Diamond \Box \neg \text{start}$$

$$\mathcal{T} \models \forall \Box \exists \Diamond \Box \neg \text{start}$$

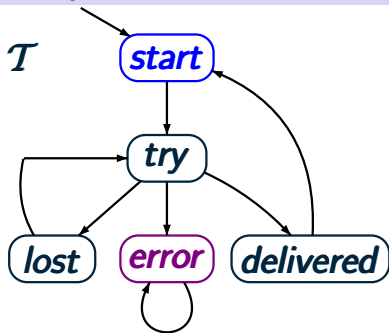
$$\mathcal{T} \models \exists \Box \exists \Diamond \Box \neg \text{start} ?$$

$$\Phi_3 = \exists \Box \exists \Diamond \Box \neg \text{start}$$

$$\text{Sat}(\forall \Box \neg \text{start}) = \{\text{error}\}$$

Example: CTL semantics

CTLSS4.1-16



$$\mathcal{T} \models \exists \Diamond \Box \neg \text{start}$$

$$\mathcal{T} \models \forall \Box \exists \Diamond \Box \neg \text{start}$$

$$\mathcal{T} \models \exists \Box \exists \Diamond \Box \neg \text{start} ?$$

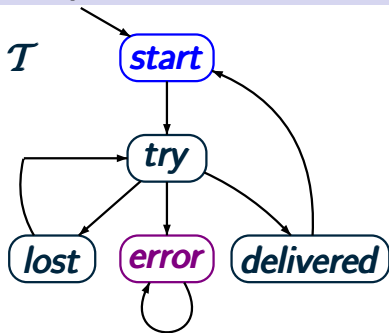
$$\Phi_3 = \exists \Box \exists \Diamond \Box \neg \text{start} \rightsquigarrow \exists \Box \exists \Diamond \Box \text{error}$$

$$\text{Sat}(\forall \Box \neg \text{start}) = \{\text{error}\}$$

$$\text{Sat}(\forall \Box \exists \Diamond \Box \neg \text{start}) = \{\text{error}\}$$

Example: CTL semantics

CTLSS4.1-16



$$\mathcal{T} \models \exists \Diamond \Box \neg \text{start}$$

$$\mathcal{T} \models \forall \Box \exists \Diamond \Box \neg \text{start}$$

$$\mathcal{T} \models \exists \Box \exists \Diamond \Box \neg \text{start} ?$$

$$\Phi_3 = \exists \Box \exists \Diamond \Box \neg \text{start} \rightsquigarrow \boxed{\exists \Box \text{error}}$$

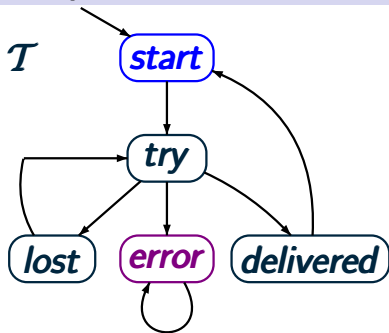
$$\text{Sat}(\forall \Box \neg \text{start}) = \{\text{error}\}$$

$$\text{Sat}(\forall \Box \exists \Diamond \Box \neg \text{start}) = \{\text{error}\}$$

$$\text{Sat}(\exists \Box \exists \Diamond \Box \neg \text{start}) = ?$$

Example: CTL semantics

CTLSS4.1-16



$$\mathcal{T} \models \exists \Diamond \Box \neg \text{start}$$

$$\mathcal{T} \models \forall \Box \exists \Diamond \neg \text{start}$$

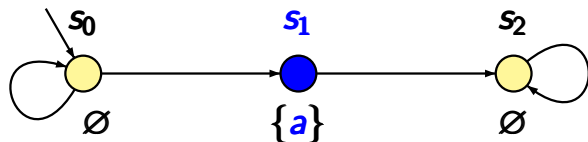
$$\mathcal{T} \not\models \exists \Box \exists \Diamond \neg \text{start}$$

$$\Phi_3 = \exists \Box \exists \Diamond \neg \text{start} \rightsquigarrow \boxed{\exists \Box \text{error}}$$

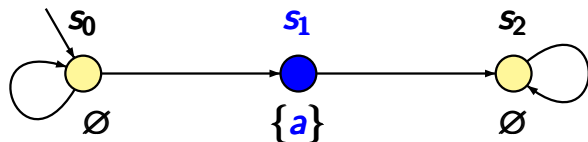
$$\text{Sat}(\forall \Box \neg \text{start}) = \{\text{error}\}$$

$$\text{Sat}(\forall \Box \exists \Diamond \neg \text{start}) = \{\text{error}\}$$

$$\text{Sat}(\exists \Box \exists \Diamond \neg \text{start}) = \{\text{error}, \text{try}\}$$



does $\mathcal{T} \models \exists \bigcirc \forall \square \neg a$ hold ?

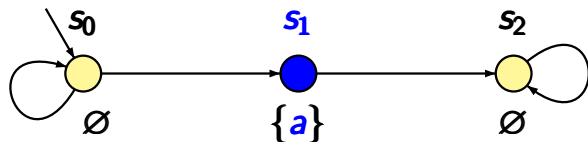


does $\mathcal{T} \models \exists \bigcirc \forall \square \neg a$ hold ?

answer: no

Example: CTL semantics

CTLSS4.1-17



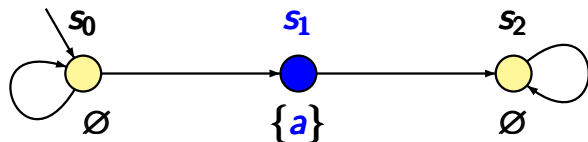
does $\mathcal{T} \models \exists \bigcirc \forall \square \neg a$ hold ?

answer: no

$$\text{Sat}(\forall \square \neg a) = \{s_2\}$$

Example: CTL semantics

CTLSS4.1-17



does $\mathcal{T} \models \exists \bigcirc \forall \square \neg a$ hold ?

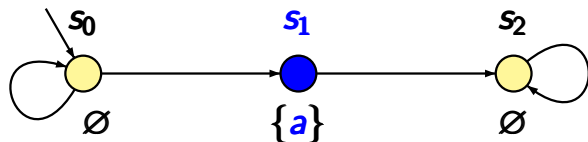
answer: no

$$\text{Sat}(\forall \square \neg a) = \{s_2\}$$

$$\text{Sat}(\exists \bigcirc \forall \square \neg a) = \{s_2, s_1\}$$

Example: CTL semantics

CTLSS4.1-17



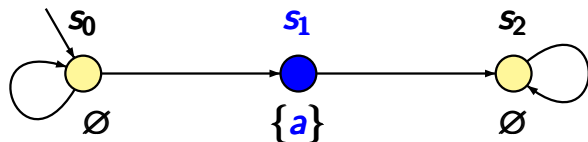
does $\mathcal{T} \models \exists \bigcirc \forall \square \neg a$ hold ?

answer: no

does $\mathcal{T} \models \forall \square \exists \bigcirc \neg a$ hold ?

Example: CTL semantics

CTLSS4.1-17

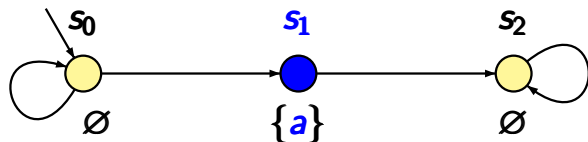


does $\mathcal{T} \models \exists \bigcirc \forall \square \neg a$ hold ?

answer: no

does $\mathcal{T} \models \forall \square \exists \bigcirc \neg a$ hold ?

answer: yes



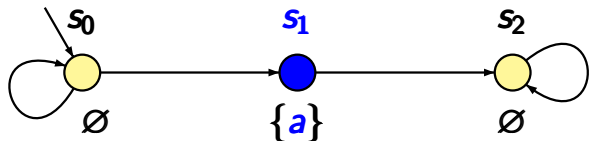
does $\mathcal{T} \models \exists \bigcirc \forall \square \neg a$ hold ?

answer: no

does $\mathcal{T} \models \forall \square \exists \bigcirc \neg a$ hold ?

answer: yes

$$\text{Sat}(\exists \bigcirc \neg a) = \{s_0, s_1, s_2\}$$



does $\mathcal{T} \models \exists \bigcirc \forall \square \neg a$ hold ?

answer: no

does $\mathcal{T} \models \forall \square \exists \bigcirc \neg a$ hold ?

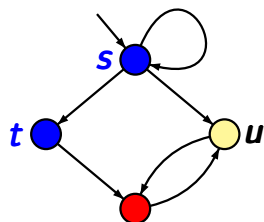
answer: yes

$$\text{Sat}(\exists \bigcirc \neg a) = \{s_0, s_1, s_2\}$$

$$\text{Sat}(\forall \square \exists \bigcirc \neg a) = \{s_0, s_1, s_2\}$$

Example: CTL semantics

CTLSS4.1-18



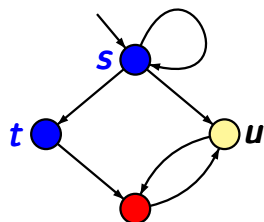
● $\hat{=} \{a\}$

● $\hat{=} \{b\}$

● $\hat{=} \emptyset$

$\mathcal{T} \models \exists \square \exists (a \cup b)$

?



● $\hat{=} \{a\}$

● $\hat{=} \{b\}$

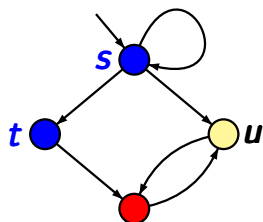
● $\hat{=} \emptyset$

$\mathcal{T} \models \exists \square \exists (a \text{ U } b)$

✓ as $s \models \exists (a \text{ U } b)$

Example: CTL semantics

CTLSS4.1-18



● $\hat{=} \{a\}$

● $\hat{=} \{b\}$

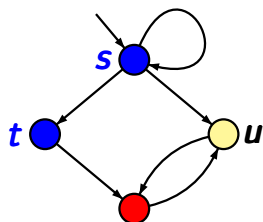
● $\hat{=} \emptyset$

$\mathcal{T} \models \exists \square \exists (a \cup b)$

✓ as $s s s \dots \models \square \exists (a \cup b)$

Example: CTL semantics

CTLSS4.1-18



● $\hat{=} \{a\}$

● $\hat{=} \{b\}$

● $\hat{=} \emptyset$

$\mathcal{T} \models \exists \Box \exists (a \cup b)$

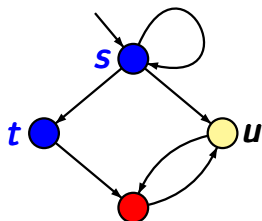
✓ as $s s s \dots \models \Box \exists (a \cup b)$

$\mathcal{T} \models \exists ((\exists \bigcirc a) \cup b)$

?

Example: CTL semantics

CTLSS4.1-18



$$\bullet \hat{=} \{a\}$$

$$\bullet \hat{=} \{b\}$$

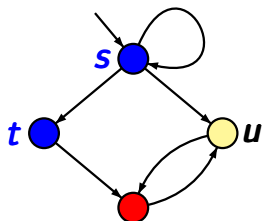
$$\bullet \hat{=} \emptyset$$

$$\mathcal{T} \models \exists \square \exists (a \cup b)$$

$$\checkmark \text{ as } s s s \dots \models \square \exists (a \cup b)$$

$$\mathcal{T} \not\models \exists ((\exists \bigcirc a) \cup b)$$

$$\text{as } t \not\models \exists \bigcirc a, u \not\models \exists \bigcirc a$$



$$\bullet \hat{=} \{a\}$$

$$\bullet \hat{=} \{b\}$$

$$\bullet \hat{=} \emptyset$$

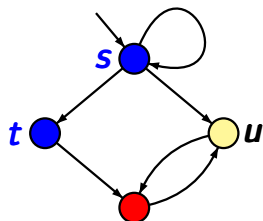
$$\mathcal{T} \models \exists \square \exists (a \cup b)$$

$$\checkmark \text{ as } s s s \dots \models \square \exists (a \cup b)$$

$$\mathcal{T} \not\models \exists ((\exists \bigcirc a) \cup b)$$

$$\text{as } t \not\models \exists \bigcirc a, u \not\models \exists \bigcirc a$$

$$\mathcal{T} \models \exists (a \cup \forall (\neg a \cup b)) \quad ?$$



$$\bullet \hat{=} \{a\}$$

$$\bullet \hat{=} \{b\}$$

$$\bullet \hat{=} \emptyset$$

$$\mathcal{T} \models \exists \Box \exists (a \cup b)$$

$$\checkmark \text{ as } s s s \dots \models \Box \exists (a \cup b)$$

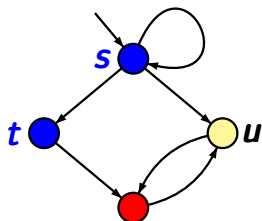
$$\mathcal{T} \not\models \exists ((\exists \circ a) \cup b)$$

$$\text{as } t \not\models \exists \circ a, u \not\models \exists \circ a$$

$$\mathcal{T} \models \exists (a \cup \forall (\neg a \cup b)) \quad \checkmark$$

Example: CTL semantics

CTLSS4.1-18



$$\bullet \hat{=} \{a\}$$

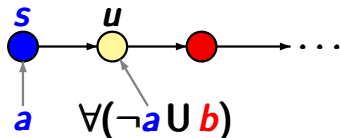
$$\bullet \hat{=} \{b\}$$

$$\bullet \hat{=} \emptyset$$

$$\mathcal{T} \models \exists \square \exists (a \cup b) \quad \checkmark \quad \text{as } s s s \dots \models \square \exists (a \cup b)$$

$$\mathcal{T} \not\models \exists ((\exists \bigcirc a) \cup b) \quad \text{as } t \not\models \exists \bigcirc a, u \not\models \exists \bigcirc a$$

$$\mathcal{T} \models \exists (a \cup \forall (\neg a \cup b)) \quad \checkmark$$



$$\models a \cup \forall (\neg a \cup b)$$

Let \mathcal{T} be a transition system and ϕ a CTL formula.
Is the following statement correct ?

if $\mathcal{T} \not\models \neg\phi$ then $\mathcal{T} \models \phi$

Let \mathcal{T} be a transition system and ϕ a CTL formula.
Is the following statement correct ?

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answer: no

Let \mathcal{T} be a transition system and ϕ a CTL formula.
Is the following statement correct ?

if $\mathcal{T} \not\models \neg\phi$ then $\mathcal{T} \models \phi$

answer: no

$\mathcal{T} \models \phi$ iff $s_0 \models \phi$ for all initial states s_0

Let \mathcal{T} be a transition system and ϕ a CTL formula.
Is the following statement correct ?

if $\mathcal{T} \not\models \neg\phi$ then $\mathcal{T} \models \phi$

answer: no

$\mathcal{T} \models \phi$ iff $s_0 \models \phi$ for all initial states s_0

$\mathcal{T} \not\models \neg\phi$ iff there exists an initial state s_0 with
 $s_0 \not\models \neg\phi$

Let \mathcal{T} be a transition system and ϕ a CTL formula.
Is the following statement correct ?

if $\mathcal{T} \not\models \neg\phi$ then $\mathcal{T} \models \phi$

answer: no

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Correct or wrong?

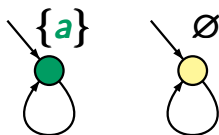
CTLSS4.1-19

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Is the following statement correct ?

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answer: no

transition system \mathcal{T} with 2 initial states:



$\mathcal{T} \not\models \exists\Box a$

$\mathcal{T} \not\models \neg\exists\Box a$

given: finite directed graph $G = (V, E)$

question: does G have a **Hamilton path**, i.e., a path that visits each node exactly once ?

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Hamilton path problem

CTLSS4.1-20

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s.t. G has a **Hamilton path** iff $\mathcal{T}_G \not\models \Phi$

finite digraph G	\rightsquigarrow	finite TS \mathcal{T}_G + CTL formula Φ
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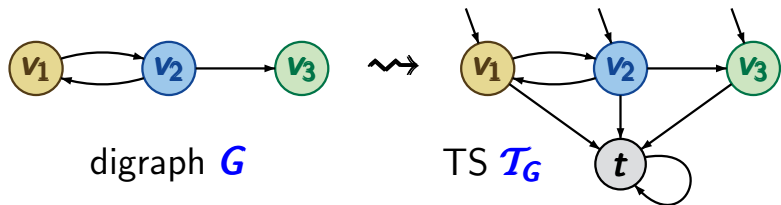


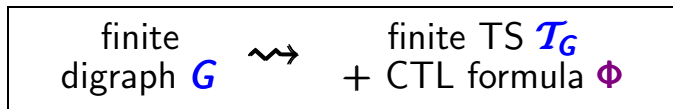
digraph G

CTL-encoding of the Hamilton path problem CTLSS4.1-20

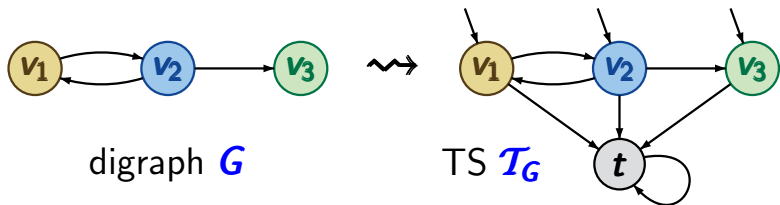
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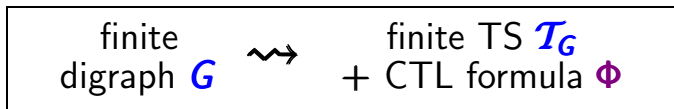


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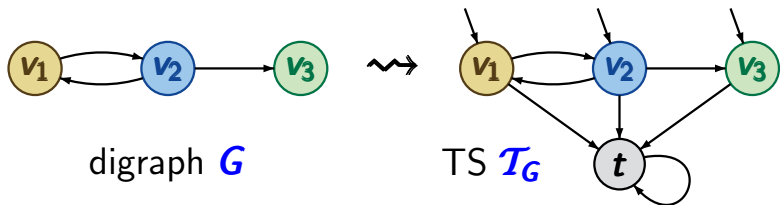


CTL formula Φ

$$\begin{aligned}
 & (v_1 \wedge \exists O(v_2 \wedge \exists O v_3)) \vee (v_1 \wedge \exists O(v_3 \wedge \exists O v_2)) \vee \\
 & (v_2 \wedge \exists O(v_1 \wedge \exists O v_3)) \vee (v_2 \wedge \exists O(v_3 \wedge \exists O v_1)) \vee \\
 & (v_3 \wedge \exists O(v_1 \wedge \exists O v_2)) \vee (v_3 \wedge \exists O(v_2 \wedge \exists O v_1))
 \end{aligned}$$



s.t. G has a Hamilton path iff $\mathcal{T}_G \not\models \Phi$



CTL formula Φ = negation of the formula

$$\begin{aligned}
 & (v_1 \wedge \exists O(v_2 \wedge \exists O v_3)) \vee (v_1 \wedge \exists O(v_3 \wedge \exists O v_2)) \vee \\
 & (v_2 \wedge \exists O(v_1 \wedge \exists O v_3)) \vee (v_2 \wedge \exists O(v_3 \wedge \exists O v_1)) \vee \\
 & (v_3 \wedge \exists O(v_1 \wedge \exists O v_2)) \vee (v_3 \wedge \exists O(v_2 \wedge \exists O v_1))
 \end{aligned}$$

$\Phi_1 \equiv \Phi_2$ iff for all transition systems \mathcal{T} :

$$\mathcal{T} \models \Phi_1 \iff \mathcal{T} \models \Phi_2$$

iff for all transition systems \mathcal{T} :

$$\text{Sat}(\Phi_1) = \text{Sat}(\Phi_2)$$

quantification over all transition systems \mathcal{T}

- without terminal states
- over AP if Φ_1 and Φ_2 are CTL formulas over AP

$\Phi_1 \equiv \Phi_2$ iff for all transition systems \mathcal{T} :

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iff for all transition systems \mathcal{T} :

$$\text{Sat}(\Phi_1) = \text{Sat}(\Phi_2)$$

Examples:

$$\neg\neg\Phi \equiv \Phi$$

$$\neg(\Phi \wedge \Psi) \equiv \neg\Phi \vee \neg\Psi$$

⋮

$$\neg\text{A}\bigcirc\Phi \equiv \text{E}\bigcirc\neg\Phi$$

Correct or wrong?

CTLSS4.1-23

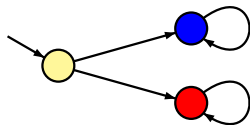
$$\exists \diamond (a \wedge b) \equiv \exists \diamond a \wedge \exists \diamond b$$

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CTLSS4.1-23

$$\exists \diamond (a \wedge b) \equiv \exists \diamond a \wedge \exists \diamond b$$

wrong, e.g.,

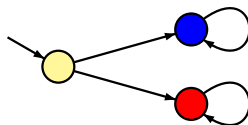


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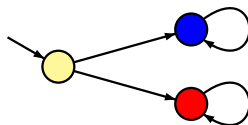
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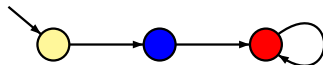
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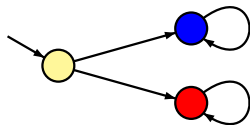
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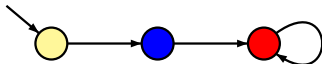
$$\exists \Diamond(a \wedge b) \equiv \exists \Diamond a \wedge \exists \Diamond b$$

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$$\forall \Diamond(a \wedge b) \equiv \forall \Diamond a \wedge \forall \Diamond b$$

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but:

$$\forall \Box(\Phi_1 \wedge \Phi_2) \equiv \forall \Box \Phi_1 \wedge \forall \Box \Phi_2$$

$$\exists \Diamond(\Phi_1 \vee \Phi_2) \equiv \exists \Diamond \Phi_1 \vee \exists \Diamond \Phi_2$$

Correct or wrong?

CTLSS4.1-24

$$\forall \alpha \forall \beta a \equiv \forall \alpha \forall \beta a$$

Correct or wrong?

CTLSS4.1-24

$$\forall x \forall y \square a \equiv \forall x \square \forall y \circ a$$

correct.

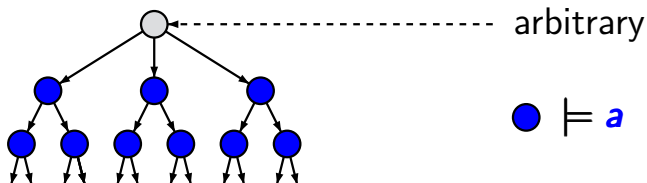
Correct or wrong?

CTLSS4.1-24

$$\forall \bigcirc \forall \square a \equiv \forall \square \forall \bigcirc a$$

correct.

both formulas require computation trees
of the form:



Correct or wrong?

CTLSS4.1-24

$$\forall \circ \forall \square a \equiv \forall \square \forall \circ a$$

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$$\exists \circ \exists \square a \equiv \exists \square \exists \circ a$$

Correct or wrong?

CTLSS4.1-24

$$\forall \circ \forall \square a \equiv \forall \square \forall \circ a$$

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Correct or wrong?

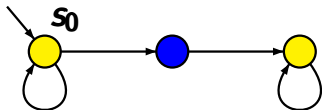
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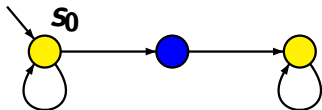
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$$\exists \bigcirc \exists \square a \equiv \exists \square \exists \bigcirc a$$

wrong, e.g.,



$$s_0 \not\models \exists \square \exists \bigcirc a$$

Correct or wrong?

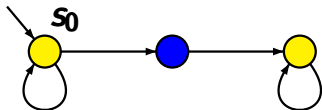
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correct.

$$\exists \bigcirc \exists \square a \equiv \exists \square \exists \bigcirc a$$

wrong, e.g.,



$$s_0 \not\models \exists \bigcirc \exists \square a$$

note: $Sat(\exists \square a) = \emptyset$

Correct or wrong?

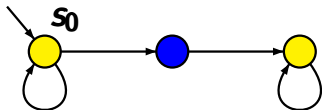
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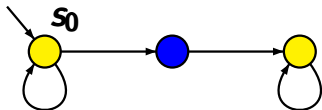
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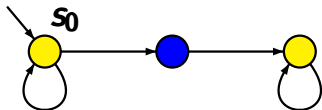
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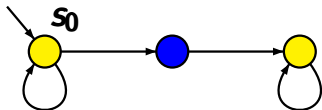
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in **LTL**: $\varphi W \psi \stackrel{\text{def}}{=} (\varphi U \psi) \vee \square\varphi$

in **CTL**: ?

in **LTL**: $\varphi \mathbf{W} \psi \stackrel{\text{def}}{=} (\varphi \mathbf{U} \psi) \vee \square\varphi$

duality of **U** and **W**:

$$\neg(\varphi \mathbf{U} \psi) \equiv (\varphi \wedge \neg\psi) \mathbf{W} (\neg\varphi \wedge \neg\psi)$$

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in **CTL**: ?

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definition of **W** in **CTL** on the basis of duality rules:

$$\exists(\phi \mathbf{W} \psi) \stackrel{\text{def}}{=} \neg\forall((\phi \wedge \neg\psi) \mathbf{U} (\neg\phi \wedge \neg\psi))$$

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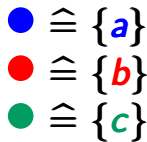
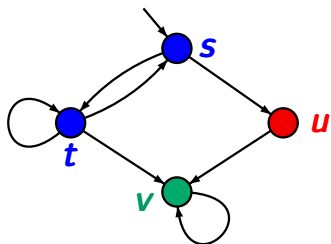
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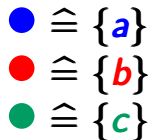
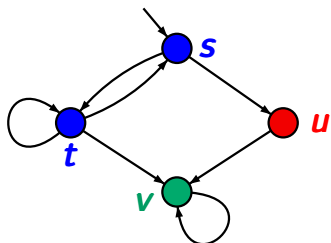
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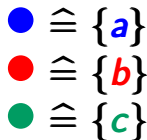
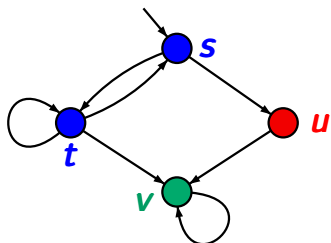
$$\forall(\Phi W \Psi) \not\equiv \forall(\Phi U \Psi) \vee \forall \square \Phi$$



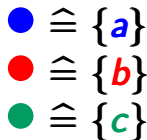
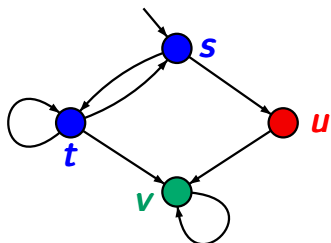
$\mathcal{T} \models \forall \Diamond \exists (a \text{ W } c)$?



$\mathcal{T} \models \forall \Diamond \exists (a \text{ W } c) \quad \checkmark \quad \text{as } s \models \exists (a \text{ W } c)$

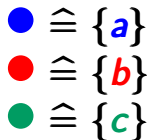
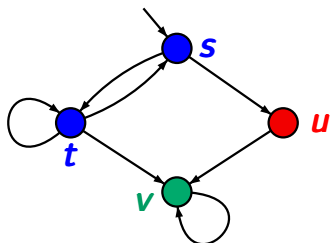


$\mathcal{T} \models \forall \diamond \exists (a \text{ W } c) \quad \checkmark \quad \text{as } s s_1 s_2 \dots \models \diamond \exists (a \text{ W } c)$



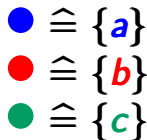
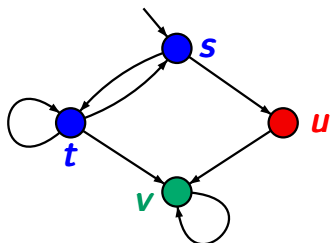
$\mathcal{T} \models \forall \Diamond \exists (a \text{ W } c) \quad \checkmark$ as $s \models \exists (a \text{ W } c)$

$\mathcal{T} \models \exists (a \text{ W } \exists \Box b) \quad ?$



$\mathcal{T} \models \forall \Diamond \exists (a \text{ W } c) \quad \checkmark \quad \text{as } s \models \exists (a \text{ W } c)$

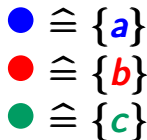
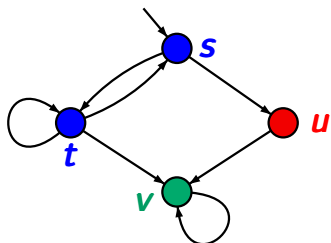
$\mathcal{T} \models \exists (a \text{ W } \exists \Box b) \quad \checkmark \quad \text{as } s \models \exists \Box a$



$\mathcal{T} \models \forall \Diamond \exists (a \text{ W } c) \quad \checkmark \quad \text{as } s \models \exists (a \text{ W } c)$

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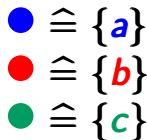
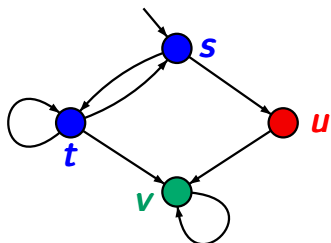
$\mathcal{T} \models \forall ((\exists \bigcirc (b \vee c)) \text{ W } (a \wedge b)) \quad ?$



$$\mathcal{T} \models \forall \Diamond \exists (a \text{ W } c) \quad \checkmark \quad \text{as } s \models \exists (a \text{ W } c)$$

$$\mathcal{T} \models \exists (a \text{ W } \exists \Box b) \quad \checkmark \quad \text{as } s \models \exists \Box a$$

$$\mathcal{T} \models \forall ((\exists \bigcirc (b \vee c)) \text{ W } (a \wedge b)) \quad \checkmark$$

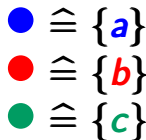
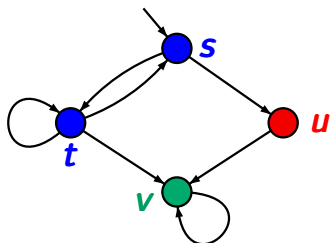


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$$\mathcal{T} \models \forall ((\exists \bigcirc (b \vee c)) \text{ W } (a \wedge b)) \quad \checkmark$$

\uparrow
 three types of paths: $(st)^\omega$ or $(st)^+ v^\omega$ or $(st)^* s u v^\omega$



$$\mathcal{T} \models \forall \Diamond \exists (a \text{ W } c) \quad \checkmark \quad \text{as } s \models \exists (a \text{ W } c)$$

$$\mathcal{T} \models \exists (a \text{ W } \exists \Box b) \quad \checkmark \quad \text{as } s \models \exists \Box a$$

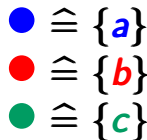
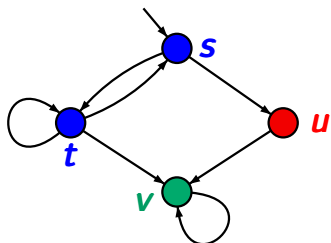
$$\mathcal{T} \models \forall ((\exists \bigcirc (b \vee c)) \text{ W } (a \wedge b)) \quad \checkmark$$

\uparrow
 three types of paths: $(st)^\omega$ or $(st)^+ v^\omega$ or $(st)^* s u v^\omega$

in all three cases: $\pi \models \Box \exists \bigcirc (b \vee c)$

Weak until W in CTL

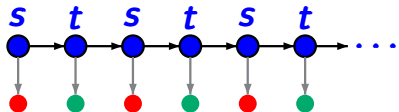
CTLSS4.1-21B



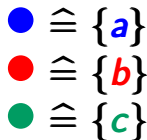
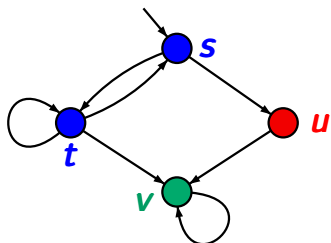
$$\mathcal{T} \models \forall \Diamond \exists (a \text{ W } c) \quad \checkmark \quad \text{as } s \models \exists (a \text{ W } c)$$

$$\mathcal{T} \models \exists (a \text{ W } \exists \Box b) \quad \checkmark \quad \text{as } s \models \exists \Box a$$

$$\mathcal{T} \models \forall ((\exists \bigcirc (b \vee c)) \text{ W } (a \wedge b)) \quad \checkmark$$



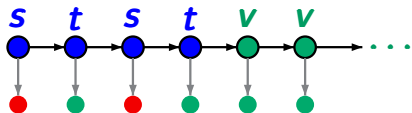
$$\models \Box \exists \bigcirc (b \vee c)$$



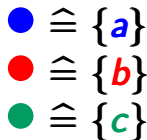
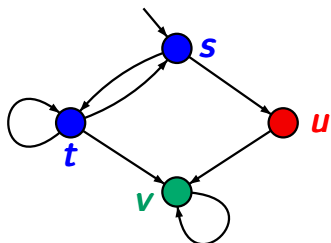
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$$\mathcal{T} \models \exists (a \text{ W } \exists \Box b) \quad \checkmark \quad \text{as } s \models \exists \Box a$$

$$\mathcal{T} \models \forall ((\exists \bigcirc (b \vee c)) \text{ W } (a \wedge b)) \quad \checkmark$$



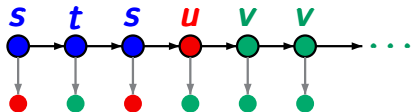
$$\models \Box \exists \bigcirc (b \vee c)$$



$$\mathcal{T} \models \forall \Diamond \exists (a \text{ W } c) \quad \checkmark \quad \text{as } s \models \exists (a \text{ W } c)$$

$$\mathcal{T} \models \exists (a \text{ W } \exists \Box b) \quad \checkmark \quad \text{as } s \models \exists \Box a$$

$$\mathcal{T} \models \forall ((\exists \bigcirc (b \vee c)) \text{ W } (a \wedge b)) \quad \checkmark$$



$$\models \Box \exists \bigcirc (b \vee c)$$

$$\exists(\phi \cup \psi) \equiv \psi \vee (\phi \wedge \exists \bigcirc \exists(\phi \cup \psi))$$

$$\exists(\phi \cup \psi) \equiv \psi \vee (\phi \wedge \exists \bigcirc \exists(\phi \cup \psi))$$

$$\forall(\phi \cup \psi) \equiv ?$$

$$\exists(\phi \cup \psi) \equiv \psi \vee (\phi \wedge \exists \circ \exists(\phi \cup \psi))$$

$$\forall(\phi \cup \psi) \equiv \psi \vee (\phi \wedge \forall \circ \forall(\phi \cup \psi))$$

Expansion laws

CTLSS4.1-26

$$\exists(\phi \cup \psi) \equiv \psi \vee (\phi \wedge \exists \text{O} \exists(\phi \cup \psi))$$

$$\forall(\phi \cup \psi) \equiv \psi \vee (\phi \wedge \forall \text{O} \forall(\phi \cup \psi))$$

$$\exists \diamond \psi \equiv \psi \vee \exists \text{O} \exists \diamond \psi$$

$$\forall \diamond \psi \equiv \psi \vee \forall \text{O} \forall \diamond \psi$$

Expansion laws

CTLSS4.1-26

$$\exists(\phi \cup \psi) \equiv \psi \vee (\phi \wedge \exists \circ \exists(\phi \cup \psi))$$

$$\forall(\phi \cup \psi) \equiv \psi \vee (\phi \wedge \forall \circ \forall(\phi \cup \psi))$$

$$\exists \diamond \psi \equiv \psi \vee \exists \circ \exists \diamond \psi$$

$$\forall \diamond \psi \equiv \psi \vee \forall \circ \forall \diamond \psi$$

$$\exists(\phi \text{ W } \psi) \equiv$$

$$\exists(\phi \cup \psi) \equiv \psi \vee (\phi \wedge \exists \text{O} \exists(\phi \cup \psi))$$

$$\forall(\phi \cup \psi) \equiv \psi \vee (\phi \wedge \forall \text{O} \forall(\phi \cup \psi))$$

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$$\forall \diamond \psi \equiv \psi \vee \forall \text{O} \forall \diamond \psi$$

$$\exists(\phi \text{ W } \psi) \equiv \psi \vee (\phi \wedge \exists \text{O} \exists(\phi \text{ W } \psi))$$

Expansion laws

CTLSS4.1-26

$$\exists(\phi \cup \psi) \equiv \psi \vee (\phi \wedge \exists \text{O} \exists(\phi \cup \psi))$$

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$$\forall \diamond \psi \equiv \psi \vee \forall \text{O} \forall \diamond \psi$$

$$\exists(\phi \text{ W } \psi) \equiv \psi \vee (\phi \wedge \exists \text{O} \exists(\phi \text{ W } \psi))$$

$$\forall(\phi \text{ W } \psi) \equiv \psi \vee (\phi \wedge \forall \text{O} \forall(\phi \text{ W } \psi))$$

Expansion laws

CTLSS4.1-26

$$\exists(\phi \cup \psi) \equiv \psi \vee (\phi \wedge \exists \text{O} \exists(\phi \cup \psi))$$

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$$\exists \diamond \psi \equiv \psi \vee \exists \text{O} \exists \diamond \psi$$

$$\forall \diamond \psi \equiv \psi \vee \forall \text{O} \forall \diamond \psi$$

$$\exists(\phi \text{ W } \psi) \equiv \psi \vee (\phi \wedge \exists \text{O} \exists(\phi \text{ W } \psi))$$

$$\forall(\phi \text{ W } \psi) \equiv \psi \vee (\phi \wedge \forall \text{O} \forall(\phi \text{ W } \psi))$$

$$\exists \square \phi \equiv ?$$

Expansion laws

CTLSS4.1-26

$$\exists(\phi \cup \psi) \equiv \psi \vee (\phi \wedge \exists \text{O} \exists(\phi \cup \psi))$$

$$\forall(\phi \cup \psi) \equiv \psi \vee (\phi \wedge \forall \text{O} \forall(\phi \cup \psi))$$

$$\exists \diamond \psi \equiv \psi \vee \exists \text{O} \exists \diamond \psi$$

$$\forall \diamond \psi \equiv \psi \vee \forall \text{O} \forall \diamond \psi$$

$$\exists(\phi \text{ W } \psi) \equiv \psi \vee (\phi \wedge \exists \text{O} \exists(\phi \text{ W } \psi))$$

$$\forall(\phi \text{ W } \psi) \equiv \psi \vee (\phi \wedge \forall \text{O} \forall(\phi \text{ W } \psi))$$

$$\exists \square \phi \equiv \phi \vee \exists \text{O} \exists \square \phi$$

Expansion laws

CTLSS4.1-26

$$\exists(\Phi \cup \Psi) \equiv \Psi \vee (\Phi \wedge \exists \text{EOE}(\Phi \cup \Psi))$$

$$\forall(\Phi \cup \Psi) \equiv \Psi \vee (\Phi \wedge \forall \text{AOA}(\Phi \cup \Psi))$$

$$\exists \diamond \Psi \equiv \Psi \vee \exists \text{EOE} \diamond \Psi$$

$$\forall \diamond \Psi \equiv \Psi \vee \forall \text{AOA} \diamond \Psi$$

$$\exists(\Phi \text{ W } \Psi) \equiv \Psi \vee (\Phi \wedge \exists \text{EOE}(\Phi \text{ W } \Psi))$$

$$\forall(\Phi \text{ W } \Psi) \equiv \Psi \vee (\Phi \wedge \forall \text{AOA}(\Phi \text{ W } \Psi))$$

$$\exists \square \Phi \equiv \Phi \vee \exists \text{EOE} \square \Phi$$

$$\forall \square \Phi \equiv \Phi \vee \forall \text{AOA} \square \Phi$$

duality of \Box and \Diamond :

$$\forall \Box \phi \equiv \neg \exists \Diamond \neg \phi$$

$$\forall \Diamond \phi \equiv \neg \exists \Box \neg \phi$$

Duality laws

CTLSS4.1-27

duality of \square and \diamond :

$$\forall \square \phi \equiv \neg \exists \diamond \neg \phi$$

$$\forall \diamond \phi \equiv \neg \exists \square \neg \phi$$

self-duality of \bigcirc :

$$\forall \bigcirc \phi \equiv \neg \exists \neg \bigcirc \neg \phi$$

$$\exists \bigcirc \phi \equiv \neg \forall \neg \bigcirc \neg \phi$$

duality of \square and \diamond :

$$\forall \square \phi \equiv \neg \exists \diamond \neg \phi$$

$$\forall \diamond \phi \equiv \neg \exists \square \neg \phi$$

self-duality of \bigcirc :

$$\forall \bigcirc \phi \equiv \neg \exists \neg \bigcirc \neg \phi$$

$$\exists \bigcirc \phi \equiv \neg \forall \neg \bigcirc \neg \phi$$

duality of **U** and **W**, e.g.:

duality of \Box and \Diamond :

$$\forall \Box \phi \equiv \neg \exists \Diamond \neg \phi$$

$$\forall \Diamond \phi \equiv \neg \exists \Box \neg \phi$$

self-duality of \bigcirc :

$$\forall \bigcirc \phi \equiv \neg \exists \neg \bigcirc \neg \phi$$

$$\exists \bigcirc \phi \equiv \neg \forall \neg \bigcirc \neg \phi$$

duality of **U** and **W**, e.g.:

$$\forall (\phi \mathbf{U} \psi)$$

duality of \Box and \Diamond :

$$\forall \Box \phi \equiv \neg \exists \Diamond \neg \phi$$

$$\forall \Diamond \phi \equiv \neg \exists \Box \neg \phi$$

self-duality of \bigcirc :

$$\forall \bigcirc \phi \equiv \neg \exists \neg \bigcirc \neg \phi$$

$$\exists \bigcirc \phi \equiv \neg \forall \neg \bigcirc \neg \phi$$

duality of **U** and **W**, e.g.:

$$\forall (\phi \mathbf{U} \psi) \equiv \neg \exists ((\phi \wedge \neg \psi) \mathbf{W} (\neg \phi \wedge \neg \psi))$$

duality of \Box and \Diamond :

$$\forall \Box \phi \equiv \neg \exists \Diamond \neg \phi$$

$$\forall \Diamond \phi \equiv \neg \exists \Box \neg \phi$$

self-duality of \bigcirc :

$$\forall \bigcirc \phi \equiv \neg \exists \neg \bigcirc \neg \phi$$

$$\exists \bigcirc \phi \equiv \neg \forall \neg \bigcirc \neg \phi$$

duality of **U** and **W**, e.g.:

$$\forall (\phi \mathbf{U} \psi) \equiv \neg \exists ((\neg \phi \wedge \neg \psi) \mathbf{W} (\neg \phi \vee \neg \psi))$$

$$\equiv \neg \exists ((\neg \psi) \mathbf{W} (\neg \phi \wedge \neg \psi))$$

duality of \Box and \Diamond :

$$\forall \Box \phi \equiv \neg \exists \Diamond \neg \phi$$

$$\forall \Diamond \phi \equiv \neg \exists \Box \neg \phi$$

self-duality of \bigcirc :

$$\forall \bigcirc \phi \equiv \neg \exists \neg \bigcirc \neg \phi$$

$$\exists \bigcirc \phi \equiv \neg \forall \neg \bigcirc \neg \phi$$

duality of **U** and **W**, e.g.:

$$\forall (\phi \mathbf{U} \psi) \equiv \neg \exists ((\phi \wedge \neg \psi) \mathbf{W} (\neg \phi \wedge \neg \psi))$$

$$\equiv \neg \exists ((\neg \psi) \mathbf{W} (\neg \phi \wedge \neg \psi))$$

$$\equiv \neg \exists ((\neg \psi) \mathbf{U} (\neg \phi \wedge \neg \psi)) \wedge \neg \exists \Box \neg \psi$$

Duality laws

CTLSS4.1-27A

duality of \Box and \Diamond :

$$\forall \Box \phi \equiv \neg \exists \Diamond \neg \phi$$

$$\forall \Diamond \phi \equiv \neg \exists \Box \neg \phi$$

self-duality of \bigcirc :

$$\forall \bigcirc \phi \equiv \neg \exists \neg \bigcirc \neg \phi$$

$$\exists \bigcirc \phi \equiv \neg \forall \neg \bigcirc \neg \phi$$

duality of **U** and **W** yields

$$\forall (\phi \mathbf{U} \psi) \equiv \neg \exists ((\neg \psi) \mathbf{U} (\neg \phi \wedge \neg \psi)) \wedge \neg \exists \Box \neg \psi$$

duality of \square and \diamond :

$$\forall \square \phi \equiv \neg \exists \diamond \neg \phi$$

$$\forall \diamond \phi \equiv \neg \exists \square \neg \phi$$

self-duality of \bigcirc :

$$\forall \bigcirc \phi \equiv \neg \exists \neg \bigcirc \neg \phi$$

$$\exists \bigcirc \phi \equiv \neg \forall \neg \bigcirc \neg \phi$$

derivation of $\forall \bigcup$ from $\exists \bigcup$ and $\exists \square$:

$$\forall (\phi \bigcup \psi) \equiv \neg \exists ((\neg \psi) \bigcup (\neg \phi \wedge \neg \psi)) \wedge \neg \exists \square \neg \psi$$

$\forall \bigcup$ and $\forall \bigcirc$ are expressible via $\exists \bigcup$, $\exists \bigcirc$ and $\exists \square$

For each **CTL** formula ψ there is an equivalent **CTL** formula ϕ built by

- operators of propositional logic
- the modalities $\exists\bigcirc$, $\exists\mathbf{U}$ and $\exists\Box$.

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$$\begin{array}{l} \phi ::= \textit{true} \mid a \mid \phi_1 \wedge \phi_2 \mid \neg\phi \mid \\ \quad \exists\bigcirc\phi \mid \exists(\phi_1 \mathbf{U} \phi_2) \mid \exists\Box\phi \end{array}$$

For each **CTL** formula Ψ there is an equivalent **CTL** formula Φ built by

- operators of propositional logic
- the modalities $\exists\bigcirc$, $\exists\mathbf{U}$ and $\exists\Box$.

$$\Phi ::= \text{true} \mid a \mid \Phi_1 \wedge \Phi_2 \mid \neg\Phi \mid \\ \exists\bigcirc\Phi \mid \exists(\Phi_1 \mathbf{U} \Phi_2) \mid \exists\Box\Phi$$

transformation $\Psi \rightsquigarrow \Phi$ relies on:

$$\forall\bigcirc\Psi \rightsquigarrow \neg\exists\bigcirc\neg\Psi$$

$$\forall(\Psi_1 \mathbf{U} \Psi_2) \rightsquigarrow \neg\exists(\neg\Psi_2 \mathbf{U} (\neg\Psi_1 \wedge \neg\Psi_2)) \wedge \neg\exists\Box\neg\Psi_2$$

CTL formula \rightsquigarrow CTL formula in \exists -normal form

$$\forall O \psi \rightsquigarrow \neg \exists O \neg \psi$$

$$\forall (\psi_1 U \psi_2) \rightsquigarrow \neg \exists (\neg \psi_2 U (\neg \psi_1 \wedge \neg \psi_2)) \wedge \neg \exists \square \neg \psi_2$$

$$\forall ((\forall O a) U \neg c)$$

Example: \exists -normal form for CTL formula

CTL formula \rightsquigarrow CTL formula in \exists -normal form

$$\forall O \psi \rightsquigarrow \neg \exists O \neg \psi$$

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$$\forall ((\forall O a) U \neg c)$$

$$\equiv \neg \exists (\neg \neg c U (\neg \forall O a) \wedge \neg \neg c) \wedge \neg \exists \square \neg c$$

Example: \exists -normal form for CTL formula

CTL formula \rightsquigarrow CTL formula in \exists -normal form

$$\forall O \psi \rightsquigarrow \neg \exists O \neg \psi$$

$$\forall (\psi_1 U \psi_2) \rightsquigarrow \neg \exists (\neg \psi_2 U (\neg \psi_1 \vee \neg \psi_2)) \vee \neg \exists \square \neg \psi_2$$

$$\forall (\forall O a) U \neg c$$

$$\equiv \neg \exists (\neg \neg c U (\neg \forall O a) \vee \neg \neg c) \vee \neg \exists \square \neg \neg c$$

$$\equiv \neg \exists (\neg c U (\neg \forall O a) \vee \neg c) \vee \neg \exists \square c$$

Example: \exists -normal form for CTL formula

CTL formula \rightsquigarrow CTL formula in \exists -normal form

$$\forall O \psi \rightsquigarrow \neg \exists O \neg \psi$$

$$\forall (\psi_1 U \psi_2) \rightsquigarrow \neg \exists (\neg \psi_2 U (\neg \psi_1 \vee \neg \psi_2)) \wedge \neg \exists \square \neg \psi_2$$

$$\forall (\forall O a) U \neg c$$

$$\equiv \neg \exists (\neg \neg c U ((\neg \forall O a) \wedge \neg \neg c)) \wedge \neg \exists \square \neg \neg c$$

$$\equiv \neg \exists (\neg c U ((\neg \forall O a) \wedge \neg c)) \wedge \neg \exists \square \neg c$$

$$\equiv \neg \exists (\neg c U ((\exists O \neg a) \wedge \neg c)) \wedge \neg \exists \square \neg c$$

- **negation** only on the level of literals
- uses for each operator (one of) its dual(s)
 - * *true* and *false*
 - * conjunction and disjunction
 - * **existential** and **universal** path quantification
 - * **until** and **weak until**
alternatively: until and release

but no additional operator for \bigcirc required

syntax of **CTL** formulas in **PNF**:

state formulas:

$$\Phi ::= \textit{true} \mid \textit{false} \mid a \mid \neg a \mid \Phi_1 \wedge \Phi_2 \mid \Phi_1 \vee \Phi_2 \mid \\ \exists \varphi \mid \forall \varphi$$

path formulas:

$$\varphi ::= \bigcirc \Phi \mid \Phi_1 \mathbf{U} \Phi_2 \mid \Phi_1 \mathbf{W} \Phi_2$$

syntax of **CTL** formulas in **PNF**:

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$$\psi ::= \bigcirc \Phi \mid \Phi_1 \mathbf{U} \Phi_2 \mid \Phi_1 \mathbf{W} \Phi_2$$

For each **CTL** formula Ψ there is an equivalent **CTL** formula Φ in **PNF**

CTL formula \rightsquigarrow CTL formula in PNF

$$\neg \text{true} \rightsquigarrow \text{false}$$

$$\neg(\phi_1 \wedge \phi_2) \rightsquigarrow \neg\phi_1 \vee \neg\phi_2 \quad \text{de Morgan laws}$$

$$\neg\exists\text{O}\phi \rightsquigarrow \forall\text{O}\neg\phi$$

$$\neg\forall\text{O}\phi \rightsquigarrow \exists\text{O}\neg\phi$$

$$\neg\exists(\phi_1 \text{U} \phi_2) \rightsquigarrow \forall((\phi_1 \wedge \neg\phi_2) \text{W} (\neg\phi_1 \wedge \neg\phi_2))$$

$$\neg\forall(\phi_1 \text{U} \phi_2) \rightsquigarrow \exists((\phi_1 \wedge \neg\phi_2) \text{W} (\neg\phi_1 \wedge \neg\phi_2))$$

CTL formula \rightsquigarrow CTL formula in PNF

$$\neg \text{true} \rightsquigarrow \text{false}$$

$$\neg(\phi_1 \wedge \phi_2) \rightsquigarrow \neg\phi_1 \vee \neg\phi_2 \quad \text{de Morgan laws}$$

$$\neg \exists O \phi \rightsquigarrow \forall O \neg \phi$$

$$\neg \forall O \phi \rightsquigarrow \exists O \neg \phi$$

$$\neg \exists (\phi_1 U \phi_2) \rightsquigarrow \forall ((\cancel{\phi_1} \wedge \neg \phi_2) W (\neg \phi_1 \wedge \neg \phi_2))$$

$$\neg \forall (\phi_1 U \phi_2) \rightsquigarrow \exists ((\cancel{\phi_1} \wedge \neg \phi_2) W (\neg \phi_1 \wedge \neg \phi_2))$$

CTL formula \rightsquigarrow CTL formula in PNF

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$$\neg\exists(\phi_1 \text{U} \phi_2) \rightsquigarrow \forall(\neg\phi_2 \text{W}(\neg\phi_1 \wedge \neg\phi_2))$$

$$\neg\forall(\phi_1 \text{U} \phi_2) \rightsquigarrow \exists(\neg\phi_2 \text{W}(\neg\phi_1 \wedge \neg\phi_2))$$

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$$\neg\forall(\phi_1 \text{U} \phi_2) \rightsquigarrow \exists(\neg\phi_2 \text{W}(\neg\phi_1 \wedge \neg\phi_2))$$

... exponential blowup possible ...