Design Methods for Irregular Repeat Accumulate Codes

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We optimize the low-complexity random-like ensemble of Irregular Repeat Accumulate (IRA) codes for binary-input symmetric channels in the large blocklength limit. IRA codes [1] are special subclasses of both irregular LDPCs and irregular Turbo codes. In IRA codes, a fraction f_i of information bits is repeated i times, for $i = 2, 3, \dots$. The distribution $\{f_i \geq 0, i = 2, 3, \dots : \sum_{i=2}^{\infty} f_i = 1\}$ is referred to as the *repetition profile*, and it is kept as a degree of freedom in the optimization of the IRA ensemble (for the sake of analysis, we will use in the following the fraction λ_i of edges connected to information bitnodes of degree i). After the repetition stage, the resulting sequence is interleaved and input to a recursive finite-state machine which outputs one bit for every a input symbols, where a is referred to as *grouping factor* and is also a design parameter.

Next, we formulate the density evolution (DE) [2] for IRA codes and we study the stability condition of the fixed-point corresponding to zero BER:

Theorem 1. *The fixed-point for the DE, , for which* Pe $= 0$*, is locally stable if and only if*

$$\lambda_2 < \frac{e^r(e^r - 1)}{a + 1 + e^r(a-1)}$$

where $r = -\log(\int e^{-z/2} dF_u(z))$ *and where* F_u *denotes the distribution of the channel observation messages.*

Thus, for every value of the channel parameter ν (ν is, for example, an indicator of the noise level in the channel), the optimal IRA ensemble parameters a and $\{\lambda_i\}$ maximize R subject to vanishing $\text{BER}(\nu) = 0$. Equivalently, they solve

$$\begin{cases} \text{maximize} & a \sum_{i=2}^{d} \lambda_i / i \\ \text{subject to} & \sum_{i=2}^{d} \lambda_i = 1, \quad \lambda_i \geq 0 \; \forall \, i \\ \text{and to} & \text{BER}(\nu) = 0 \end{cases}$$

for whose solution we can resort to the numerical techniques in [2]. However, the constraint $\text{BER}(\nu) = 0$ is given directly in terms of the fixed-point of the DE recursion, and makes optimization computationally very intensive.

A variety of methods have been developed in order to simplify the code ensemble optimization [1, 3]. They consist of replacing the DE with a dynamical system defined over the reals (rather than over the space of distributions), whose trajectories and fixed-points are related in some way to the trajectories and fixed-point of the DE. Several recent works show that DE can be accurately described in terms of the evolution of the mutual information between the variables associated with the bitnodes and their messages (see [4, 5]). A key idea in order to approximate DE by mutual information evolution is to describe each computation node in the BP-decoding by a (EXIT) *mutual information transfer function* [4].

We approximate the densities of the messages exchanged in the message passing decoder using mutual information and propose four different optimization methods. First we make use of the reciprocal channel approximation [3] and assume that the distributions at every iteration are either Gaussian (GA) (method 1) or the output of a BEC (method 2). This yields closed-form DE approximations. Next the EXIT function of a decoding block is obtained by Monte Carlo simulation by assuming either Gaussian or BEC distributed input messages (method 3 and 4 respectively). These approximated DE yields four optimization methods that can be formulated as linear programs.

Then, we show some properties of the approximated DE derived above:

Theorem 2. *The local stability condition of the approximated DE with Method 1 is the same as that of the exact DE.*

Proposition 3. *The local stability condition of the approximated DE with Method 2 is less stringent than that of the exact DE.*

Theorem 3. *The DE approximations of Methods 2 and 4 have unique fixed-point with vanishing BER only if the IRA ensemble coding rate* R *satisfies* $R < C = \mathfrak{I}(F_u)$*.*

Because of space limitations, in this summary we omit our results for the BSC and only present numerical results for the BIAWGNC, where the codes were optimized according to the proposed methods, and the thresholds evaluated by using the full DE. The Gaussian a priori approximation turns out to be more attractive since the codes designed under this assumption have the smallest gap to Shannon limit. Even if the best LDPC codes [6] may slightly outperform our designed codes, we believe that these IRA codes are of interest because of their inherent extremely low encoding and decoding complexity.

Method	1	2	3	4
Rate	0.50183	0.49697	0.50154	0.49465
a	8	8	8	8
d	7.94153	8.09755	7.95087	8.17305
$\text{SNR}_{gap}(\text{DE})$	0.059	0.406	0.075	0.306
$\text{SNR}_{gap}(\text{approx.})$	-0.025	0.040	-0.021	0.071

References

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