History-Preserving Bisimilarity for Higher-Dimensional Automata via Open Maps

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Executive Summary

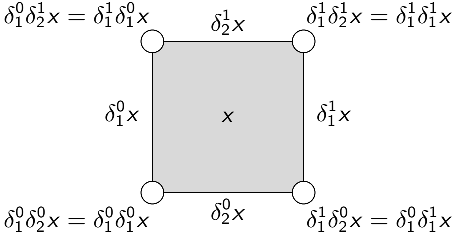
- History-preserving bisimilarity is, "morally", a relation on *paths*
- But we can show that for higher-dimensional automata, it is equivalent to a relation on *states* and (higher-dimensional) *transitions*
  - Easy consequence: decidability of HPB for *finite* HDA (generalizing a result for safe Petri nets)
- This adds weight to the claim that HDA are a natural and useful (and beautiful!) formalism for concurrency

1. Higher-Dimensional Automata
   - Definition
   - Labels (and why we can ignore them)
   - Higher-dimensional paths
   - Open-maps bisimulation
2. History-Preserving Bisimilarity
   - Homotopy
   - History-preserving bisimilarity & main result
   - Unfoldings
3. Conclusion and Discussion

Higher-dimensional automata

A precubical set:

- a graded set $X = \{X_n\}_{n\in\mathbb{N}}$
- in each dimension n, $2n$ face maps $\delta_k^0, \delta_k^1 : X_n \to X_{n-1}$ $(k = 1, \dots, n)$
- the precubical identity: $\delta_k^\nu \delta_\ell^\mu = \delta_{\ell-1}^\mu \delta_k^\nu$ for all $k < \ell$



A higher-dimensional automaton: a pointed precubical set (i.e. precubical set with initial state)

Higher-dimensional automata

HDA as a model for concurrency:

- points $x \in X_0$: states
- edges $a \in X_1$: transitions
- n-squares $\alpha \in X_n$ ($n \geq 2$): independency relations

- van Glabbeek (2006): Up to history-preserving bisimilarity, HDA generalize "the main models of concurrency proposed in the literature"

Morphisms and labels

- HDA: pointed precubical set
- morphism of HDA: pointed precubical morphisms (i.e. respects face maps)
- labeled HDA: HDA X plus labeling $\lambda : X_1 \to \Sigma$ s.t. $\lambda\delta_1^0 x = \lambda\delta_1^1 x$ and $\lambda\delta_2^0 x = \lambda\delta_2^1 x$ for all $x \in X_2$ (*opposite edges have same label*)
- morphism of labeled HDA: HDA morphism plus label morphism plus commutativity
- trick (Goubault 2002): use *higher-dimensional tori* for labeling. Then all involved functions become precubical morphisms

$\Rightarrow$ labeled HDA = pointed arrow category

$\Rightarrow$ results for unlabeled HDA generalize easily to labeled HDA

Higher-dimensional paths

- a computation in a HDA: a *cube path*: sequence $x_1, \dots, x_n$ of cubes connected by face maps, i.e. s.t. $x_i = \delta_k^0 x_{i+1}$ or $x_{i+1} = \delta_k^1 x_i$
- $x_i = \delta_k^0 x_{i+1}$: *start* of a new concurrent event
- $x_{i+1} = \delta_k^1 x_i$: *end* of a concurrent event
- HDP $\hookrightarrow$ HDA: subcategory of cube path objects and path extensions (not full!)

Open-maps bisimulation

- HDA morphism $f : X \to Y$ open if right-lifting w.r.t. HDP
- HDA X, Y om-bisimilar if span $X \leftarrow Z \to Y$ of open maps

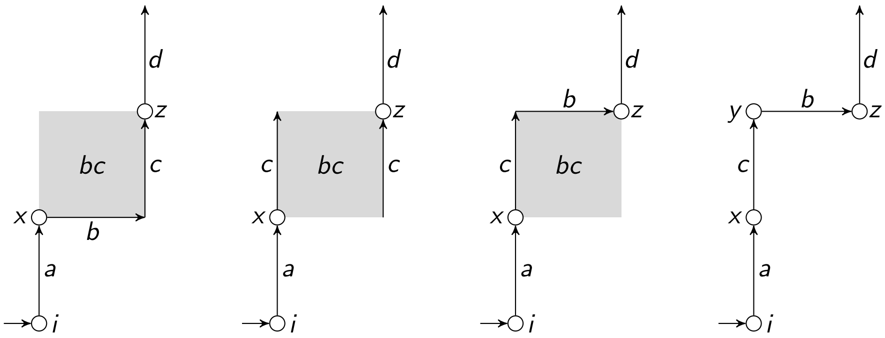
- Theorem: HDA X, Y om-bisimilar iff exists *pointed precubical subset* $R \subseteq X \times Y$ s.t. for all reachable $x \in X$, $y \in Y$ with $(x, y) \in R$:
  - for all $x = \delta_k^0 x'$, there is $y = \delta_k^0 y'$ with $(x', y') \in R$
  - for all $y = \delta_k^0 y'$, there is $x = \delta_k^0 x'$ with $(x', y') \in R$

- This is *beautiful*! But is it *useful*?

Homotopy

- hp-bisimilarity is a relation on computations which respects (Mazurkiewicz) trace equivalence
- for HDA: homotopy
- cube paths $x_1, \dots, x_n$, $y_1, \dots, y_n$ adjacent if $x_i = y_i$ for *all but one i*, and
  - x_i and y_i are distinct lower faces of x_{i+1}, or
  - x_i and y_i are distinct upper faces of x_{i-1}, or
  - x_{i-1}, x_{i+1} are lower and upper faces of x_i, and y_i is an upper face of x_{i-1} and a lower face of x_{i+1}, or
  - vice versa
- homotopy $\sim$: reflexive, transitive closure of adjacency

Example



History-preserving bisimilarity

HDA X, Y hp-bisimilar if exists relation R between cube paths in X and cube paths in Y s.t.

- the empty paths are related,
- for all $(\rho, \sigma) \in R$:
  - for all $\rho \rightsquigarrow \rho'$, there is $\sigma \rightsquigarrow \sigma'$ with $(\rho', \sigma') \in R$,
  - for all $\sigma \rightsquigarrow \sigma'$, there is $\rho \rightsquigarrow \rho'$ with $(\rho', \sigma') \in R$,
  - for all $\rho \sim \rho'$, there is $\sigma \sim \sigma'$ with $(\rho', \sigma') \in R$,
  - for all $\sigma \sim \sigma'$, there is $\rho \sim \rho'$ with $(\rho', \sigma') \in R$,

Main result: HDA are hp-bisimilar iff they are om-bisimilar.

Unfoldings

Goal: categorical setting for hp-bisimilarity

Tool: unfoldings of HDA

- unfolding up to homotopy, AKA *universal covering*
- unfolding of HDA X is $\tilde{X}$, set of homotopy classes of cube paths in X
- with suitable face maps (lower faces not trivial to define!)
- and a *projection* $\pi_X : \tilde{X} \to X$
- unfoldings are higher-dimensional trees

HP-morphisms

- A hp-morphism $f : X \curvearrowright Y$ is a morphism of unfoldings

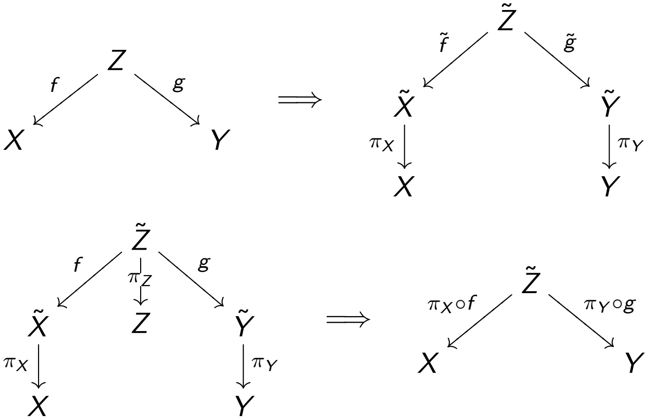
$$X \xleftarrow{\pi_X} \tilde{X} \xrightarrow{f} \tilde{Y} \xrightarrow{\pi_Y} Y$$

- $\mathsf{HDA_h}$: category of HDA and hp-morphisms
- embedding $\mathsf{HDP} \approx \mathsf{HDP_h} \hookrightarrow \mathsf{HDA_h}$
- hp-morphism $f : X \curvearrowright Y$ hp-open if right-lifting w.r.t. HDP
- Theorem: HDA X, Y hp-bisimilar iff span $X \curvearrowleft Z \curvearrowright Y$ of hp-open maps

Connecting the dots

Theorem: HDA X, Y om-bisimilar iff hp-bisimilar

Proof:



Conclusion and Discussion

- "Morally", the good *morphisms* for HDA are hp-morphisms of unfoldings
- But *hp-bisimilarity* has a simple precubical characterization
- This fits well in the "geometric" work on HDA

- Coalgebraic characterization?
- Relation to Staton-Winskel's (LICS 2010) unfolding of HDA into presheaves over event structures?
- *Hereditary* hp-bisimilarity?