ITERATIVE MULTI-USER ALGORITHM FOR CONVOLUTIONALLY CODED ASYNCHRONOUS DS-CDMA SYSTEMS: TURBO-CDMA

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ABSTRACT

A low-complexity and iterative receiver is proposed for the CDMA uplink, consisting of a low-complexity equalizer and single-user decoders. Iterations are used to feed the equalizer with more reliable data, which improves performance. This receiver called Turbo-CDMA is aimed to eliminate MUI almost completely in the presence of multipath and to be robust to the near-far effect. Simulation results show that the performance approaches single-user performance at moderate and also low signal-to-noise ratios, even for severe distortion channel.

Keywords: Coded CDMA, iterative methods, turbo-equalizer, multi-user detection, interference cancellation.

1. INTRODUCTION

Direct-Sequence Spread-Spectrum code-division multiple access (DS-CDMA) has raised a lot of interest in mobile communications and is already standardized (IS-95, UMTS). The main advantage of this multiple access technique is its large spectral efficiency [10]. In DS-CDMA, all users access the same channel by modulating orthogonal spreading sequences. Since we consider high rate communication over band-limited channels, this transmitted signal is subject to intersymbol interference (ISI) due to multipath propagation as well as multi-user interference (MUI) due to the non-orthogonal received sequences.

Under this assumption of non-orthogonal received sequences, the Rake receiver [6] is not optimal. Optimality is achieved through maximum-likelihood multi-user detector [9]. Unfortunately, the complexity of this algorithm grows exponentially with the number of users and the channel length, which forbids its use. Several suboptimal solutions have been proposed, among them zero forcing and minimum mean square error equalization [5]. We propose here to enhance the performance of those techniques in taking into account that most often (UMTS for instance), the data symbols transmitted over the channel are coded by a channel encoder. The CDMA transmitter is described in Figure 1:

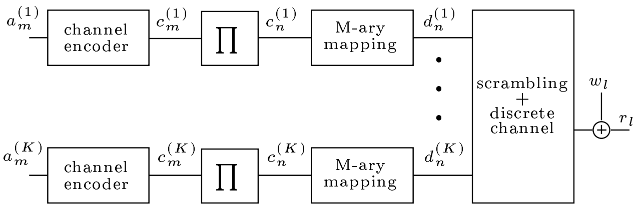

Figure 1: Received signal model ($\prod$: interleaver)

Considering that the convolutional code and channel form a serial concatenation of convolutions, an iterative soft-input/soft-output maximum a posteriori equalizer and decoder can be derived by following the idea of turbo-codes ([3] for single user and [1] for multi-user). Because of the complexity of such a detector, we propose in the sequel a suboptimal detector based on an interference canceler (IC) to generalize the single user turbo-equalizer [4] to the CDMA case.

In the paper, vectors and matrices are in boldface. The symbols $(.)^T$, $(.)^\dagger$ and $\|.\|$ stand for transposition, trans-conjugation and the vector norm.

2. SYSTEM MODEL

We consider here a block transmission DS-CDMA system where K users share the same temporal and spectral resources (See Figure 1). Each user employs convolutional coding and non-uniform interleaving. So, each user transmits a symbol sequence at rate $1/T_s$. We denote

$$\begin{aligned}\mathbf{d}^{(k)} &= [d_1^{(k)}, d_2^{(k)} \cdots d_n^{(k)} \cdots d_N^{(k)}]^T, k = 1 \cdots K \\ \mathbf{d} &= [\mathbf{d}^{(1)T}, \mathbf{d}^{(2)T} \cdots \mathbf{d}^{(k)T} \cdots \mathbf{d}^{(K)T}]^T\end{aligned}$$

N symbols taken from a complex alphabet of size M. Each data symbol $d_n^{(k)}$ $(n = 1...N, k = 1...K)$ is modulated by a spreading sequence consisting of Q complex elements at chip interval T_c with $T_c = T_s/Q$ and $Q \leq K$. Then,the resulting chip sequences of all users are transmitted over K multipath different channels of chip-length W. The K users are asynchronous and have different average powers. Finally, the CDMA channel adds zero-mean complex white gaussian noise $w_l (l = 1...QN{+}W{-}1)$ with variance σ_w^2. The transmission of the data sequence $\mathbf{d}^{(k)}$ over the CDMA channel can be expressed by the received sequence $\mathbf{r}$:

$$\mathbf{r} \;=\; [r_1, r_2 \cdots r_l \cdots r_{QN+W-1}]^T$$

$$\mathbf{r} \;=\; \underbrace{\left.\left[\begin{array}{c|c|c} \overset{N}{\longleftrightarrow} & & \overset{N}{\longleftrightarrow} \\ \mathbf{H_1} & \cdots & \mathbf{H_K} \\ & & \end{array}\right]\right\}_{QN+W-1}}_{\mathbf{H}} \; \underbrace{\left.\begin{bmatrix} d_1^{(1)} \\ \vdots \\ d_N^{(1)} \\ \hline \vdots \\ \hline d_1^{(K)} \\ \vdots \\ d_N^{(K)} \end{bmatrix}\right\}_{KN}}_{\mathbf{d}} + \mathbf{w}$$

where spreading and convolution by the channel are put together in the matrix $\mathbf{H}$ of size $(QN{+}W{-}1, KN)$.

3. TURBO-CDMA RECEIVER

In this section, we derive the iterative receiver to be called turbo-CDMA. We describe first the iterative aspects of the equalizer/decoder before discussing the Multi-user/Inter-symbol interference canceler.

3.1. Turbo-CDMA principle

The turbo-CDMA is implemented in a modular pipelined structure with p identical iterations, where each module consists of (see Figure 2) the concatenation of a Soft Input/Soft Output CDMA equalizer and for each equalizer output $\mathbf{y}^{(k)} = [y_1^{(k)}...y_N^{(k)}]^T$ $(k = 1...K)$ a deinterleaver, Soft Input/Soft Output decoder and interleaver. The K outputs of the p^{th} iteration are used together with the observation $\mathbf{r}$ to feed the next iteration, and so on.

For each user, decoding is carried out by using the MAP algorithm [2]. The decoder gives estimates of the information bits $\hat{a}_m^{(k)}$ and the corresponding BER. The decoder also computes information about the coded

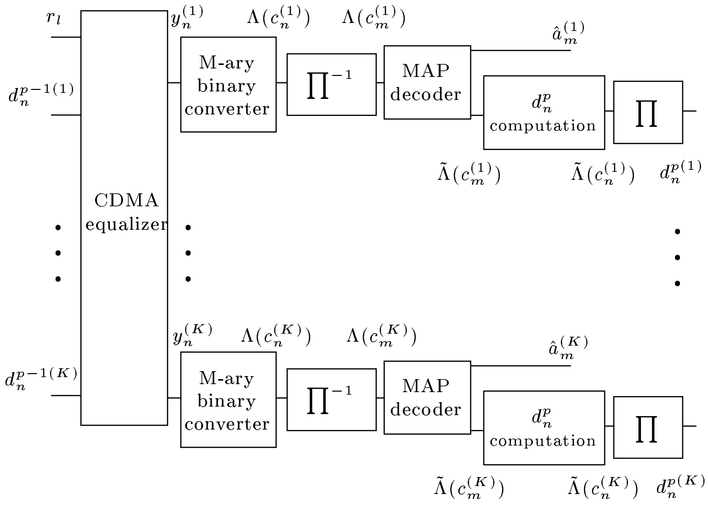

Figure 2: Module p of the turbo-cdma ($\prod$: interleaver,$\prod^{-1}$: deinterleaver)

bits in the form of log-likelihood ratios:

$$\tilde{\Lambda}(c_m^{(k)}) = \ln \frac{\mathcal{P}(c_m^{(k)} = 1 \mid y_n^{(k)})}{\mathcal{P}(c_m^{(k)} = 0 \mid y_n^{(k)})}$$

Thanks to the decoder, the bits have been corrected and $\tilde{\Lambda}(c_m^{(k)})$ is hopefully more reliable than $\Lambda(c_m^{(k)})$. After deinterleaving an estimate $d_n^{p\,(k)}$ of the mean value of the estimated symbols $d_n^{(k)}$ is calculated as in [4]. In the case of a binary modulation (BPSK):

$$d_n^{p\,(k)} = \mathcal{P}(c_m^{(k)} = 1 \mid y_n^{(k)}) - \mathcal{P}(c_m^{(k)} = 0 \mid y_n^{(k)})$$

All $d_n^{p\,(k)}$ $(k = 1...K)$ feed the equalizer of the following iteration.

3.2. Multi-user/Inter-symbol interference canceler

We propose here to use a Multi-user /Inter-symbol interference canceler (CDMA-IC) as for equalizer. It consists of two filters (see Figure 3), a feedforward filter ($\mathbf{F}$) and a "feedback filter" ($\mathbf{B}$) to filter the $d_n^{p\,(k)}$. $\mathbf{F}$ has taps spaced at the chip interval T_c and is fed by the received signal r_l. $\mathbf{B}$ has taps spaced at the symbol interval T_s to process the symbol sequence $d_n^{p-1\,(k)}$ $(k = 1...K)$ of the previous iteration.

The CDMA-IC filters the entire block of data at once, so that there is no problem of causality (as in regular DFE) and filter $\mathbf{B}$ can remove the interference caused by not only previous but also future symbols of any user. It allows us to choose for $\mathbf{F}$ the filter that maximizes the output signal-to-noise ratio (SNR), i.e. the filter matched to $\mathbf{H}$. Hence, provided that fed back

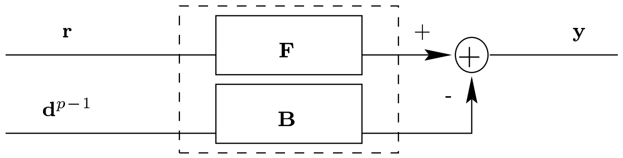

Figure 3: Multi-user/Inter-symbol IC
($\mathbf{y} = [\mathbf{y}^{(1)T} ... \mathbf{y}^{(K)T}]^T$)

symbols are correct (i.e. $\mathbf{d}^{p-1} = \mathbf{d}$), the filter that results in complete elimination of interference (MUI and ISI) is the autocorrelation of $\mathbf{H}$ where the diagonal terms have been suppressed:

$$\mathbf{y} = \mathbf{F}\,\mathbf{r} - \mathbf{B}\,\mathbf{d}^{p-1}, \begin{cases} \mathbf{F} = \mathbf{H}^{\dagger} \\ \mathbf{B} = \mathbf{H}^{\dagger}\mathbf{H} - diag(\mathbf{H}^{\dagger}\mathbf{H}) \end{cases} \quad (1)$$

Let us expand the expression for the CDMA-IC output under the assumption that $\mathbf{d}^{p-1} = \mathbf{d}$ and using (1):

$$\begin{aligned} y_n^{(k)} &= Q\|\mathbf{h}^{(k)}\|^2 d_n^{(k)} + \xi_n^{(k)}, & (2) \\ \text{with } \xi_n^{(k)} &= (\mathbf{s}^{(k)} \star \mathbf{h}^{(k)})^{\dagger}[w_{(n-1)Q+1} \cdots w_{nQ+W}]^T & \\ &\Rightarrow \sigma_\xi^2 = \|\mathbf{h}^{(k)}\|^2 Q \sigma_w^2 & \\ &\Rightarrow SNR(y_n^{(k)}) = \frac{\|\mathbf{h}^{(k)}\|^2 Q \sigma_d^2}{\sigma_w^2} & (3) \end{aligned}$$

where $\mathbf{h}^{(k)}$ is the chip-channel of user k defined as in [5] from $\mathbf{H}$. $\mathbf{s}^{(k)}$ is the corresponding spreading sequence. Equation (2) shows that the CDMA-IC results in complete elimination of all interferences (MUI an ISI), provided that the previous and future fed back symbols are correct. Let us consider the non-spread transmission of symbol $d_n^{(k)}$ over the non-dispersive AWGN channel with amplification $\|\mathbf{h}^{(k)}\|^2$. Let us note n, the noise of the transmission. The SNR of the non-spread output is:

$$SNR_{\text{non spread}} = \frac{\|\mathbf{h}^{(k)}\|^2 \sigma_d^2}{\sigma_n^2} \quad (4)$$

Moreover the spread and non-spread noise power are related by: $\sigma_w^2 = \sigma_n^2 Q$, so that $SNR(y_n^{(k)})$ is the same as that of the non-spread AWGN transmission without interferences and the BER is the same as in the AWGN case. This explains why the performance at the output of the CDMA-IC is given by the additive noise only. Moreover, the noise is colored. But, thanks to the deinterleaver, the noise feeding the decoder is whitened, which explains that the performance at the output of the decoder is the same as the performance of the AWGN non-spread coded channel.

Note that the equalizer of the very first iteration is a Zero-Forcing CDMA-DFE [5] that processes the observation only because $\mathbf{d}^0$ is not accessible.

4. SIMULATIONS

In the following simulations, we use for each user a rate $R_c = \frac{1}{2}$ systematic convolutional code with 64 states and generator polynomials (133,171) [7]. The interleaver is a (64,64) matrix read non-uniformly. The modulation is a BPSK. The spreading code sequences are orthogonal variable spreading factor codes (OVSF), as in UMTS. Spreading is followed by scrambling. The number of users is $K = 4$ and the spreading factor is $Q = 16$. The channel chip-length W is put to 36, so that ISI occurs over more than 2 symbols. Moreover the users' transmissions are asynchronous.

The near-far robustness of our receiver is investigated. 3 users (user 1 to 3) are set to maximal power 1 and one user (user 4) to a minimal power ($P_{user\,4} = 0.2025$), resulting in about 7 dB difference. The signal-to-multi-user-interference ratio (SIR) of user 4 after despreading is about 0 dB, where

$$SIR(user\,4) = 10\log_{10}\left(\frac{QP_{user\,4}}{\sum_{i=1,i\neq4}^{4} P_{user\,i}}\right)$$

When the SNR per symbol is 0 dB (BPSK and code rate $R_c = \frac{1}{2}$), the MUI (after despreading) is as powerful as the additive noise and as the symbol of interest. Since the SIR has to be compared with the SNR per symbol in order to measure the influence of MUI, the performance of the Turbo-CDMA receiver (evaluated after decoding) are plotted versus the SNR per symbol. Remember that for BPSK case and code with $R_c = 1/2$, the SNR per information bit ($\frac{E_b}{N_o}$) is related to the SNR per symbol ($\frac{E_s}{N_o}$) by: $\frac{E_b}{N_o} = \frac{E_s}{N_o} + 3$ dB.

Figure 4 shows the performance of the Turbo-CDMA receiver after decoding for user 1 and Figure 5 for user 4. As a benchmark, the error rate for the AWGN channel with no ISI but with code is also shown. The curves show that the Turbo-CDMA receiver yields a significant improvement in performance relative to a non-iterative CDMA receiver that consists of a Zero-Forcing CDMA DFE followed by a decoder (i.e. the first iteration in Figures 4 and 5). This improvement can be called the "Turbo CDMA gain".

Second, a comparison of these results with those of the single-user case (Figure 6) leads us to conclude that users 1 and 4 approach the single-user performance. (Note that users 2 and 3 are equivalent to user 1). In the case of user 1, the SIR worths 8.6 dB to be compared with the SNR per symbol. For user 1, MUI has therefore little influence on the performance, whereas it has a lot of influence for user 4, since MUI is larger than the additive noise, for $\frac{E_s}{N_0} \geq 0$ dB. Figure 5 shows that even user 4 with a 7 dB power-difference approaches the single-user case. Our method is robust to the near-

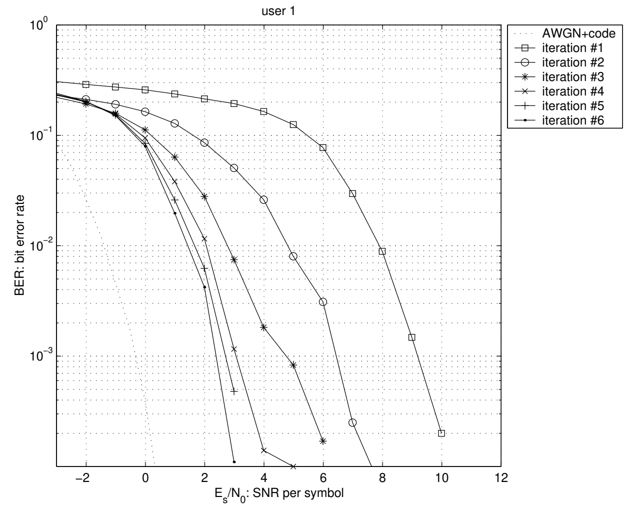

Figure 4: Performance of Turbo-CDMA: user 1.

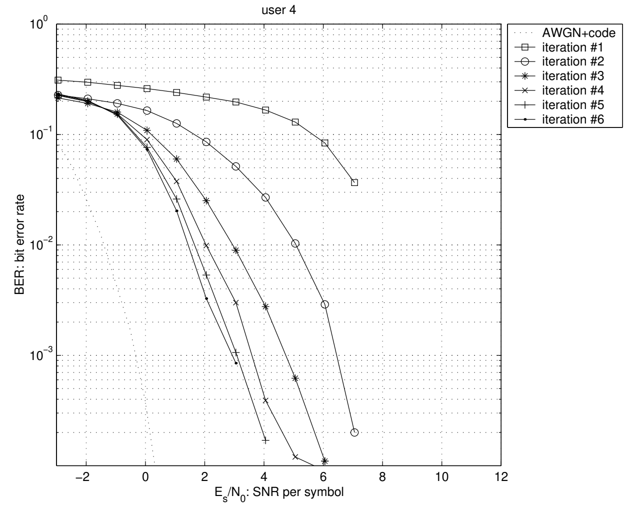

Figure 5: Performance of Turbo-CDMA: user 4.

far effect since one user of a multi-user system can not outperform the performance of a single-user system. The turbo-CDMA results therefore in almost complete elimination of the MUI.

These results indicate that there is still a degradation in performance because the users do not approach the AWGN coded channel case. However the single-user performance shows the same gap, which is due to the residual ISI that can not be eliminated in the case of a severe distortion channel (see [8]).

5. CONCLUSION

In this paper, we have proposed a multi-user receiver for the uplink CDMA channel, that results in almost complete elimination of the multi-user interference even when near-far effect occurs.

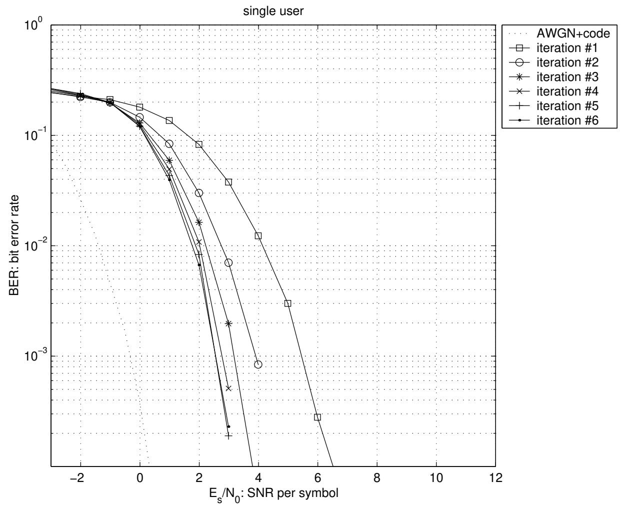

Figure 6: Single-user performance.

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