IMPROVED INTERFERENCE CANCELLATION FOR TURBO-EQUALIZATION

I. Fijalkow[1], *A. Roumy*[1,2], *S. Ronger*[1], *D. Pirez*[2], *P. Vila*[2]

[1]Equipe de Traitement des Images et du Signal
ETIS / ENSEA-UCP, 6 av du Ponceau
95014 Cergy-Pontoise Cdx, France
`fijalkow, roumy@ensea.fr`

[2]Thomson-CSF Communications
BP 156, 66 rue du Fossé Blanc
92231 Genneviliers Cdx, France
`Pierre.VILA@tcc.thomson-csf.com`

ABSTRACT

Sub-optimal joint equalization and decoding is performed by iterated equalization and decoding. This processing is named turbo-equalization with reference to the turbo-decoding of serially concatenated codes. We propose to optimize the equalizer structure (an interference canceler) thanks to the training sequence available to estimate the channel impulse response. The interference canceler, optimized at each iteration, permits to reduce the number of iterations needed to achieve a given performance. The gain is all the more important that the channel is hard to equalize.
Keywords: Joint equalization and decoding, turbo-equalization, optimal DFE, Wiener filtering.

1. INTRODUCTION

Digital communications receivers must fight the perturbations introduced by the propagation conditions and the electronics. Each device in the receiver is built so to combat a specific perturbation. The equalizer removes the intersymbol interference (frequency fading). The deinterleaver fights flat fading, and finally the decoder removes the residual errors due to noise and non-optimal performance of the previous devices. The equalizer that can be based either on the Viterbi algorithm or on a Decision Feedback Equalizer (DFE) structure, is based on a hard-decision making even if the signal just before decision is very far from an input signal alphabet member. Some information available before the decision making is therefore lost.

Actually, the optimal receiver that could be thought of is a Viterbi algorithmm applied to the global system including coder-interleaver and channel. Unfortunatly the exponential complexity of the algorithm with respect to the system lenght forbids from such a solution. Several sub-optimal solutions have been proposed, among them the turbo-detector in [3] is of great interest. Considering that the convolutive coder and the channel form a serial concatenation of convolutions, [3] proposed to follow the idea of turbo-codes by iterating soft-input / soft-output *maximum a posteriori* equalizer and decoder. It can however not be used for either long channels or for large size alphabets of modulation (8-PSK, 16-QAM,...). In those cases, Laot and Glavieux proposed to use an interference canceler for equalization optimized in the absence of errors from the previous iteration, see [4]-[5].

In this paper, we propose to optimize the interference canceler at each iteration so to improve performance in a reduced number of iterations.

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2. THE TURBO-EQUALIZER

In this section, we present the structure of the turbo-equalizer proposed in [4]-[5] and explain its limitations due to the fixed interference canceler.

2.1. Principle

The received signal is assumed to be the output of the baseband discret equivalent model:

$$r_n = \sum_{j=0}^{Q} h_j a_{n-j} + w_n$$

where a_n is the symbol input sequence, (h_k) is the channel finite impulse response and w_n is the additive white gaussian noise. The turbo-equalizer performs iterated soft-input / soft-output equalization and decoding as in figure 1.

The output of the p^{th} iteration is used together with the channel output to feed the next iteration, and so on. Note that the equalizer of the very first iteration is a DFE (or a MAP equalizer when possible, see [7])

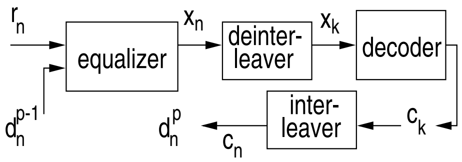

Figure 1: Turbo-equalizer $p^{\,th}$ iteration.

that processes the output of the channel only since d_n^0 is not accessible.

Decoding is carried out by using the (MAP) algorithm [1]. The decoder computes information about the coded bits in the form of log-likelihood ratios:

$$\Lambda(c_k) = \ln \frac{\mathcal{P}(c_k = 1 \mid x_k)}{\mathcal{P}(c_k = 0 \mid x_k)}$$

The decoder also gives estimates of the information bits $\hat{c}_k$ and the corresponding Bit Error Rate (BER). After interleaving the mean value of the estimated symbols d_n is computed, in the BPSK case, as,

$$d_n^p = \mathcal{P}(c_n = 1 \mid x_n) - \mathcal{P}(c_n = 0 \mid x_n)$$

This value feeds the following iteration.

2.2. The fixed interference canceler

[4]-[5] propose to use an interference canceler (see figure 2) to perform the equalization in the second and following iterations of the turbo-equalizer.

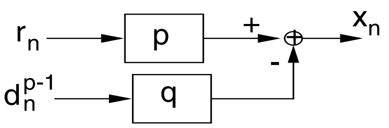

Figure 2: Interference canceler structure

If the previous iteration provides us with the correct symbols only $d_n^{p-1} = d_n$, the filters $\mathbf{p}$ and $\mathbf{q}$ are found by minimization of the input/output Mean Square Error (MSE), with the constraint $q(0) = 0$ (to prevent from suppression of the signal of interest). The MSE, $\mathrm{E}\left[|x_n - d_n|^2\right]$, miminimum is described by

$$\begin{aligned} p_0(n) &= \alpha h^\star(-n), \text{ where } \alpha = \frac{\sigma_d^2}{\sigma_d^2 + \sigma_w^2}, \\ q_0(n) &= \alpha(\gamma_h(n) - \gamma_h(0)\delta(n)), \end{aligned} \qquad (1)$$

with $\gamma_h(n) = \sum_k h(k)h^*(k-n)$. p_0 is the filter matched to the channel h and q_0 is the channel intercorrelation where the middle term has been suppressed.

This interference canceler can be viewed as the optimal non-causal Decision Feedback Equalizer (DFE) described when correct decisions are taken (see [2] for a review on DFEs). The optimal DFE isn't used in practice since one doesn't know about future symbols. Nevertheless, in our case since q_0 is fed by the data coming from the previous iteration, d_n^{p-1}, there is no need of causality and the optimal DFE / interference canceler can be used.

If $d_n^{p-1} = d_n$, the equalizer output is $x_n = \alpha\gamma_h(0)d_n + \alpha p_0 * w_n$. Note that the signal to noise ratio $\gamma_h(0)\sigma_d^2/\sigma_w^2$ is not changed by the interference canceler. The BER after deinterleaving and decoding is also similar to that of the channel without intersymbol interference corrupted by white gaussian noise only and corrected by the coding gain. The performance achieved at the output of the decoder is optimal in the sense it corresponds to the best achievable performance for a transmission at a given rate and Signal to Noise ratio (SNR), see figure 3.

Indeed, the assumption of correct decisions isn't satisfied. The loss due to the non-perfect cancellation of remaining interferences can be viewed in figure 3 where d_n^{p-1} is the output of a DFE.

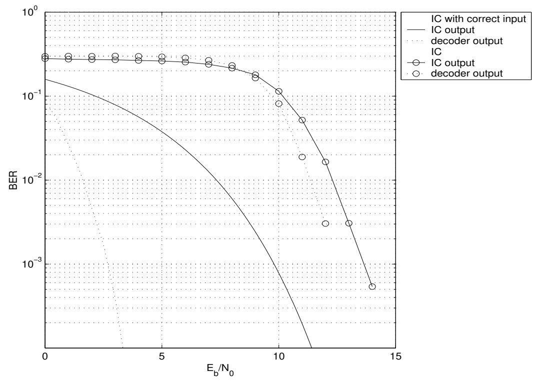

Figure 3: Performances of the fixed Interference Canceller (IC) for the Proakis C channel. The previous iteration equalizer is a DFE.

In the sequel, we propose to take into account the values of d_n^{p-1} in order to improve the interference canceler.

3. THE IMPROVED INTERFERENCE CANCELER

In this section, we propose to optimize the interference canceler at each iteration by using all knowledge we got of the input data, i.e. the training sequence used to estimate the channel impulse response. First, we find the exact expression of the optimal filters. In the sequel, we will propose a procedure to get an estimate from a finite size training sequence.

3.1. Optimal interference canceler

At first we assume to know both true input data d_n and the output of the previous iteration. Optimization is performed using the MSE criterion but taking into account that $d_n^{p-1} = d_n + \epsilon_n$ where ϵ_n depends on both observations r_n and true input d_n. Performing the calculation in the frequency domain, we observe that the optimal filters should have an infinite impulse response which is not desirable in practice. Therefore, we choose to constrain the filters to have a finite impulse response. We set:

$$\mathbf{p} = [p(-L), p(-L+1), \cdots, p(L)]^\top,$$
$$\tilde{\mathbf{q}} = [q(-L), \cdots, q(-1), q(1), \cdots, q(L)]^\top,$$

where L is choosen to be large enough compared to the channel length. $^\top$ stands for transposition and † for conjugate-transposition. Note that $\tilde{\mathbf{q}}$ includes the constraint $q_0(0) = 0$.

The interference canceler optimization can be seen as a Wiener filtering problem where the filter is $(\mathbf{p}^\dagger, \tilde{\mathbf{q}}^\dagger)^\dagger$, the observed signal $(\mathbf{r}^\dagger, -\tilde{\mathbf{d}}^{p-1\dagger})^\dagger$, and the desired signal d_n. Here $\tilde{\mathbf{d}}^{p-1} = (d_{n+L}^{p-1}, \cdots, d_{n+1}^{p-1}, d_{n-1}^{p-1}, \cdots, d_{n-L}^{p-1})^\top$ and $\mathbf{r} = (r_{n+L}, \cdots r_{n-L})^\top$. The Minimum MSE is then achieved when $\mathbf{p}_\epsilon$ and $\tilde{\mathbf{q}}_\epsilon$ are given by a Wiener-like equation,

$$\mathcal{R}\begin{pmatrix} \mathbf{p}_\epsilon \\ \tilde{\mathbf{q}}_\epsilon \end{pmatrix} = \mathbf{b}, \tag{2}$$

where $\mathcal{R} = \mathrm{E}\left[\begin{pmatrix} \mathbf{r} \\ -\tilde{\mathbf{d}}^{p-1} \end{pmatrix}(\mathbf{r}^\dagger, -\tilde{\mathbf{d}}^{p-1\dagger})\right]$ is the correlation matrix of the interference canceler input vector. $\mathbf{b} = \mathrm{E}\left[d_n \begin{pmatrix} \mathbf{r} \\ -\tilde{\mathbf{d}}^{p-1} \end{pmatrix}^*\right]$ represents the intercorrelation between the reference and the input vector.

3.2. Sub-optimal interference canceler

The optimal solution requires many samples to be estimated accuratly so that first simulations didn't show a clear benefit from the estimated optimal solution. We propose therefore to approximate the optimal solution assuming a small error variance in terms of the solution found when correct decisions are assumed.

We notice that $\mathcal{R} = \mathcal{R}_0 + \mathcal{R}_\epsilon$ and $\mathbf{b} = \mathbf{b}_0 + \mathbf{b}_\epsilon$ where $\mathcal{R}_0$ and $\mathbf{b}_0$ correspond to the correct decision assumption, i.e. $d_n^{p-1} = d_n$,

$$\mathcal{R}_0 = \sigma_d^2 \mathcal{T}(h)\mathcal{T}(h)^\dagger + \sigma_w^2 I,\ b_0 = \sigma_d^2 \mathcal{T}(h)\delta_0 \tag{3}$$

where $\mathcal{T}(h) = \begin{pmatrix} h_0 & h_1 & \cdots & h_Q & 0 & \cdots & \\ 0 & h_0 & \cdots & & h_Q & 0\cdots & \\ & & & \ddots & & & \\ & & \cdots & 0 & h_0 & \cdots & h_Q \end{pmatrix}$ is the channel convolution matrix and δ_0 the kronecker vector
with its central term equal to 1. The terms due to non-zero ϵ_n are $\mathcal{R}_\epsilon = \begin{pmatrix} \mathbf{0} & -R_{r,\tilde{\epsilon}} \\ -R_{\tilde{\epsilon},r} & R_{\tilde{\epsilon},\tilde{\epsilon}} + R_{\tilde{\epsilon},\tilde{d}} + R_{\tilde{d},\tilde{\epsilon}} \end{pmatrix}$, and $\mathbf{b}_\epsilon = \begin{pmatrix} \mathbf{0} \\ -cal R^*_{\tilde{\epsilon},d} & \delta_0 \end{pmatrix}$.

Then using a first order Taylor expansion of $\mathcal{R}^{-1} \approx (\mathbf{I} - \mathcal{R}_0^{-1}\mathcal{R}_\epsilon)\mathcal{R}_0^{-1}$, we propose the new interference canceler approximation:

$$\begin{pmatrix} \mathbf{p}_\epsilon \\ \tilde{\mathbf{q}}_\epsilon \end{pmatrix} \approx \begin{pmatrix} \mathbf{p}_0 \\ \tilde{\mathbf{q}}_0 \end{pmatrix} - \mathcal{R}_0^{-1}\left(\mathcal{R}_\epsilon \begin{pmatrix} \mathbf{p}_0 \\ \tilde{\mathbf{q}}_0 \end{pmatrix} + \mathbf{b}_\epsilon\right) \tag{4}$$

where only the second order terms related to ϵ_n remain. The terms involving ϵ appear as correction to the main term $\mathbf{p}_0, \tilde{\mathbf{q}}_0$. Such an approximation should be valid since iterating the turbo-equalizer with the interference canceler is useful only when the first iteration equalizer achieves good enough performance to induce the "turbo-effect", see [7]. Therefore, the new interference canceler should be efficient to increase the breakdown effect when the turbo-effect is triggered.

The improvement due to the optimization of the interference canceler can be described by the MSE corresponding to (4), denoted $\mathrm{MSE}(\mathbf{p}_\epsilon, \tilde{\mathbf{q}}_\epsilon)$. Simple calculation shows that $\mathrm{MSE}(\mathbf{p}_\epsilon, \tilde{\mathbf{q}}_\epsilon)$

$$\begin{aligned} &= (\mathbf{p}_\epsilon^\dagger, \tilde{\mathbf{q}}_\epsilon^\dagger)\mathcal{R}\begin{pmatrix} \mathbf{p}_\epsilon \\ \tilde{\mathbf{q}}_\epsilon \end{pmatrix} - 2\mathrm{Re}\{(\mathbf{p}_\epsilon^\dagger, \tilde{\mathbf{q}}_\epsilon^\dagger)\mathbf{b}\} + \sigma_d^2 \\ &= \mathrm{MSE}(\mathbf{p}_0, \tilde{\mathbf{q}}_0) + (\delta\mathbf{p}_\epsilon^\dagger, \delta\tilde{\mathbf{q}}_\epsilon^\dagger)\mathcal{R}\begin{pmatrix} \delta\mathbf{p}_\epsilon \\ \delta\tilde{\mathbf{q}}_\epsilon \end{pmatrix} \\ &\quad + 2\mathrm{Re}\left\{(\delta\mathbf{p}_\epsilon^\dagger, \delta\tilde{\mathbf{q}}_\epsilon^\dagger)\left(\mathcal{R}\begin{pmatrix} \mathbf{p}_\epsilon \\ \tilde{\mathbf{q}}_\epsilon \end{pmatrix} - \mathbf{b}\right)\right\} \\ &\approx \mathrm{MSE}(\mathbf{p}_0, \tilde{\mathbf{q}}_0) - (\delta\mathbf{p}_\epsilon^\dagger, \delta\tilde{\mathbf{q}}_\epsilon^\dagger)\mathcal{R}\begin{pmatrix} \delta\mathbf{p}_\epsilon \\ \delta\tilde{\mathbf{q}}_\epsilon \end{pmatrix} \end{aligned} \tag{5}$$

where for a vector $\mathbf{x}$, $\delta\mathbf{x}_\epsilon = \mathbf{x}_\epsilon - \mathbf{x}_0$. Therefore, the gain in terms of MSE, $\left\| \begin{matrix} \delta\mathbf{p}_\epsilon \\ \delta\tilde{\mathbf{q}}_\epsilon \end{matrix} \right\|_{\mathcal{R}}^2$, is all the more important than new interference canceler (4) is different from the fixed one, i.e. in the case of very difficult channels.

3.3. Implementation

$\mathbf{p}_0, \tilde{\mathbf{q}}_0$ are calculated through a straightforward estimation of the channel impulse response, $h(n)$, using the training sequence, [6]. Once the channel is estimated by $\hat{h}$, the expressions in (3) provide estimates $\hat{\mathcal{R}}_0$ and $\hat{b}_0$.

For each iteration, $\hat{\mathcal{R}}$ is estimated using the entire sequence, whereas $\hat{b}$ can be estimated during the training period only. $\hat{\mathcal{R}}_\epsilon$ and $\hat{b}_\epsilon$ are then deduced from $\hat{\mathcal{R}}$ and $\hat{b}$ by subtraction of $\hat{\mathcal{R}}_0$ and $\hat{b}_0$. The proposed interference canceler is then calculated using (4).

4. SIMULATIONS

The simulations were performed with:
- a recursive systematic convolutional code with constraint length 7 and generators $[1, \frac{171}{133}]$,
- interleaver: 59×59 uniform matrix,
- BPSK modulation,
- channel : Proakis B and C [6, page 616],
- the filters $\mathbf{p}$ and $\mathbf{q}$ are constrainted to have the same size as $\mathbf{p}_0$ and $\mathbf{q}_0$[1],
- simulation over 10^6 information bits and long training sequences of 512 bits.

On the following simulations (Figures 4 and 5), we focus on an SNR zone just following the turbo-equalizer trigger point, to see the effect of the optimized interference canceler on the BER.

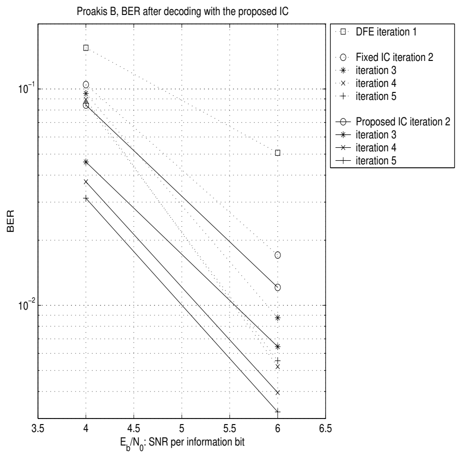

Figure 4: Performance of the proposed turbo-equalizer for channel Proakis B.

The proposed interference canceler allows to reduce the number of iterations: 2 instead of 4 for Proakis B, and 2 instead of 5 for Proakis C, (see Figure 4 and 5) to achieve a given performance. As expected from (5), the gain is all the more important than the channel is difficult, in our case channel Proakis C.

5. CONCLUSION

We have proposed to optimize the turbo-equalizer interference canceler in order to reduce the processing computation cost. The required number of iterations is therefore significantly reduced. The new processing allows also to deal with very difficult channels such as Proakis C. Further work will consist in improving the estimation procedure recursively on several training sequences.

[1] Indeed, more degrees of freedom on the filters lenght could only improve the performance.

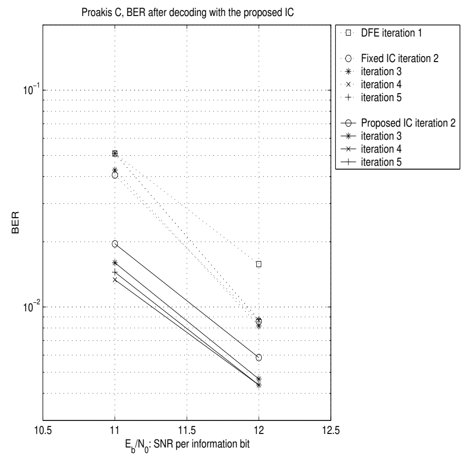

Figure 5: Performance of the proposed turbo-equalizer for channel Proakis C.

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