Chain-Monadic Second Order Logic over Regular Automatic Trees and Epistemic Planning Synthesis

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Abstract

We consider infinite relational structures that have a finite presentation by means of a finite tuple of finite-state automata, and already known as *automatic structures*. While it is well established that model checking against first-order logic is decidable over automatic structures, we show how this seminal result can be adapted for a restricted class of automatic structures called *regular automatic trees* and an extension of the logic based on *chain-MSO*, and written cMSO (where MSO stands for monadic second-order logic). The logic cMSO, as chain-MSO, is interpreted over trees, and its second-order quantifiers range over subsets of branches. In the setting of regular automatic trees, we relate cMSO and logics of knowledge and time, among which the branching epistemic linear-time mu-calculus $\mathrm{BL}_\mu^{\mathsf{lin}}\mathrm{K}$. We finally apply our results to dynamic epistemic logic and its related epistemic planning problem: when restricting to event models with propositional preconditions and postconditions, the relational structures arising from epistemic planning problems turn out to be regular automatic trees. This already established latter central property allows us to derive the (already known) decidability of the epistemic planning problem as a mere corollary, but also to enlarge the class of decidable epistemic planning problems to goals expressed in $\mathrm{BL}_\mu^{\mathsf{lin}}\mathrm{K}$, with an effective way of computing the set of all successful (possibly infinite) plans.

Keywords: Automatic structures, chain Monadic Second-Order logic, Epistemic planning synthesis.

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1 Introduction

In artificial intelligence, classical planning consists in generating a finite sequence of actions for achieving a given goal. The so-called *epistemic planning* [6] generalizes classical planning with epistemic goal (agent a knows that...) and complex actions (public announcements, private announcements, etc.). Epistemic planning is based on Dynamic epistemic logic ([2], [25]) and is undecidable in the general case [7].

Actions are represented by event models [4] that are Kripke structures of events, instead of possible worlds, equipped with a precondition and a postcondition. When event preconditions and postconditions are propositional (Boolean) only (intuitively, announcements are Boolean formulas and uncertainty is only about the content of the messages and physical changes), epistemic planning has been shown to be decidable ([30],[15]).

Regrettably, epistemic planning as considered so far concerns finite plans, and is in essence a reachability problem. However, real-life applications may challenge infinite plans. We give three examples of planning goals that require an infinite plan.

(i) safety properties, e.g. 'invariantly, an intruder a does not know the location of the piece of jewelry more than 3 consecutive steps'.

(ii) recurring bounded properties, e.g. 'all drones know that the region is safe every 20 steps';

(iii) strategic reasoning, e.g. 'with the current plan, the drone a never knows the region is safe but every 10 steps, there is a(nother) plan to let the drone a eventually know the region is safe'.

Nevertheless, infinite plans have been considered in [15] in the DEL (Dynamic Epistemic Logic) setting. In this approach, *DEL structures*, those arising from iterating ad infinitum the triggering of events, are seen as infinite relational structures. Maubert has shown that, when all event preconditions and postconditions are propositional, DEL structures are *automatic structures* [5,18]. Automatic structures are relational structures which have a presentation by means of a finite family of finite-state automata. As noticed by Maubert, the decidability result of epistemic planning (for epistemic goals) relies on the fact that model checking against first-order logic is decidable over automatic structures: in a DEL structure, the skeleton is a tree whose nodes (or equivalently finite branches) are finite plans, i.e. sequences of triggered events, and this tree structure is augmented with "transverse" binary relations between nodes in possibly different branches. These extra relations denote epistemic relations between histories. In DEL structures, the existence of a plan reduces to the existence of a point in this automatic (tree) structure where the epistemic planning goal (hence expressible in FO) holds. After all, the whole epistemic planning problem can be expressed in FO whose outermost quantifier is ex-

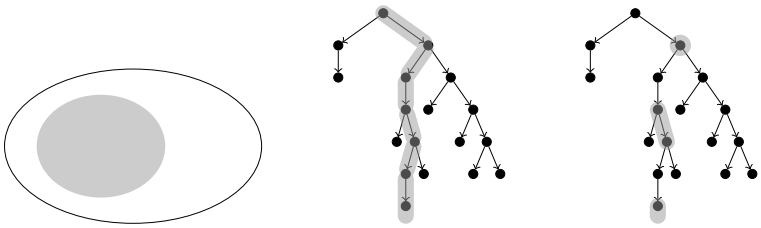
[4] also called sometimes "action models" in the literature.

istential. Additionally, automata constructions that answer the query of an FO-formula on an automatic structure allow to synthesize an automaton that recognizes the set of all (finite) plans achieving the goal. Regarding infinite plans, Maubert relies on the involved theory of *uniform strategies* he has developed [15]. This theory enables him to deal with epistemic planning instances whose goals are expressed in the branching-time epistemic logic CTL*K, the extension of temporal logic CTL* [12] with epistemic modalities. The setting does lead to synthesizing infinite plans, but the synthesis relies on a bottom-up technique over the goal formula that prevents the setting from being extended to logics featuring fix-points; while statements (ii) and (iii) above cannot be expressed in CTL*K, a well-tuned logic with fix-point or monadic with second-order quantifiers would suffice.

In an attempt to enlarge the specification language for epistemic planning goals, that goes beyond CTL*K while remaining decidable, one should observe that using the full *monadic second-order logic* (MSO) is hopeless: by [22], model checking against MSO over the binary tree equipped with the transverse binary relation "equal level" is undecidable. Relation "equal level" is an instance of an agent epistemic relation where the agent does not observe anything from the current finite sequence of triggered events but its length. However, there are variants of MSO, where second-order quantifications are constrained enough to yield a decidable model checking problem over infinite trees of the kind of DEL structures. For example, relying on [12], and as depicted in Figure 1, there exist mainly three interpretations of the second-order quantifiers, yielding full MSO, path-MSO, and chain-MSO. The full MSO logic can be interpreted over arbitrary relational structures and the second-order quantifiers range over all subsets of the structure domain. On the contrary, the path-MSO logic is interpreted on (possibly infinite) trees and the quantifiers range over paths/branches [5] of the tree. Finally, chain-MSO, also interpreted on trees only, requires the second-order quantifier to range over *chains*, that are subsets of paths [12]. It is known that the expressive power of chain-MSO strictly subsumes the one of path-MSO, the former allowing to describe arbitrary ω-regular properties of branches, while the latter captures only star-free properties, see [12].

In this paper, we extend chain-MSO of [21,12] with relations in the tree, e.g. interpreting epistemic modalities, making statements (ii) and (iii) expressible. This extension is written cMSO. We introduce *regular automatic trees*, a strict but large subclass of automatic structures that encompass DEL structures with propositional events. We then show that model checking against cMSO is decidable over *regular automatic trees*. Our decidability result is strongly inspired from the proof technique in [23, Th. 5.2] to show that chain-MSO with the "equal level" predicate is decidable over n-ary trees. Our proof yields automata constructions that can be exploited to achieve the synthesis of plans in epistemic planning.

[5] The difference between paths and branches is irrelevant by [12].



(a) MSO
quantification over any subset.

(b) path-MSO
quantification over any path in a tree.

(c): cMSO
quantification over any chain in a tree.

Figure 1. Different restrictions of second-order quantifications.

Noticeably, the logic cMSO captures the linear-time mu-calculus ([4], [26]) enriched with path quantifiers and epistemic modalities, here written $\mathsf{BL}_{\mu}^{\mathsf{lin}}\mathrm{K}$ (the acronym stands for '**B**ranching **L**ogic of the **lin**ear-time μ-calculus with **K**nowledge). In essence, the logic $\mathsf{BL}_{\mu}^{\mathsf{lin}}\mathrm{K}$ is an epistemic extension of the logic ECTL*, for "extended CTL*", of [12], but in a mu-calculus style rather than with the use of automata modalities, in the sense of [29]. Since CTL*K can easily be embedded into $\mathsf{BL}_{\mu}^{\mathsf{lin}}\mathrm{K}$ (the hard-coded linear-time temporal operators of CTL* are based on fix-points expressible in the linear-time mu-calculus), our result significantly exceeds the one of [15] whose proof technique does not allow for considering arbitrary fix-point formulas.

The presented contribution is derived from the preliminary work available in [10]. In Section 2, we recall the notion of automatic structures and results on model checking against FO-formulas and MSO-formulas. In Section 3, we present the logic cMSO, as well as the subclass of so-called *regular automatic trees*. We prove that model checking against cMSO is decidable over this subclass (Theorem 3.9). In Section 4, we compare cMSO with some logics of knowledge and time, namely CTLK, CTL*K and $\mathsf{BL}_{\mu}^{\mathsf{lin}}\mathrm{K}$, with increasing expressive power. Section 5 is dedicated to the application of our results to the *generalized epistemic planning problem* (Definition 5.7) where goals are arbitrary $\mathsf{BL}_{\mu}^{\mathsf{lin}}\mathrm{K}$-statements and instances of the planning domain rely on DEL specifications with propositional events.

2 Automatic structures and decidable theories

In this section we recall seminal results on automatic structures: namely that first-order logic FO is decidable over these structures while monadic second-order logic MSO is not. The interested reader may refer to [5,18] for further details.

2.1 Structures and logics

Logics FO and MSO are interpreted over relation structures.

Definition 2.1 A *relational structure* is a structure of the form $\mathcal{A} = \langle D, R_1 \dots R_p \rangle$ where D is a non-empty set called the *domain*; $R_1 \dots R_p$ are relations over D of arity $r_1 \dots r_p$, respectively; namely $R_i \subseteq D^{r_i}$.

The set of symbols $\{R_1 \dots R_p\}$ is called the *signature* of $\mathcal{A}$. We take the convention to write $R_i(d_1, \dots, d_{r_i})$ for $(d_1, \dots, d_{r_i}) \in R_i$. We distinguish the particular class of structures that are infinite trees of fixed finite degree n, called *n-ary trees*, that are central in our contribution. These structures represent computation trees (with a root of address ε, i-successor relations, and prefix binary relation between addresses) augmented with additional relations, such as epistemic relations.

Definition 2.2 Given a finite set $E := \{1, \dots, n\}$ of directions, we call *n-ary tree* a structure $\mathcal{T} = \langle D, r, S_1, \dots, S_n, R_1, \dots, R_p \rangle$ where:

- D is a prefix-closed subset of E^* (the addresses of the nodes);
- r is a unary relation that holds only for ε, the root address of the tree;
- $S_e(x, xe)$ whenever $xe \in D$;
- $R_1, \dots, R_p$ are additional relations on D.

In the particular case of a tree, elements of D are *nodes*.

Notice that in Definition 2.2 of a tree no condition was imposed on the relations $R_1 \dots R_p$. They may therefore correspond to transversal links between nodes just as epistemic relations would do, e.g. the "equal level" binary relation.

Example 2.3 The structure $\mathcal{T}_2 = \langle E^*, S_1, S_2 \rangle$ is a 2-ary tree called the *infinite full binary tree*. Also the structure $\mathcal{T}_2^{\mathsf{el}} = \langle E^*, S_1, S_2, \mathsf{el} \rangle$ obtained by augmenting $\mathcal{T}_2$ with the binary relation "at equal level in the tree" ($\mathsf{el}(x, y)$ holds if, and only if, $|x| = |y|$) is another example of a 2-ary tree.

First-order logic (FO) over relational structures $\mathcal{A} = \langle D, R_1 \dots R_p \rangle$ concerns formulas that conform to the following syntax: $\varphi ::= R_i(x_1 \dots x_{r_i}) \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \exists x\varphi$, where x, x_1, x_{r_i} are first-order variables whose interpretation ranges over the domain of relational structures.

When turning to the more expressive monadic second-order logic (MSO), one allows for second-order variables that range over subsets of the domain of relational structures. Formally, the syntax of MSO is given by: $\varphi ::= R_i(x_1 \dots x_{r_i}) \mid x \in X \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \exists x\varphi \mid \exists X\varphi$, where x is a first-order variable and X is a *second-order* variable whose interpretation ranges over subsets of the domain. In the following, we let $\mathcal{V}_1$ and $\mathcal{V}_2$ denote respectively the set of first-order variables and second-order variables.

As usual, in a formula, a variable $x \in \mathcal{V}_1$ (resp. $X \in \mathcal{V}_2$) is *free* if it is not under the scope of a quantifier, namely some Qx (resp. QX) with $Q \in \{\forall, \exists\}$. A formula is *closed* if it contains no free variables. An *assignment* in domain D is a function σ that maps a variable of $\mathcal{V}_1$ onto an element of D and a variable of $\mathcal{V}_2$ onto a subset of D. Given an assignment σ in D, a variable $x \in \mathcal{V}_1$ and $d \in D$, we let $\sigma[x \mapsto d]$ be the assignment on D that coincides with σ but maps x onto d. Similarly, we define $\sigma[X \mapsto D']$ where $D' \subseteq D$ for the second-order variables.

Due to limited space, we do not provide the semantics of MSO, see for example [11].

2.2 Automatic presentations

We now describe how some relational structures can be encoded using formal languages, following the presentation of [18] . By *alphabet* we mean a finite set of symbols, named *letters*. A *word* over Σ is a finite sequence of letters. We denote by Σ^* the set of such sequences. For $u \in \Sigma^*$ and $n \geq 0$, let $u[n]$ be the $n+1$-th letter of u (when defined). We assume familiarity with the basic definitions of automata theory and the properties of regular languages.

Let $\Box$ be a fresh padding symbol. If Σ is an alphabet, we define $\Sigma_\Box$ as $\Sigma \uplus \{\Box\}$. The following definition describes an encoding for k-tuples of words as a word on the product alphabet $\Sigma_\Box^n$; since words of the tuple may have different length, the padding symbol $\Box$ is used to align the words.

Definition 2.4 If $u, v \in \Sigma^*$, their *convolution* $u \otimes v$ is the word of $(\Sigma_\Box \times \Sigma_\Box)^*$ of length $\max(|u|, |v|)$ such that $(u \otimes v)[n] = (u[n], v[n])$ if $n < \min(|u|, |v|)$; $(u \otimes v)[n] = (u[n], \Box)$ if $|v| \leq n < |u|$; and $(u \otimes v)[n] = (\Box, v[n])$ if $|u| \leq n < |v|$. Convolution is defined similarly for k-tuples of words.

Example 2.5 The convolution of $\mathsf{aaba} \otimes \mathsf{b} \otimes \mathsf{ba}$ over alphabet $\Sigma = \{\mathsf{a}, \mathsf{b}\}$ is the four-letter word $\binom{\mathsf{a}}{\mathsf{b}}\binom{\mathsf{a}}{\Box}\binom{\mathsf{b}}{\Box}\binom{\mathsf{a}}{\Box}$ over alphabet $\Sigma_\Box^3$.

Definition 2.6 Let $\mathcal{A} = \langle D, R_1 \dots R_p \rangle$ be a relational structure. An *automatic presentation* of A is a tuple of regular languages $(L_D, L_1, \dots, L_n)$ meeting the following conditions: (1) there exists a one-to-one *encoding function* $\mathtt{enc} : D \to L_D$, and (2) for all R_i (arity r_i), $L_i = \{u_1 \otimes \dots \otimes u_{r_i} \mid \forall j, u_j \in L_D \text{ and } (\mathtt{enc}^{-1}(u_1), \dots, \mathtt{enc}^{-1}(u_{r_i})) \in R_i\}$.

The alphabet Σ of L_D is called the *encoding alphabet*. The inverse $\mathtt{enc}^{-1}$ of the encoding function is the *decoding function*. We write $\mathtt{enc}(D)$ for L_D, and more generally, $\mathtt{enc}(R)$ for the set $\{\mathtt{enc}(d_1) \otimes \dots \otimes \mathtt{enc}(d_r) \mid (d_1, \dots, d_r) \in R\}$, for an arbitrary relation $R \subseteq D^r$.

A structure is *automatic* if it has (at least) an automatic presentation. A presentation can effectively be described by a tuple of finite automata $(\mathcal{M}_D, \mathcal{M}_1, \dots, \mathcal{M}_p)$ recognizing $(L_D, L_1, \dots, L_n)$.

Remark 2.7 We may assume that equality is among the relations R_i, represented by the regular language $\{u \otimes u \mid u \in L_D\}$. In the literature, the standard definition of automatic presentations allows an element to have several encodings, whenever equality can be presented by some regular language. However, both definitions enable to present the same structures [5].

Example 2.8 (i) Finite structures are automatic, since finite languages are regular.

(ii) $\langle \mathbb{N}, \leq \rangle$ is automatic, with an automatic presentation over the unary alphabet $\{1\}$ by letting $\mathtt{enc}(n) = 1^n$. Automaton $\mathcal{M}_\leq$ of Figure 2 verifies there are less 1's in the first component than in the second component.

(iii) Both trees $\mathcal{T}_2$ and $\mathcal{T}_2^{\mathsf{el}}$ are automatic.

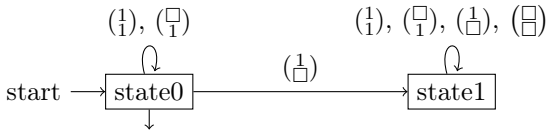


Figure 2. The finite-state automaton $\mathcal{M}_{\leq}$ of Example 2.8 (ii).

We recall the known fundamental theorem of [18, Th. 3.1] regarding the model checking problem against FO over automatic structures.

Definition 2.9 MODEL CHECKING AGAINST FO

Input : A presentation $(\mathcal{M}_D, \mathcal{M}_1, \ldots, \mathcal{M}_p)$ of a relational structure $\mathcal{A}$ and a closed FO-formula φ over the corresponding signature.

Output : *Yes* if $\mathcal{A} \models \varphi$, *no* otherwise.

Theorem 2.10 *[18, Th. 3.1] Model checking against FO is decidable.*

The main ingredient of the proof of Theorem 2.10 relies on the construction of Proposition 2.11 which heavily relies on the closure of regular languages by intersection, negation and projection over components.

Let $\varphi(x_1 \ldots x_n) \in \mathsf{FO}$ where $x_1 \ldots x_n \in \mathcal{V}_1$ are the only free variables, and let $\mathcal{A}$ be a relational structure. We write $\varphi^{\mathcal{A}} := \{(d_1 \ldots d_n) \in D^n \mid \mathcal{A}, [x_i \mapsto d_i]_{1 \leq i \leq n} \models \varphi[x_1 \ldots x_n]\}$ for the set of tuples that satisfy φ.

Proposition 2.11 *There is an algorithm that given an automatic presentation of a relational structure $\mathcal{A}$ with encoding function $\mathtt{enc}$, and a first-order formula $\varphi(x_1, \ldots, x_n)$, outputs an automaton that recognizes $\mathtt{enc}(\varphi^{\mathcal{A}})$.*

Theorem 2.10 can sometimes be extended to MSO. For example, there are very specific cases where MSO can be decided, among which the typical example of the automatic structure $(\mathbb{N}, <)$.

Proposition 2.12 *[3] The MSO-theory of a structure having an automatic presentation based on a unary encoding alphabet is decidable.*

Also, by Rabin's Theorem [17], MSO is decidable over the full binary tree with signature r, S_1, S_2. However, MSO becomes undecidable over the full binary relation augmented with the "equal level" relation el (see Example 2.3), although the obtained structure is automatic.

Theorem 2.13 *[22] Model checking against MSO over the binary tree $\mathcal{T}_2^{\mathsf{el}}$ is undecidable.*

We now focus on a subclass of automatic structures where the undecidability frontier can be pushed back.

3 Model checking against cMSO over RA trees

We consider the variant of MSO stemming from "chain MSO", written cMSO in this paper, which contrary to FO and MSO, can only be interpreted over

infinite trees. The second-order quantifiers range over *chains*, namely subsets of nodes along a branch, as depicted in Figure 1(c). The logic cMSO has already been defined in [21,12,23] for n-ary trees and with a signature restricted to the successor relations $S_1, \ldots, S_n$; the main results concern the decidability of the model checking of chain-MSO over the full binary tree, and its expressivity. In our setting, the signature of cMSO can be arbitrary, so that properties involving e.g. knowledge and time can be expressed.

We also exhibit a subclass of automatic structures called *regular automatic trees* that are automatic trees where the encoding of a node is its address in the tree. This subclass is large and encompasses infinite models arising from the unfolding of finite-state systems but also in dynamic epistemic logic. We establish that model checking against cMSO is decidable over regular automatic trees (Theorem 3.9).

3.1 The logic cMSO over trees

We recall a tree structure $\mathcal{T} = \langle D, r, S_1, \ldots, S_n, R_1, \ldots, R_p \rangle$ over a finite set $E = \{1, \ldots, n\}$ of directions (Definition 2.2), where D is a prefix-closed subset of E^* describing the nodes through their addresses in the tree. In the following, we let S^* be the reflexive and transitive closure of the *generalized successor relation* $S := \bigcup_{i=1}^{n} S_i$.

Definition 3.1 In a tree structure $\mathcal{T}$ of domain $D \subseteq E^*$, a subset $C \subseteq D$ is a *chain* if it is totally ordered with respect to S^*, i.e. for all $d, d' \in C$, either $S^*(d, d')$ or $S^*(d', d)$ holds.
We denote by $Ch(\mathcal{T})$ the set of chains of the tree $\mathcal{T}$.

Example 3.2 In the full binary tree $\mathcal{T}_2$, the set $\{1^{2n} \mid n \in \mathbb{N}\}$ is a chain, whereas $\{1, 2\}$ is not.

The notion of chains enables one to consider an interpretation of MSO formulas in trees, named cMSO, where the second-order quantifiers are restricted to chains.

Definition 3.3 The logic cMSO is interpreted in a tree $\mathcal{T} = \langle D, r, S_1, \ldots, S_n, R_1, \ldots, R_p \rangle$ with an assignment σ in D for free variables as follows.

$\mathcal{T}, \sigma \models R_i(x_1 \ldots x_{r_i})$	iff $(\sigma(x_1) \ldots \sigma(x_{r_i})) \in R_i$;
$\mathcal{T}, \sigma \models x \in X$	iff $\sigma(x) \in \sigma(X)$;
$\mathcal{T}, \sigma \models \neg\varphi$	iff $\mathcal{T}, \sigma \not\models \varphi$;
$\mathcal{T}, \sigma \models (\varphi \wedge \psi)$	iff $\mathcal{T}, \sigma \models \varphi$ and $\mathcal{T}, \sigma \models \psi$;
$\mathcal{T}, \sigma \models \exists x \varphi$	iff there exists $d \in D$ such that $\mathcal{T}, \sigma[x \mapsto d] \models \varphi$;
$\mathcal{T}, \sigma \models \exists X \varphi$	iff there exists $C \in Ch(\mathcal{T})$ s. th. $\mathcal{T}, \sigma[x \mapsto C] \models \varphi$.

For a closed formula φ, we simply write $\mathcal{T} \models \varphi$ without specifying the assignment.

The logic cMSO is close to the logic path-MSO [21,12] which restricts second-order quantification to maximal branches only. Actually the latter is a subsystem of the former: the cMSO-formula $\mathsf{pathfrom}[X, x_0] := x_0 \in X \wedge \forall x \{x \in X \rightarrow [(\exists y S(x,y) \rightarrow \exists y(S(x,y) \wedge y \in X)) \wedge \neg S(x, x_0)]\}$ expresses that chain X is a maximal path starting at note x_0, which corresponds to the very quantifier in path-MSO. This translation has already been noticed many times in the literature.

3.2 Regular automatic trees

We now turn to a particular class of trees, called *regular automatic trees*, which are automatic trees for the most intuitive encoding of nodes, that is by their address in the tree. Since the domain is required to be regular, such trees arise from the unfolding of some finite-state transition systems, hence the terminology of *regular trees*.

Definition 3.4 A tree $\mathcal{T} = \langle D, r, S_1, \ldots, S_n, R_1, \ldots, R_p \rangle$ on the set of directions $E = \{1, \ldots, n\}$ is a *regular automatic (RA) tree* if the identity function $\mathtt{enc} : D \rightarrow E^*$ describes an automatic presentation of $\mathcal{T}$ over the encoding alphabet E. This presentation is called the *canonical presentation* of $\mathcal{T}$.

Since the encoding function of a RA tree is the identity function, the regularity of $S_1, \ldots, S_n$ and $\preceq$ directly follows from that of D. Thus, a tree is a regular automatic tree if, and only if, $D \subseteq E^*$ is regular, and for all R_i (of arity r_i), $L_i = \{d_1 \otimes \cdots \otimes d_{r_i} \mid (d_1, \ldots, d_{r_i}) \in R_i\}$ is regular.

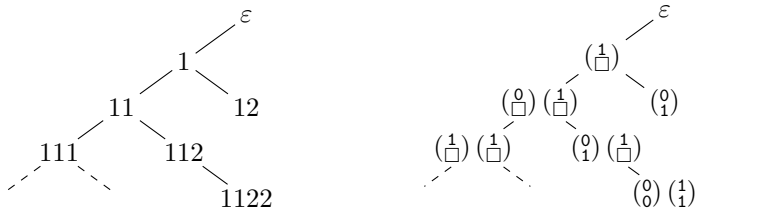
Example 3.5 The tree $\mathcal{T}_2^{\mathsf{el}}$ of Example 2.3 is a regular automatic tree.

As stated in the following proposition, RA trees form a strict subclass of automatic trees.

Proposition 3.6 *There are automatic trees that are not RA trees.*

Proof

Let $E = \{1, 2\}$ we consider the binary tree $\mathcal{T}^{\text{auto}}_{\text{notreg}} = \langle D, r, S_1, S_2, \mathsf{el} \rangle$ of Figure 3(a) where $D = \{1^m 2^k \mid 0 \leq k \leq m\}$ and el is the "equal level" relation of Example 2.3.



(a) The tree $\mathcal{T}^{\text{auto}}_{\text{notreg}}$. (b) Encodings of nodes of $\mathcal{T}^{\text{auto}}_{\text{notreg}}$.

Figure 3. An automatic tree that is not regular.

As D is not a regular language, $\mathcal{T}^{\text{auto}}_{\text{notreg}}$ is not a RA tree, but it is automatic: consider the encoding alphabet $\Sigma = (\{\mathtt{0}, \mathtt{1}\}_\square)^2$ and let $\mathtt{enc}(1^m 2^k) = \mathtt{bin}(m) \otimes$

$\texttt{bin}(k)$ be the encoding function, as presented in Figure 3(b), where $\texttt{bin}(m)$ (resp. $\texttt{bin}(k)$) the binary representation of m (resp. k) with least significant digit first. For example, $\texttt{enc}(112) = \texttt{enc}(1^2 2^1) = \texttt{bin}(2) \otimes \texttt{bin}(1) = \binom{0}{1}\binom{1}{\square}$. The reader should easily get convinced that $\texttt{enc}(D)$, $\texttt{enc}(S_1)$, $\texttt{enc}(S_2)$ and $\texttt{enc}(\mathsf{el})$ are regular languages. □

When restricting to RA trees, some usual relations over trees can be easily captured by automata over the canonical encoding of nodes. We recall the following additional binary relations over trees.

- The reflexive and transitive closure S^* of the generalized successor binary relation S;
- The binary relation $(d, d') \in \preccurlyeq$ whenever node d is not deeper than node d' in $\mathcal{T}$;
- The binary relation el, where $(d, d') \in \mathsf{el}$ whenever nodes d and d' are at the same depth in the tree.
- The binary equality relation $=$.

Lemma 3.7 *Let $\langle D, r, S_1, \ldots, S_n, R_1, \ldots, R_p\rangle$ be a RA tree. Then the relational structure $\langle D, r, S_1, \ldots, S_n, R_1 \ldots R_p, S^*, \preccurlyeq, \mathsf{el}, =\rangle$ is also an RA tree.*

Restricting to the class of RA trees allows for decidability results of model checking against logics that go beyond FO.

3.3 Model checking against cMSO

We now describe the main result of this section, as stated by Theorem 3.9, where the model checking problem against cMSO is as follows.

Definition 3.8 Model checking against cMSO

Input : $\langle \mathcal{M}_D, \mathcal{M}_r, (\mathcal{M}'_i)_{1\leq i\leq n}, (\mathcal{M}_i)_{1\leq i\leq p}\rangle$ the canonical presentation of a RA tree $\mathcal{T}$ and a closed cMSO-formula φ over the corresponding signature.

Output : *Yes* if $\mathcal{T} \models \varphi$, *no* otherwise.

Theorem 3.9 can be read as a variant of Theorem 2.10 where, on the one hand, the logic FO is extended, and, on the other hand, the class of automatic structures is restricted. It can also be read as a generalization of [23, Th. 5.2].

Theorem 3.9 *Model checking against cMSO is decidable over RA trees.*

The rest of this section is dedicated to the proof of Theorem 3.9. This proof goes over the proof made in [23, Th. 5.2] to generalize the result to arbitrary RA trees.

Before starting the proof, we may assume without loss of generality, that all the variables occurring in the formulas are second-order variables. This means to add an extra unary predicate Sing() which conveys that a set is a singleton, and to add inclusion formulas of the form $X \subseteq Y$ so as to capture previous formulas of the form $x \in X$. The reader may refer to [23] for this routine

transformation. The syntax then becomes as follows.

$$\varphi ::= \mathsf{Sing}(X) \mid X \subseteq Y \mid R_i(X_1 \dots X_{r_i}) \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \exists X \varphi, \quad \text{where } X, Y, X_1, \dots \in \mathcal{V}_2 \tag{1}$$

The new syntax requires to adjust the semantics of the formulas, and in particular of those of the form $R_i(X_1, \dots, X_{r_i})$, by imposing that each X_j is interpreted as a singleton, this is routine. The logic resulting from (1) is expressively equivalent to cMSO to the extent that a singleton is a chain. Therefore, in the following, we still use cMSO when using the syntax of (1).

A key idea to develop automata construction to model check against cMSO is that a set of addresses is a chain if, and only if, it is contained in the set of all prefixes of some infinite word. Given a chain $C \subseteq E^*$, we let $\mathsf{Branches}(C) := \bigcap\{uE^\omega \mid u \in C\}$ be the set of infinite words whose set of prefixes contains C. In particular, $\mathsf{Branches}(C)$ is a singleton if, and only if, C is infinite.

For $v \in \mathsf{Branches}(C)$, we write $\mathsf{mbranch}(v, C)$ for the infinite word over $E \times \{0, 1\}$ whose first component is v and whose second component is marked by 1 if the current prefix of v belongs to C, and by 0 otherwise. Formally, $\mathsf{mbranch}(v, C) = m[0]m[1]m[2]\dots$ with $m[i] = (v[i], b)$ where $b = 1$ if $v[0]v[1]\dots v[i] \in C$, and $b = 0$ otherwise.

The words $\mathsf{mbranch}(v, C)$ are used to encode the chain C. We will overload the encoding identity function $\mathtt{enc}$ of RA trees to denote the encoding of chains. We letting $\mathtt{enc}(C) := \{\mathsf{mbranch}(v, C) \mid v \in \mathsf{Branches}(C)\}$. Also, we extend the notation to $\mathtt{enc}(C_1, \dots, C_m)$ that contains infinite words over alphabet $(E \times \{0, 1\})^m$ by letting $\mathtt{enc}(C_1, \dots, C_m) := \mathtt{enc}(C_1) \otimes \dots \otimes \mathtt{enc}(C_m)$ where $\otimes$ is the convolution extended to sets of infinite words in a natural manner. For $m = 0$, $\mathtt{enc}()$ is the singleton containing the unique infinite word over the singleton alphabet $(\Sigma \times \{0, 1\})^0$.

We now provide an automata-theoretic approach for model checking against cMSO formulas. We first establish the existence of a Büchi automaton $\mathcal{C}_m$ that verifies if an infinite word over alphabet $(E \times \{0, 1\})^m$ denotes a m-tuple of chains, as stated by Lemma 3.10.

Lemma 3.10 *One can effectively construct a Büchi automaton $\mathcal{C}_m$ that recognizes the encoding of m-tuples of chains, namely the language* $\bigcup_{C_1,\dots,C_m \in Ch(\mathcal{T})} \mathtt{enc}(C_1, \dots, C_m)$.

The automaton $\mathcal{C}_m$ of Lemma 3.10 runs m copies of the domain automaton $\mathcal{M}_D$. Each copy reads one of the m infinite words over E extracted from the infinite input word over $(E \times \{0, 1\})^m$. Automaton $\mathcal{C}_m$ rejects if one of these copies rejects.

Let $\mathcal{T} = \langle D, r, S_1, \dots, S_n, R_1, \dots, R_p\rangle$ be a RA tree over the set of directions $E = \{1, \dots, n\}$ with $T = \langle \mathcal{M}_D, \mathcal{M}_r, (\mathcal{M}'_i)_{1\le i\le n}, (\mathcal{M}_i)_{1\le i\le p}\rangle$ its canonical presentation, and let φ be a cMSO-formula[6], with free variables $X_1, \dots, X_m$.

[6] in the language defined by the grammar (1)

We define the set

$$\texttt{enc}(\varphi^{\mathcal{T}}) := \bigcup_{\substack{C_1,\dots,C_m \in Ch(\mathcal{T}) \\ \mathcal{T}, [X_i \to C_i]_{1\leq i\leq m} \models \varphi}} \texttt{enc}(C_1,\dots,C_m) \tag{2}$$

Proposition 3.11 *Given φ a cMSO-formula[7] with free variables $X_1,\dots,X_m$, one can effectively construct a Büchi automaton $\mathcal{B}_{\varphi,\mathcal{T}}$ that recognizes $\texttt{enc}(\varphi^{\mathcal{T}})$.*

We can now conclude the proof of Theorem 3.9. Observe that model checking against formula φ over the RA tree $\mathcal{T}$ amounts to deciding whether the language of the Büchi automaton $\mathcal{B}_{\varphi,\mathcal{T}}$ of Proposition 3.11 is non-empty. Indeed, this language is not empty if, and only if, there is some assignment σ of the second order-variables $X_1,\dots,X_m$ such that $\mathcal{T},\sigma \models \varphi$. In particular, if φ is closed ($m = 0$), by convention we have:

- If $\mathcal{T} \not\models \varphi$, the union in Equation (2) is empty as it ranges over an empty set;
- If $\mathcal{T} \models \varphi$, the union in Equation (2) is not empty as it is equal to $\texttt{enc}()$.

Notice that Proposition 2.12 from [3] regarding the decidablity of the full MSO-theory (and not only cMSO) for automatic presentations based on a singleton alphabet is now a direct corollary of Theorem 3.9 since every set of words over a singleton alphabet is a chain.

From Theorem 3.9 on the decidability of cMSO over RA trees, and because binary relations $S^*, \preccurlyeq, \mathsf{el}, =$ come for free with automata constructions (Lemma 3.7), we have the following.

Corollary 3.12 *Model checking against cMSO$[r, S_1,\dots,S_n, R_1,\dots,R_p, S^*, \preccurlyeq, \mathsf{el}, =]$ is decidable over a RA trees.*

Notice that if we restrict to trees with only relations $S_1 \dots S_n$, model checking against the full logic MSO becomes decidable (Rabin's Theorem [17]). However, considering extra relations (e.g. "equal level") makes the model checking against MSO undecidable over RA trees, as we already saw in Theorem 2.13 of Section 2, which is not the case for cMSO by Corollary 3.12.

Also, since every path quantifier in a path-MSO-formula can be translated in cMSO, and since considering extra arbitrary relations in trees, such as epistemic relations, does not harm this translation, we obtain the following result, which to our knowledge has not been considered in the literature.

Corollary 3.13 *Given a RA tree $\mathcal{T} = \langle D, r, S_1,\dots,S_n, R_1,\dots,R_p\rangle$ and a close path-MSO-formula φ over the signature of $\mathcal{T}$, it can be decided if $\mathcal{T} \models \varphi$.*

We now turn to logical formalisms that combine knowledge and time in the spirit of [13].

[7] with syntax of (1)

4 Logics of knowledge and time over RA trees

We now exploit the peculiarities of RA trees to consider extensions of several classic formalisms and study their expressivity. We first recall our criterion for expressivity.

Definition 4.1 A logic $\mathcal{L}$ is *embedded into* a logic $\mathcal{L}'$, whenever there is an effective translation that maps every $\mathcal{L}$-formula onto an equivalent $\mathcal{L}'$-formula, namely for every regular automatic tree $\mathcal{T}$, we have: $\mathcal{T} \models \varphi$ if, and only if, $\mathcal{T} \models \varphi'$.

Notice that when $\mathcal{L}$ is embedded into $\mathcal{L}'$, the decidability of the model checking against $\mathcal{L}'$ entails the decidability of the model checking against $\mathcal{L}$.

In tree structures, *time* is captured by the generalized successor binary relation $S := \bigcup_{i=1}^{n} S_i$, as in computation trees for temporal logics such as CTL [8], CTL^* [12], etc. Depending on their arities, the other relations $R_1, \ldots, R_p$ have different roles. Unary relations label nodes with atomic propositions that range over some fixed set $AP = \{p, q, \ldots\}$. Binary relations, written $K_1 \ldots K_m$ as the epistemic modalities in multi-agent epistemic logic, will play the role of *knowledge* modalities as in classic epistemic logic. On this basis, we shortly write cMSOK for the logic $\mathsf{cMSO}[r, S_1, \ldots, S_n, (p)_{p\in AP}, K_1 \ldots K_m]$.

The ability to quantify over chains allows us to capture properties along branches of the trees that are naturally stated in linear-time mu-calculus. Recall that MSO along linear orders, hence branches, is expressively equivalent to Büchi automata that capture all ω-regular properties, as shown by [24], and so does the linear-time mu-calculus [9]. We therefore introduce the logic $\mathrm{BL}_\mu^{\mathsf{lin}}\mathrm{K}$ that is based on the linear-time mu-calculus [4,27], equipped with knowledge modalities but also with second order quantifications over branches of the tree, just as CTL^* extends LTL to the branching-time semantics.

The logic $\mathrm{BL}_\mu^{\mathsf{lin}}\mathrm{K}$ is called the *branching epistemic linear-time mu-calculus*; in the spirit of CTL^*, it relies on two kinds of formulas: *state formulas* (Φ) and *path formulas* (φ).

Definition 4.2 The syntax of $\mathrm{BL}_\mu^{\mathsf{lin}}\mathrm{K}$ is as follows.

- State formulas: $\Phi ::= p \mid K_i\Phi \mid \neg\Phi \mid (\Phi \wedge \Phi) \mid \mathsf{E}\varphi$ where $i \in \{1, \ldots, m\}$.
- Path formulas: $\varphi ::= Z \mid \Phi \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \mathsf{X}\varphi \mid \mu Z.\ \varphi[Z]$ where φ is closed in $\mathsf{E}\varphi$, $Z \in \mathcal{V}$ is under the scope of an even number of negations in $\varphi[Z]$.

State formula $K_i\Phi$ is read as 'Agent i knows Φ' and state formula $\mathsf{E}\varphi$ is read as 'there is a path starting from the current state that satisfies φ'. Path formula $\mathsf{X}\varphi$ is read as 'φ holds in the next state'. Path formula $\mu Z.\ \varphi[Z]$ is the linear mu-calculus fix-point construction. State formula $\mathsf{A}\varphi$ is an abbreviation of $\neg\mathsf{E}\neg\varphi$ and is read as 'φ holds in all paths starting from the current state'. All formulas of $\mathrm{BL}_\mu^{\mathsf{lin}}\mathrm{K}$ are interpreted in a tree $\mathcal{T} = \langle D, r, S_1, \ldots, S_n, K_1 \ldots K_m, (p)_{p\in AP}\rangle$. State formulas are interpreted on nodes, while path formulas are interpreted on branches.

- State formulas:

$\mathcal{T}, d \models p$	iff	$d \in p$;
$\mathcal{T}, d \models K_i\Phi$	iff	for all $d' \in D$ s.t. $(d, d') \in K_i$ we have $\mathcal{T}, d' \models \Phi$;
$\mathcal{T}, d \models \neg\Phi$	iff	$\mathcal{T}, d \not\models \Phi$;
$\mathcal{T}, d \models (\Phi \wedge \Psi)$	iff	$\mathcal{T}, d \models \Phi$ and $\mathcal{T}, d \models \Psi$;
$\mathcal{T}, d \models \mathsf{E}\varphi$	iff	there exists a maximal chain C with least element d (i.e. a branch starting from d) such that $\mathcal{T}, C \models \varphi$;

- Path formulas: we add a valuation $\sigma : \mathcal{V} \to 2^{\mathbb{N}}$ for the free second-order variables in the formulas, and write σ^{+i} for the valuation $\sigma^{+i}(Z) = \{n \mid n+i \in \sigma(Z)\}$.

$\mathcal{T}, \pi, \sigma \models Z$	iff	$0 \in \sigma(Z)$
$\mathcal{T}, \pi, \sigma \models \Phi$	iff	$\mathcal{T}, \pi(0) \models \Phi$;
$\mathcal{T}, \pi, \sigma \models (\varphi \wedge \psi)$	iff	$\mathcal{T}, \pi, \sigma \models \varphi$ and $\mathcal{T}, \pi, \sigma \models \psi$;
$\mathcal{T}, \pi, \sigma \models \neg\varphi$	iff	$\mathcal{T}, \pi, \sigma \not\models \varphi$;
$\mathcal{T}, \pi, \sigma \models \mathsf{X}\varphi$	iff	$\mathcal{T}, \pi(1)\pi(2)\ldots, \sigma^{+1} \models \varphi$;
$\mathcal{T}, \pi, \sigma \models \mu Z.\ \varphi[Z]$	iff	0 is in the least-fix point of $[\![\varphi[Z]]\!]^{\sigma}_{\pi}$

where $\begin{cases} [\![\varphi[Z]]\!]^{\sigma}_{\pi} : 2^{\mathbb{N}} \to 2^{\mathbb{N}} \\ \qquad P \mapsto \{i \in \mathbb{N} \mid \pi(i)\pi(i+1)\ldots, (\sigma[Z \mapsto P])^{+i} \models \varphi[Z]\}. \end{cases}$

A tree (resp. a forest) satisfies a state formula if it satisfies it from its root node (resp. from all its distinguished root nodes).

Notice that the least fix-point always exists. Indeed, the function $[\![\varphi[Z]]\!]^{\sigma}_{\pi}$ is monotone (recall that we have required that every variable to occur under the scope of an even number of negations) and the Knaster-Tarski Theorem [20] applies in the complete lattice $(2^{\mathbb{N}}, \subseteq)$. Besides, we can alternatively define the least fix-point of $[\![\varphi[Z]]\!]^{\sigma}_{\pi}$ as $\bigcap\{F \subseteq 2^{\mathbb{N}} \mid [\![\varphi[Z]]\!]^{\sigma}_{\pi}(F) \subseteq F\}$.

Example 4.3
- $\mathcal{T}, \pi \models \mu Z.\ p$ if, and only if, $\mathcal{T}, \pi(0) \models p$.
- $\mathcal{T}, \pi \models \mu Z.\ (\mathsf{X}Z \vee p)$ if, and only if, there exists a node $\pi(i)$ along π such that $\mathcal{T}, \pi(i) \models p$.
- $\mathcal{T}, \pi \models \mu Z.\ (\psi \vee (\varphi \wedge \mathsf{X}Z))$ if, and only if, there exists a node $\pi(j)$ such that $\mathcal{T}, \pi(j) \models \psi$ and $\mathcal{T}, \pi(i) \models \varphi$ for every $0 \leq i < j$.

By means of fix-points in $\mathsf{BL}^{\mathsf{lin}}_{\mu}\mathrm{K}$, we capture the linear-time logic LTL [16], and since we also have the existential path quantifier E, we can embed the full logic CTL^* of [12] in $\mathsf{BL}^{\mathsf{lin}}_{\mu}\mathrm{K}$.

Now the epistemic modalities K_i allow us to capture $\mathsf{CTL}^*\mathrm{K}$, the logic CTL^* equipped with knowledge modalities. More precisely, we introduce the following macro for 'φ until ψ': $\varphi\mathsf{U}\psi$ is an abbreviation of formula $\mu Z.(\psi \vee (\varphi \wedge \mathsf{X}Z))$.

Definition 4.4 $\mathsf{CTL}^*\mathrm{K}$ is the syntactic fragment of $\mathsf{BL}^{\mathsf{lin}}_{\mu}\mathrm{K}$ defined by:

- State formulas: $\Phi ::= \Phi \wedge \Phi \mid \neg\Phi \mid \mathsf{E}\varphi \mid K_i\Phi \mid p$ where $i \in \{1, \ldots, m\}$.
- Path formulas: $\varphi ::= \Phi \mid \neg\varphi \mid \varphi \wedge \varphi \mid \mathsf{X}\varphi \mid \varphi\mathsf{U}\varphi$.

The syntactic fragment CTLK of $\mathsf{CTL}^*\mathrm{K}$ has only state formulas:

$$\Phi ::= \Phi \wedge \Phi \mid \neg\Phi \mid \mathsf{EX}\varphi \mid \mathsf{E}(\Phi\mathsf{U}\Phi) \mid \mathsf{A}(\Phi\mathsf{U}\Phi) \mid K_i\Phi \mid p$$

Proposition 4.5 *CTLK is embedded into* $\mathsf{FO}[K_1 \dots K_m, (p)_{p\in AP}, S, S^*, \preccurlyeq, =]$.

We end this section with the main Theorem 4.6 that states the decidability of model checking against $\mathrm{BL}_\mu^{\mathsf{lin}}$Kover RA trees. First of all, it should be clear that the following holds.

Theorem 4.6 *$BL_\mu^{lin}K$ is embedded into cMSOK.*

Corollary 4.7 *Model checking against $BL_\mu^{lin}K$ is decidable over automatic regular trees.*

Figure 4 is a recap of the expressivity results we have obtained (arrows resulting from transitivity are omitted), where an arrow $\mathcal{L} \to \mathcal{L}'$ means that $\mathcal{L}$ is embedded into $\mathcal{L}'$.

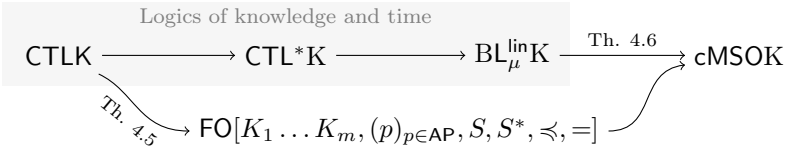


Figure 4. Embeddings between the various logics.

5 Application to epistemic planning synthesis

As announced, we show that Theorem 3.9 together with Proposition 3.11 have an interesting impact in the domain of epistemic planning. The epistemic planning setting as introduced by [6] relies on Dynamic epistemic logic, that we recall here. Next we explain how the original problem can be generalized to attain more expressive planning goals, such as statements (i)–(iii) of the introduction, while maintaining the decidability frontier for free. Our results generalize the ones obtained in [15,1], but make great use of the already established property [15, Lemma 22, p. 109] that, under the assumption that preconditions and postconditions of events are propositional, the relational structure that contains all plan candidates is automatic, and actually it is a RA tree (see Theorem 5.5).

5.1 Preliminaries on Dynamic epistemic logic

As earlier, AP denotes the set of atomic propositions with typical elements $p, q, \dots$, and Ag is finite set of agents with typical elements $a, b, \dots$. The propositional language is denoted by $\mathscr{L}_{Prop}$ and the language of multi-agent epistemic modal logic is denoted by $\mathscr{L}_{EL}$.

Definition 5.1 A Kripke model $\mathcal{M} = (W, (K_a)_{a\in Ag}, V)$ is defined by a non-empty set W of epistemic worlds, epistemic relations $(K_a)_{a\in Ag} \subseteq W \times W$ and a valuation function $V : W \to 2^{AP}$.

A pair $(\mathcal{M}, w)$ is called a *pointed epistemic model*.

Figure 5 bottom left shows a pointed epistemic model.

The dynamic of the system is captured by *event models*. An event model is like a Kripke model but possible worlds are instead events equipped with a

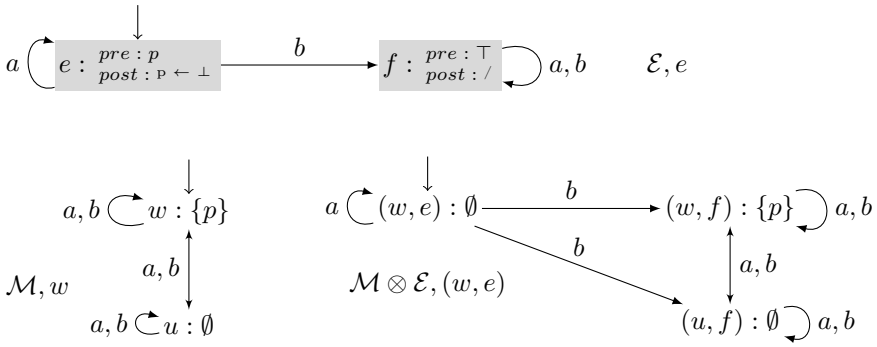


Figure 5. Example of a DEL product.

precondition and a postcondition. We intentionally denote the set of events E with the same notation that the set of directions in trees (Definition 2.2), as they will denote directions in the DEL structure (see Definition 5.4).

Definition 5.2 An *event model* $\mathcal{E} = (E, (K_a^{\mathcal{E}})_{a\in Ag}, pre, post)$ is defined by a non-empty set of *events* E, epistemic relations $(K_a^{\mathcal{E}})_{a\in Ag} \subseteq E \times E$, a precondition function $pre : E \to \mathscr{L}_{EL}$ and a postcondition function $post : E \times AP \to \mathscr{L}_{Prop}$. When $pre : E \to \mathscr{L}_{Prop}$, the event model $\mathcal{E}$ is *propositional*.

A pair $(\mathcal{E}, e)$ is called a *pointed event model*, where e represents the actual event.

Definition 5.3 Let $\mathcal{M} = (W, (K_a)_{a\in Ag}, V)$ be a Kripke model. Let $\mathcal{E} = (E, (K_a^{\mathcal{E}})_{a\in Ag}, pre, post)$ be an event model. The *product* of $\mathcal{M}$ and $\mathcal{E}$ is $\mathcal{M} \otimes \mathcal{E} = (W', (K_a)', V')$ where:

- $W' = \{(w, e) \in W \times E \mid \mathcal{M}, w \models pre(e)\}$;
- $((w, e), (w', e')) \in K'_a$ iff $(w, w') \in K_a$ and $(e, e') \in K_a^{\mathcal{E}}$;
- $V'((w, e)) = \{p \in AP \mid \mathcal{M}, w \models post(e, p)\}$.

Figure 5 top shows an example of a pointed event model with two events e and f. The actual event is e but agent b imagines event f as the sole possible event. Also, the product is given in bottom right.

5.2 DEL structures

In this subsection, following [15], we incorporate the initial epistemic model $\mathcal{M}$ and the infinitely many the products $\mathcal{M} \otimes \mathcal{E}^n := \mathcal{M} \otimes \underbrace{\mathcal{E} \otimes \cdots \otimes \mathcal{E}}_{n \text{ times}}$ into a single structure, called a *DEL structure*, denoted by $\mathcal{M}\mathcal{E}^*$. Worlds of $\mathcal{M} \otimes \mathcal{E}^n$, called *histories*, are naturally denoted by words of the form $we_1 \dots e_n$ where w is a world of $\mathcal{M}$ and $e_1, \dots, e_n$ are events of $\mathcal{E}$. E.g. the world $((w, e_1), e_2)$ is denoted we_1e_2. The pair $(\mathcal{M}, \mathcal{E})$ is called the *DEL presentation* of $\mathcal{M}\mathcal{E}^*$. A DEL presentation $(\mathcal{M}, \mathcal{E})$ is *propositional* if $\mathcal{E}$ is propositional.

Definition 5.4 [15, , Def. 54, p. 105] The *DEL structure* denoted by $(\mathcal{M}, \mathcal{E})$ is the structure $\mathcal{M}\mathcal{E}^* = (\mathcal{H}, (S_e)_{e\in E}, (K_a)_{a\in Ag}, (p)_{p\in AP})$, where $\mathcal{H}$ is the disjoint union of the sets of worlds of $\mathcal{M} \otimes \mathcal{E}^n$ – namely, the *histories*; $(h, h') \in S_e$ whenever $h' = he$ for all events e; $(h, h') \in K_a$ whenever h and h' are worlds of $\mathcal{M} \otimes \mathcal{E}^n$, for some n, and $(h, h') \in K'_a$ where K'_a is the epistemic relation for agent a in $\mathcal{M} \otimes \mathcal{E}^n$; and finally, $h \in p$ whenever $p \in V(h)$ in $\mathcal{M} \otimes \mathcal{E}^n$.

From the proof of [15, Lemma 22, p. 109], one can easily show that:

Theorem 5.5 *Propositional DEL structures $\mathcal{M}\mathcal{E}^*$ are finite sets (forests) of RA trees, and their presentation is effectively computable from $(\mathcal{M}, \mathcal{E})$.*

We now turn to the epistemic planning problems.

5.3 Generalized epistemic planning and plan synthesis

The *epistemic planning problem*, as originally stated by [6] is defined as follows.

Definition 5.6 Epistemic planning problem

Input: a pointed epistemic model $\mathcal{M}, w$, an event model $\mathcal{E}$, an $\mathscr{L}_{EL}$-formula ψ;

Output: Yes, if there is a sequence of events $e_1, \ldots, e_n$ in $\mathcal{E}$ such that $\mathcal{M}\mathcal{E}^n, we_1 \ldots e_n \models \psi$.

We generalize this epistemic planning problem by considering a model checking problem against $\mathsf{BL}_\mu^{\mathsf{lin}}\mathrm{K}$ over DEL structures.

Definition 5.7 Generalized Epistemic planning problem

Input: a pointed epistemic model $\mathcal{M}, w$, an event model $\mathcal{E}$, an $\mathsf{BL}_\mu^{\mathsf{lin}}\mathrm{K}$-formula φ;

Output: Yes, if $\mathcal{M}\mathcal{E}^* \models \varphi$.

The problem of Definition 5.7 is indeed a generalization of epistemic planning problem of Definition 5.6 in the sense that the latter can be reduced to the former by model checking the formula $\varphi := \mathsf{EF}\psi$, where ψ is the planning goal.

The following example shows the relevance of generalized epistemic planning.

Example 5.8 We give $\mathsf{BL}_\mu^{\mathsf{lin}}\mathrm{K}$-formulas φ for the three statements (i)–(iii) of the introduction.

(i) 'invariantly, an intruder a does not know the location of the piece of jewelry more than 3 consecutive steps' can be expressed as $\mathsf{EG}(\Theta \to (\mathsf{X}\neg\Theta \vee \mathsf{X}^2\neg\Theta \vee \mathsf{X}^3\neg\Theta)$ where $\Theta := \bigvee_{\ell\in Loc} K_a\mathrm{pieceOfJewelleryIn}(\ell)$.

(ii) 'all drones know that the region is safe every 20 steps' can be expressed as $\mathsf{E}\nu Z.(\bigwedge_{a\in Ag} K_a\mathrm{regionSafe} \wedge \mathsf{X}^{20}Z)$.

(iii) 'with the current plan, the drone a never knows the region is safe but every 10 steps, there is a(nother) plan to let the drone a eventually know the region is safe' can be expressed as $\mathsf{EG}[\neg K_a\mathrm{regionSafe} \wedge \nu Z.(\mathsf{EF}K_a\mathrm{regionSafe} \wedge \mathsf{X}^{10}Z)]$.

The following result subsumes the decidability result for the epistemic planning problem for propositional DEL presentations ([30],[15]), and is a mere corollary of Theorems 5.5, 4.6 and 3.9.

Theorem 5.9 *The generalized epistemic planning problem is decidable for propositional DEL presentations.*

In other terms, in the literature, decidability was established for reachability goals and Theorem 5.9 says that the problem remains decidable for goals that are arbitrary $\mathsf{BL}_\mu^{\mathsf{lin}}\mathrm{K}$-formulas φ. However, when considering planning, only formulas φ of the form $\mathsf{E}\varphi'$ are relevant. For such formulas, we can effectively build an automaton that recognizes exactly all plans achieving φ'. This automaton arises from the automata constructions for $\mathsf{BL}_\mu^{\mathsf{lin}}\mathrm{K}$-formulas, made possible by the automata constructions for cMSO-formulas, and the fact that every $\mathsf{BL}_\mu^{\mathsf{lin}}\mathrm{K}$-formula can be effectively translated into a cMSO-formula. The algorithm to obtain this "plan automaton" is as follows.

(i) Compute the $\mathsf{cMSO}\mathrm{K}$-formula χ equivalent to φ' (Th. 4.6). By the translation of $\mathsf{BL}_\mu^{\mathsf{lin}}\mathrm{K}$ into cMSO, formula χ has a single free second-order variable X, interpreted as a path, i.e. a plan;

(ii) Compute $\mathcal{B}_\chi$ (Proposition 3.11) which accepts all paths achieving goal χ, or equivalently φ'.

Actually, automaton $\mathcal{B}_\chi$ reads infinite words over alphabet $E \times \{0,1\}$, where the second component of the letters is always equal to 1, since the specified chain is required to be a path. Projecting a word accepted by $\mathcal{B}_\chi$ onto the first components of its letters provides a plans.

6 Discussion

We have adapted the seminal result of the decidability of the FO-theory of every automatic structure by augmenting the logic up to cMSO, a logic relying on $\mathsf{chain\text{-}MSO}$ of [12,21], but by restricting to the subclass of RA trees. A nice application of this result is the decidability of the generalized epistemic planning problem for propositional DEL presentations.

Regarding the computational complexity of our algorithms, it should be observed that the automata construction to model check against cMSO (Proposition 3.11) is non elementary in the number of alternations between existential and universal quantifiers in the formula (each underlying negation yields an exponentiation blow-up for the complementation). Still, it would be relevant to investigate in practice, if the automata have particular shapes that do not reach this non-elementary worst-case upper-bound complexity.

Compared to the work of Maubert [15] with $\mathsf{CTL}^*\mathrm{K}$, we offer the entire logic cMSO (or $\mathsf{BL}_\mu^{\mathsf{lin}}\mathrm{K}$) that strictly subsumes $\mathsf{CTL}^*\mathrm{K}$, and yet allowing to solve the epistemic planning problems over the same class of planning domains (i.e. propositional DEL presentations). However, it should be noticed that [15] also considers a different problem called the *epistemic protocol synthesis problem*, which differs from the generalized epistemic planning problem: in the

epistemic protocol synthesis problem, one has to prune the tree structure so that the remaining satisfies a $\mathsf{CTL}^*\mathrm{K}$-goal. This subject is out of the scope of this paper, but suggests some comments.

The existence of a pruning, i.e. a sub-tree, that satisfies the protocol goal, requires the use of second-order quantifiers ranging over arbitrary subsets of nodes, that is the full logic MSO which cannot be model checked (Theorem 2.13). It is a open question whetherthere is a logic in between cMSO and full MSO to solve the epistemic protocol synthesis problem with $\mathrm{BL}_\mu^{\mathsf{lin}}\mathrm{K}$ specifications. The existence of a pruning of the tree that satisfies the protocol goal, requires the use of second-order quantifiers ranging over arbitrary subsets of nodes, that is the full logic MSO which cannot be model checked (Theorem 2.13). It is an open question whether or not there is room for a logic in between cMSO and full MSO to solve the epistemic protocol synthesis problem with $\mathrm{BL}_\mu^{\mathsf{lin}}\mathrm{K}$ specifications. Also, the synthesis technique deployed by [15] actually applies to a family of tree structures that is larger than the one of RA trees. At the moment, the technique does not adapt to logics which involve arbitrary fix-points. It is an open question whether or not the work of [15] can be faithfully extended to deal the entire alternation-free fragment of $\mathrm{BL}_\mu^{\mathsf{lin}}\mathrm{K}$.

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