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Turbo multiuser detection for coded asynchronous DS-CDMA over frequency selective channels

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Abstract — **In order to achieve performant but low-complexity multiuser detection for coded signals, we propose to use a Full Turbo-CDMA structure. Noticing that a good order of demodulation is very effective in improving performance, we propose a new criterion for the order of demodulation in an asynchronous and frequency selective CDMA context. It takes into account both Intersymbol- and Multiuser interferences.**

I. Introduction

Direct-Sequence Spread-Spectrum code-division multiple access (DS-CDMA) has raised a lot of interest in mobile communications and is already standardized (IS-95). The main advantage of this multiple access technique is its large spectral efficiency [15]. In DS-CDMA, all users share the same resources (time & frequency) and are assigned orthogonal spreading sequences. Since we consider asynchronous high rate communication over band-limited channels (i.e. uplink transmission), the transmitted signal is subject to intersymbol interference (ISI) due to multipath propagation as well as multiuser interference (MUI) due to the non-orthogonal received sequences. There may be a large gap in performance between the single user and the multiuser receiver, so the most performant approach is multiuser.

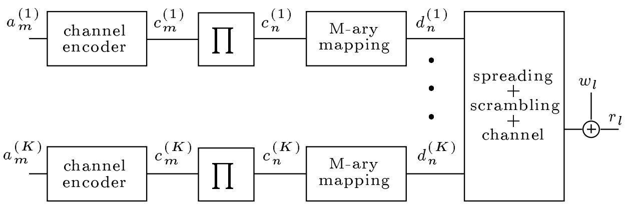

Fig. 1: Received signal model ($\prod$: interleaver)

Because most wireless systems use error control coding, performance can be enhanced by taking into account that data are coded, when equalization is done. In this coded context, optimality [4] is achieved through an highly complex algorithm. Considering that the convolutional code and channel form a serial concatenation of convolutions, an iterative soft-input/soft-output maximum a posteriori equalizer and decoder can be derived by following the idea of Turbo-codes ([3] for single user and [1] for multiuser). Because of the complexity of such a detector, we proposed in [12] a sub-optimal detector based on an interference canceler (IC) to generalize the single user Turbo-equalizer [5] to the CDMA case. This structure [12] can be improved at the expense of increasing the complexity as in [16] or by using a successive scheme rather than a parallel one as all schemes mentioned before. The "Full Turbo" structure we first proposed in [13] improves performance of [12] at the expense of an extra delay of time.

In this paper, we propose to improve the performance of the previous Full Turbo multiuser scheme [13]. This can be done by demodulating the users according to a criterion that takes into account the MUI. We propose here to show the influence of this criterion on the demodulator but also on the global system.

Throughout this paper, scalars are lower case whereas vectors are also lower case but in boldface and matrices are upper case and in boldface. The symbols $(.)^T$, $(.)^\dagger$ and $\|.\|$ stand respectively for transposition, trans-conjugation and L_2 norm. Variables have subscripts that refer to the time increment, superscripts that refer to the iteration and superscripts in brackets to the user.

II. System model

We model here the uplink of a coded DS-CDMA communication system, where K asynchronous users access a common frequency selective channel to transmit information (see Figure 1). Each user employs convolutional coding, non-uniform interleaving and bit-to-symbol mapping. So, each user transmits a symbol sequence at symbol rate $1/T_s$. We denote

$$\mathbf{d}^{(k)} = [d_1^{(k)}, d_2^{(k)} \cdots d_n^{(k)} \cdots d_N^{(k)}]^T, k = 1 \cdots K,$$

N symbols taken from a complex alphabet of finite size.

Then, the DS-CDMA system is used, where K users transmit over frequency selective channels with additive Gaussian noise (as in [8]). The received sequence $\mathbf{r}$ is the concatenation of the samples of the received signal at chip rate $1/T_c$ with $T_c = T_s/Q$ and $Q \leq K$. Q is called the spreading factor.

$$\mathbf{r} = [r_1, r_2 \cdots r_l \cdots r_{QN+W-1}]^T$$

$$\mathbf{r} = \underbrace{\left[\begin{array}{c|c|c} \overset{N}{\longleftrightarrow} & & \overset{N}{\longleftrightarrow} \\ \mathbf{H}^{(1)} & \cdots & \mathbf{H}^{(K)} \end{array} \right]}_{QN+W-1 \text{ rows}} \underbrace{\left[\begin{array}{c} \mathbf{d}^{(1)} \\ \hline \vdots \\ \hline \mathbf{d}^{(K)} \end{array} \right]}_{KN} + \mathbf{w},$$

$$\text{where } \mathbf{H}^{(k)} = [\mathbf{h}_1^{(k)} \cdots \mathbf{h}_N^{(k)}] \ ,$$

$$\mathbf{h}_n^{(k)} = [\, 0 \cdots 0 \quad \underset{\underset{(n-1)Q+1}{\uparrow}}{g_1^{(k)}} \quad \cdots \quad \underset{\underset{nQ+W}{\uparrow}}{g_{Q+W-1}^{(k)}} \quad 0 \cdots 0 \,]^T.$$

$\mathbf{r}$ contains the contribution of all users and an additive white gaussian noise $\mathbf{w}$ with variance σ_w^2. The contribution of one user (k) can be written as a matrix-vector product $\mathbf{H}^{(k)}\mathbf{d}^{(k)}$, where each column $\mathbf{h}_n^{(k)}$ of $\mathbf{H}^{(k)}$ is a shifted version of

$$\mathbf{g} = [g_1^{(k)}, g_2^{(k)} \cdots g_l^{(k)} \cdots g_{Q+W-1}^{(k)}]^T,$$

where $\mathbf{g}$ contains the spreading code, all equipment filtering (that is transmitter power, chip pulse waveform, transmitter filter and its matched filter in the receiver) and propagation effects (multipath, time delay).

Since we consider a block transmission where symbols before and after the block are known, they can be canceled, that is to say they can be set to 0. So first and last vectors in $\mathbf{H}^{(k)}$ are not truncated. Moreover, the columns in $\mathbf{H}^{(k)}$ may differ due to the non-stationarity of the propagation effects and due to the scrambling codes that can modulate different symbols.

III. Full Turbo-CDMA principle

The Turbo-principle can be successfully applied to the case of multiuser detection. We consider here the Full Turbo CDMA receiver that we proposed in [13]. Let k refers to the user of interest.

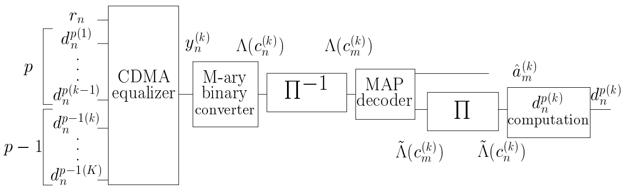

Fig. 2: Module p of the Full Turbo-CDMA for user (k).
(Π: interleaver, Π^{-1}: deinterleaver)

Each module of this Full Turbo-CDMA receiver consists of (see Figure 2) the concatenation of a Soft Input/Soft Output CDMA equalizer, a deinterleaver, a Soft Input/Soft Output decoder and an interleaver. Note that the equalizer of user k and of the pth iteration is fed by the output of iteration p for already decoded users (i.e. for users 1 to $k-1$) and with the output of iteration $p-1$ for the others as in Figure 2.

Decoding is carried out by using the MAP algorithm [2]. The decoder is fed by soft information about the coded bits in the form of log-likelihood ratios:

$$\Lambda(c_n^{(k)}) = \log \frac{\mathcal{P}(c_n^{(k)} = 1 \mid \mathbf{y}^{(k)})}{\mathcal{P}(c_n^{(k)} = 0 \mid \mathbf{y}^{(k)})}.$$

Under the assumptions that there is no ISI left at the output of the equalizer (which corresponds to the Gaussian approximation in [16]) $y_n^{(k)} = \beta_n^{(k)} c_n^{(k)} + \xi_n^{(k)}$, that the modulation is binary and that the noise $\xi_n^{(k)}$ is independent of $c_n^{(k)}$, the log-likelihood ratio can be rewritten as:

$$\Lambda(c_n^{(k)}) = \frac{4}{2\sigma_\xi^2} \mathcal{R}e\{y_n^{(k)} \beta_n^{(k)*}\}.$$

The decoder output gives estimates of the information bits $\hat{a}_m^{(k)}$. The decoder also computes information about the coded bits:

$$\tilde{\Lambda}(c_m^{(k)}) = \log \frac{\mathcal{P}(c_m^{(k)} = 1 \mid \mathbf{y}^{(k)})}{\mathcal{P}(c_m^{(k)} = 0 \mid \mathbf{y}^{(k)})}.$$

Thanks to the decoder, the bits have been corrected and $\tilde{\Lambda}(c_m^{(k)})$ is hopefully more reliable than $\Lambda(c_n^{(k)})$. After deinterleaving, it is used to estimate $d_n^{p\,(k)}$ that feeds the equalizer [5]. In the case of a binary modulation (BPSK):

$$d_n^{p\,(k)} = \mathcal{P}(c_n^{(k)} = 1 \mid \mathbf{y}^{(k)}) - \mathcal{P}(c_n^{(k)} = 0 \mid \mathbf{y}^{(k)}).$$

In the next Section, we recall the CDMA equalizer for this Full-Turbo structure and propose a criterion to order the demodulation.

IV. Full DFE and order of demodulation

In [13], we derive a DFE rather than linear equalizers (as in [16]) for the equalizer of the first iteration. We have actually shown earlier [11] that the better the very first equalizer is, the better the asymptotic performance is: the "Turbo" effect is triggered earlier in terms of E_b/N_0. Therefore using a DFE rather than a linear equalizer helps to achieve better performance.

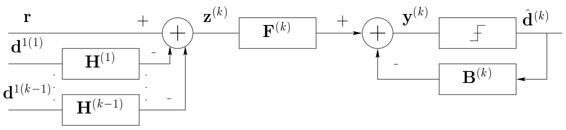

Fig. 3: DFE of for user (k) and for the Full Turbo-cdma receiver.

We proceed now to consider an equalizer that is well suited to the Full structure since it processes the data user blocks. Let k refers to the user of interest. This DFE (see Figure 3) first cancels the contribution of the already decoded users of the current iteration:

$$\mathbf{z}^{(k)} = \mathbf{r} - \sum_{j=1}^{k-1} \mathbf{H}^{(j)} \mathbf{d}^{1(j)}.$$

Then, it is aimed to minimize the **user block** Mean Square Error (MSE) $E[\|\mathbf{y}^{(k)} - \mathbf{d}^{(k)}\|^2]$, where $\mathbf{y}^{(k)}$ is the output of the DFE:

$$\mathbf{y}^{(k)} = \mathbf{F}^{(k)} \mathbf{z}^{(k)} - \mathbf{B}^{(k)} \hat{\mathbf{d}}^{(k)}$$

If correct decisions are considered relatively to the already decoded users (1 to $k-1$),

$$\mathbf{z}^{(k)} = \mathbf{H}^{(k)} \mathbf{d}^{(k)} + \sum_{j=k+1}^{K} \mathbf{H}^{(j)} \mathbf{d}^{(j)} + \mathbf{w} = \mathbf{H}^{(k)} \mathbf{d}^{(k)} + \mathbf{n}.$$

The next users are considered as "noise" with covariance matrix

$$\mathbf{R}_n = E[\mathbf{n}\mathbf{n}^\dagger] = \sigma_w^2 \mathcal{I} + \sigma_d^2 \sum_{j=k+1}^{K} \mathbf{H}^{(j)} \mathbf{H}^{(j)\dagger}.$$

The minimization of the **user block** MSE under the assumption that fed back symbols of the kth user are correct is derived in a similar way to [14, page 377] and leads to

$$\left\{ \begin{array}{lcl} \mathbf{F}^{(k)} & = & \mathbf{\Sigma}^{-2} \mathbf{L}^{-1\dagger} \mathbf{H}^{(k)\dagger} \mathbf{R}_n^{-1} \\ \mathbf{B}^{(k)} & = & \mathbf{L} - \mathcal{I} \end{array} \right.$$

where $\mathbf{H}^{(k)\dagger} \mathbf{R}_n^{-1} \mathbf{H}^{(k)} + \sigma_d^{-2} \mathcal{I} = (\mathbf{\Sigma}\mathbf{L})^\dagger \mathbf{\Sigma}\mathbf{L}$ is the Cholesky decomposition.

In the DFE (Figure 3), previously detected symbols feed the matrix $\mathbf{B}^{(k)}$. However, detected symbols may not correspond to the correct ones. Since the DFE is derived under the assumption of correct feedback, it fails to take into account this effect of decisions errors. This leads to the well known problem of error propagation. Note that a usual approach consists of demodulating the users in the order of decreasing received powers. However, it fails to consider all interferences. So, we propose a most appropriate order of demodulation.

In order to reduce error propagation, we want to subtract the contributions of users with less decision errors. In the case of synchronous transmission, this is done in [14, page 345] by

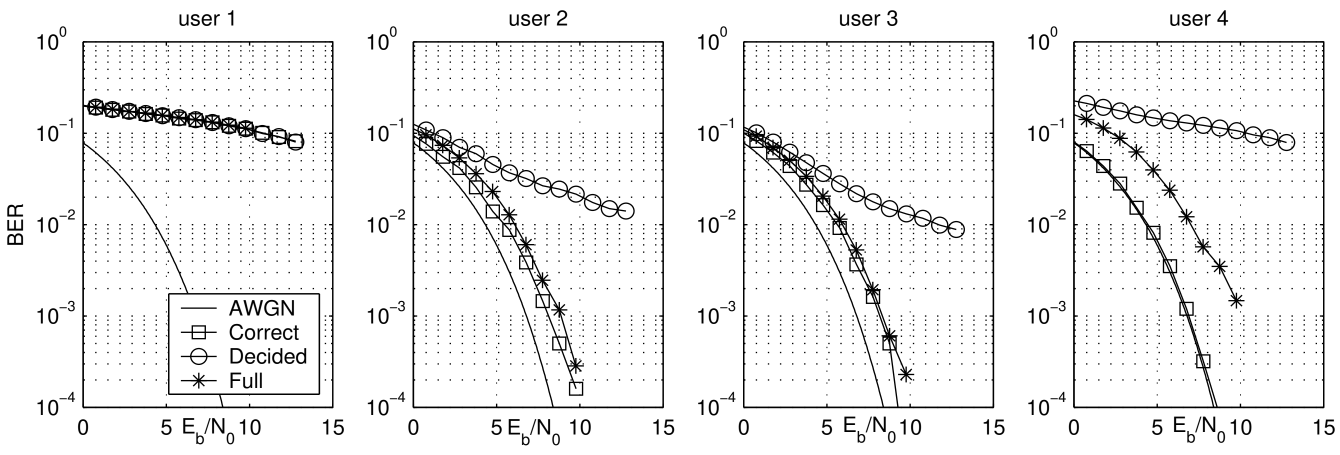

Fig. 4: Performance of the **user block** MMSE DFE, with order of demodulation (1/2/3/4).

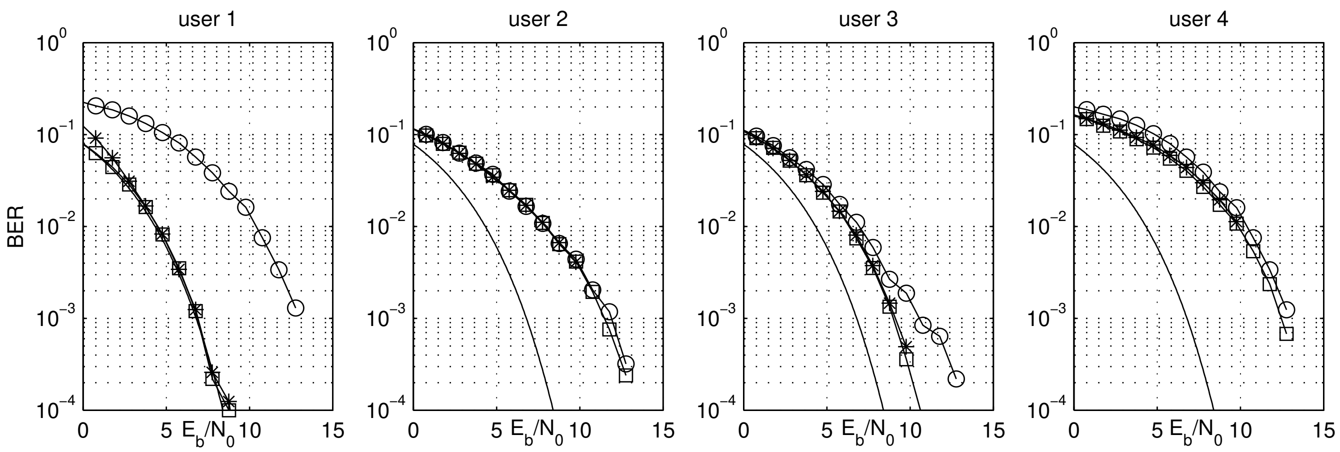

Fig. 5: Performance of the **user block** MMSE DFE, with best order of demodulation (2/3/4/1). See legend in Figure 4.

ordering the users according to a criterion that takes into account the crosscorrelations among users. When asynchronism and frequency selectivity occurs, a further improvement is to define a criterion that takes into account ISI and MUI. Consequently, we generalize the criterion proposed in [7] to the multiuser case and propose to demodulate the users in decreasing order of:

$$\sum_{n=1}^{N}\Big(\underbrace{\mathbf{h}_n^{(k)\dagger}\mathbf{h}_n^{(k)}}_{\text{received power}} - \sum_{m,m\neq n} \underbrace{|\mathbf{h}_n^{(k)\dagger}\mathbf{h}_m^{(k)}|}_{\text{ISI}} - \sum_{j,j\neq k}\sum_{m} \underbrace{|\mathbf{h}_n^{(k)\dagger}\mathbf{h}_m^{(j)}|}_{\text{MUI}}\Big) \quad (1)$$

(1) is the average value of the received power for user (k) minus all interferences. Note that (1) can be interpreted as the distance between the 2 nearest states at the output of the Rake receiver relative to user k.

Next, we will show the importance of the order of demodulation on the performance of the DFE. This is all the more important that we know that the equalizer of the first iteration is crucial for the Turbo-receiver to be triggered.

V. Simulation settings and DFE's Performance

In the following simulations, we use for each user a rate $R_c = \frac{1}{2}$ systematic convolutional code with 64 states and generator polynomials (133,171) [9]. The interleaver is a (64,64) matrix read non-uniformly. The modulation is a BPSK. The spreading code sequences are orthogonal variable spreading factor codes (OVSF), as in UMTS. Spreading is followed by scrambling. The number of users is $K = 4$ and the spreading factor is $Q = 16$. The channel chip-length W is set to 14. Moreover the users' transmissions are asynchronous. All users have equal received powers, so that the usual order of demodulation is arbitrary.

Figure 4 shows the performance of the DFE for an arbitrary order of demodulation, when data from the previous users are either correct, decided at the output of this equalizer or decoded thanks to the Full structure. Note that the behavior of the users are very different even if they have equal received powers. This suggests that the received powers are not sufficient to determine the order of demodulation. Moreover, this very bad order of demodulation yields error propagation, since performance with correct and detected feedback are so different.

The performance of the Full DFE (i.e. with decoded fed back data) is also plotted in Figure 4. The improvement of the Full structure (decoded fed back data instead of decided data) yields to significant reduction of error propagation. Yet, since the order of demodulation is not the most appropriate, the performance of user 1 are still very poor. This can be improved by using the ordering criterion (1) as is shown in Figure 5. Note that error propagation is very few (little distance between correct and decided) and they can be almost completely eliminated with the Full structure. We get almost the best achievable performance after DFE (correct fed back data).

VI. Full IC

In order to show the effect of the ordering in the Full-CDMA receiver, we recall the structure of the Multiuser /Intersymbol interference canceler (CDMA-IC) proposed in [13] as for equalizer for its low-complexity. The output of this IC, $\mathbf{y}^{(k)} = [y_1^{(k)} \cdots y_n^{(k)} \cdots y_N^{(k)}]$ can be expressed as:

$$\mathbf{y}^{p\,(k)} = \mathbf{F}^{(k)}\mathbf{r} - \sum_{j=1}^{k-1} \mathbf{B}_j^{(k)} \mathbf{d}^{p\,(j)} - \sum_{j=k}^{K} \mathbf{B}_j^{(k)} \mathbf{d}^{p-1\,(j)}. \qquad (2)$$

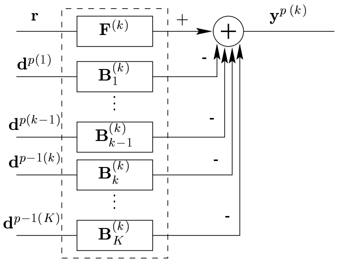

Fig. 6: Proposed multiuser IC

This leads to the structure shown in Figure 6. It consists of a "feedforward" matrix $\mathbf{F}^{(k)}$ and a bank of "feedback" matrices $(\mathbf{B}_1^{(k)} \cdots \mathbf{B}_j^{(k)} \cdots \mathbf{B}_K^{(k)})$. These matrices are chosen so to minimize the symbol criterion: $E[\|y_n^{(k)} - d_n^{(k)}\|^2]$, under the assumption that all fed back inputs are correct. Because of this assumption, whatever the scheme is, Full or not, the expressions are the same ([12] and [13]):

$$\left\{ \begin{array}{lcl} \mathbf{F}^{(k)} & = & \mathbf{D}^{(k)}\mathbf{H}^{(k)\dagger}, \\ \mathbf{B}_j^{(k)} & = & \mathbf{D}^{(k)}\left(\mathbf{H}^{(k)\dagger}\mathbf{H}^{(j)} - diag(\mathbf{H}^{(k)\dagger}\mathbf{H}^{(j)})\right), \\ \mathbf{D}^{(k)} & = & Diag(\alpha_1^{(k)} \cdots \alpha_n^{(k)} \cdots \alpha_N^{(k)}), \\ \alpha_n^{(k)} & = & \frac{\sigma_d^2}{\sigma_d^2\|\mathbf{h}_n^{(k)}\|^2 + \sigma_w^2}, \end{array} \right.$$

where $Diag(x_1 \cdots x_K)$ denotes a diagonal matrix whose diagonal elements are $x_1 \cdots x_K$, whereas $diag(\mathbf{A})$ is a diagonal matrix whose diagonal elements are the diagonal elements of the square matrix $\mathbf{A}$.

Note that the CDMA-IC processes the entire block of data at once. Therefore, there is no problem of causality (as in regular DFE) and the set of matrices $\{\mathbf{B}_j^{(k)}\}_j$ can remove the interference caused by the previous but also by the future symbols of any user. This allows $\mathbf{F}^{(k)}$ to be the matched matrix to $\mathbf{H}^{(k)}$ which maximizes the output signal-to-noise ratio (SNR) and which leads to better performance than the DFE.

Note also that the Full CDMA-IC leads to a successive structure, since the users have to be processed (equalizer+decoder) one after each other. So, this CDMA-IC is aimed to eliminate the interference due to $k-1$ "previous" users and to $K-(k-1)$ "future" users. Since the "previous" users have been decoded once more than the "future" ones, they are more reliable. The better the previous are, the less error propagation they yield and the better the performance is. So, the order of demodulation should also influence the global performance of the Turbo-receiver. This is what we propose to discuss in the next section.

VII. Performance of the Full Turbo-CDMA

The behavior of the Full Turbo-CDMA receiver is similar to that of the Turbo-equalizer (single user case) we have studied in [11]. Indeed, there is a trigger point followed by a breakdown effect, i.e. performance improves as the iteration step grows.

Figure 7 shows the performance of this Full Turbo-CDMA receiver where the users were demodulated in an arbitrary order (that is not the best one or even the worst one). Figure 8 shows the performance of the same Full Turbo-CDMA receiver where the users were demodulated in the order of decreasing levels defined in (1). The performance of users 2 (as 3) is only slightly better since it was already (see Figure 7) very close to the performance of a one user transmission over an AWGN. This bound is called code+AWGN in the Figures. In contrast, users 4 and 1 take advantage of this good order in which users are demodulated. Here the influence of the order of demodulation is more important. The amelioration is so important that for user 1, the 1st iteration of the well ordered Full receiver is already better than the 6th iteration of the arbitrary ordered one. In this case, the Full Turbo-CDMA leads, with users demodulated in the order of decreasing level (1), to much better asymptotic performance.

However, simulations show that even this Turbo receiver never achieves the bound derived with the multiuser IC fed by correct data. We analyzed earlier in [11] this gap between simulation performance and the AWGN+code bound. First, AWGN+code is not the most tight bound since it is not achievable when severe distortion occurs. Second, the gap is also due to the suboptimal Turbo process.

VIII. Conclusion

The main contribution of this paper is the derivation of a new criterion to order the demodulation of the users. This criterion is well suited for all CDMA equalizers that take into account both Intersymbol- and Multiuser interferences in an asynchronous and mutipath context. This criterion is very useful to the a Full Turbo CDMA receiver, where demodulation is carried out successively.

Simulations show that the Full Turbo-CDMA receiver with the demodulation order criterion approaches the performance of AWGN channel. The gain in number of iterations required to achieve a given performance is all the more important that the user is difficult to equalize.

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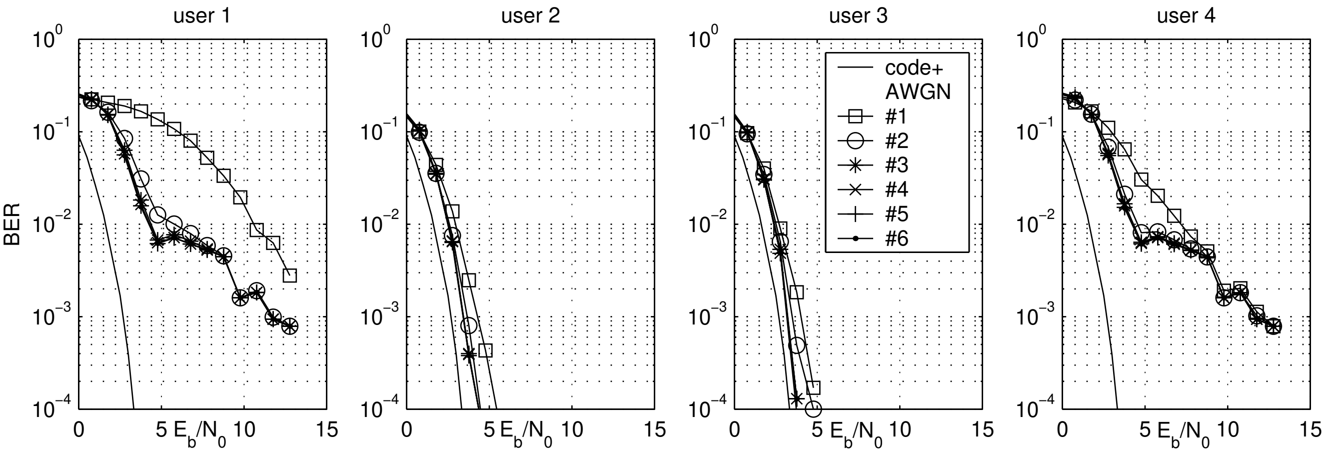

Fig. 7: Full Turbo-CDMA performance, with multiuser **user block** MMSE DFE and order of demodulation (1/2/3/4).

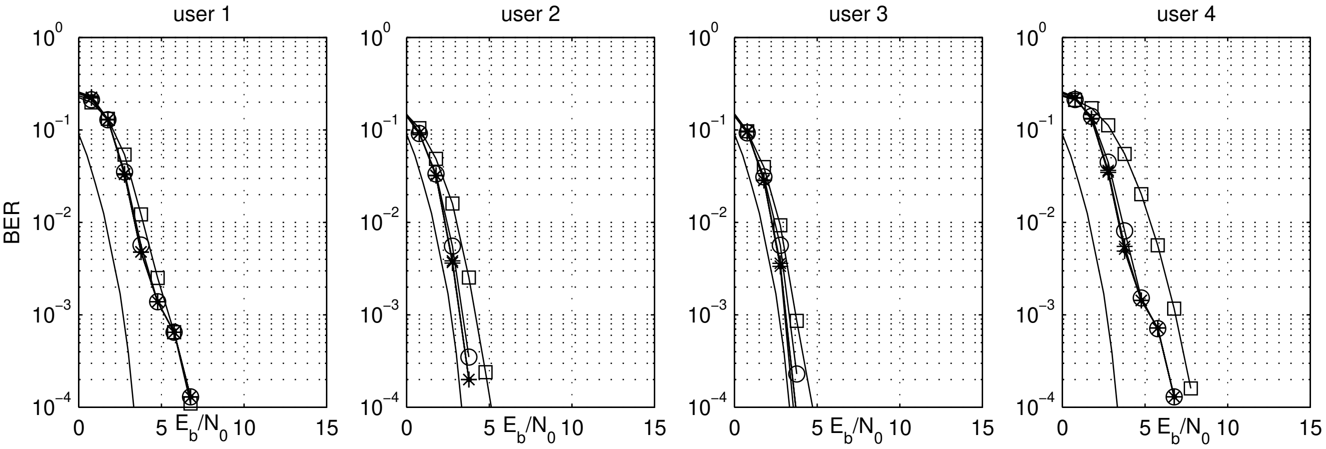

Fig. 8: Full Turbo-CDMA performance, with multiuser **user block** MMSE DFE and best order of demodulation (2/3/4/1). See legend in Figure 7.

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