Full Turbo receiver for coded asynchronous DS-CDMA over multipath channel

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Abstract: *In order to improve the Turbo-CDMA receiver [11] without increasing complexity, we propose a new structure called Full-Turbo CDMA since it fully uses the decoded information. This Full receiver enhances performance at the expense of an extra delay of time.*

Keywords: Turbo equalization, multiuser receiver, Code division multiacces (CDMA), interference suppression, frequency selective channels.

1. Introduction

In the DS-CDMA uplink transmission, asynchronism yields inter-user interference and channel distortion yields inter-user as well as inter-symbol interference. In this context, there may be a large gap in performance between the single-user and the multiuser receiver, so the most appropriate approach is multiuser.

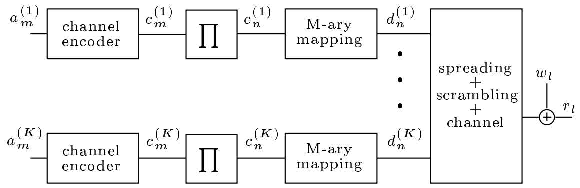

Figure 1: Received signal model ($\prod$: interleaver)

Because most wireless systems use error control coding, performance can be enhanced in taking into account that data are coded, when equalization is done. In this coded context, optimality [4] is achieved through an highly complex algorithm. Considering that the convolutional code and channel form a serial concatenation of convolutions, an iterative soft-input/soft-output maximum a posteriori equalizer and decoder can be derived by following the idea of turbo-codes ([3] for single user and [1] for multiuser). Because of the complexity of such a detector, we proposed in [11] a suboptimal detector based on an interference canceler (IC) that generalizes the single user Turbo-equalizer [5] to the CDMA case.

In this paper, we propose to fully use the multiuser receiver so to improve performance in a reduced number of iterations.

Throughout this paper, scalars are lower case whereas vectors are also lower case but in boldface and matrices are upper case and in boldface. The symbols $(.)^T$, $(.)^\dagger$ and $\|.\|$ stand respectively for transposition, trans-conjugation and L_2 norm. Variables have subscripts that refer to the time increment, superscripts that refer to the iteration and superscripts in brackets to the user.

2. System model

We model here the uplink of a coded DS-CDMA communication system, where K asynchronous users access a common frequency selective channel to transmit information (see Figure 1). Each user employs convolutional coding, non-uniform interleaving and bit-to-symbol mapping. So, each user transmits a symbol sequence at symbol rate $1/T_s$:

$$\mathbf{d}^{(k)} = [d_1^{(k)}, d_2^{(k)} \cdots d_n^{(k)} \cdots d_N^{(k)}]^T, k = 1 \cdots K.$$

Then, the DS-CDMA system is used, where K users transmit over frequency selective channels with additive Gaussian noise (as in [7]). The received sequence $\mathbf{r}$ is the concatenation of the samples of the received signal at chip rate $1/T_c$ with $T_c = T_s/Q$ and $Q \leq K$. Q is called the spreading factor.

$$\mathbf{r} = [r_1, r_2 \cdots r_l \cdots r_{QN+W-1}]^T$$

$$\mathbf{r} = \underbrace{\left[\begin{array}{c|c|c} \overbrace{\mathbf{H}^{(1)}}^{N} & \cdots & \overbrace{\mathbf{H}^{(K)}}^{N} \end{array} \right]}_{QN+W-1 \text{ rows}} \underbrace{\left[\begin{array}{c} \mathbf{d}^{(1)} \\ \hline \vdots \\ \hline \mathbf{d}^{(K)} \end{array} \right]}_{KN \text{ rows}} + \mathbf{w},$$

where $\mathbf{H}^{(k)} = [\mathbf{h}_1^{(k)} \cdots \mathbf{h}_N^{(k)}]$,

$$\mathbf{h}_n^{(k)} = [\, 0 \cdots 0 \quad \underset{\substack{\uparrow \\ (n-1)Q+1}}{g_1^{(k)}} \quad \cdots \quad \underset{\substack{\uparrow \\ nQ+W}}{g_{Q+W-1}^{(k)}} \quad 0 \cdots 0 \,]^T.$$

$\mathbf{r}$ contains the contribution of all users and an additive white gaussian noise $\mathbf{w}$ with variance σ_w^2. The contribution of one user (k) can be written as a matrix-vector product $\mathbf{H}^{(k)}\mathbf{d}^{(k)}$, where each column $\mathbf{h}_n^{(k)}$ of $\mathbf{H}^{(k)}$ is a shifted version of

$$\mathbf{g} = [g_1^{(k)}, g_2^{(k)} \cdots g_l^{(k)} \cdots g_{Q+W-1}^{(k)}]^T,$$

where $\mathbf{g}$ contains the spreading code, all equipment filtering (that is transmitter power, chip pulse waveform, transmitter filter and its matched filter in the receiver) and propagation effects (multipath, time delay).

3. Turbo-CDMA principle

The Turbo-principle can be successfully applied to the case of multiuser detection. Take, as an example, the Turbo-detector. [9] (as [6]) proposes to use the Turbo-detector in a chip- and symbol-synchronous CDMA context and proceeds to design an algorithm devoted to the asynchronous CDMA channel in [1]. Owing to the complexity of the detector, both papers propose a method to reduce it in using the M-algorithm.

Because the complexity of the detectors used in Turbo-detection is still high, we consider in the sequel the suboptimal detector based on an IC that we proposed in [11] (see Figure 2, where k refers to the user of interest). It can be seen as a generalization of the single user Turbo-equalizer [5] to the CDMA case.

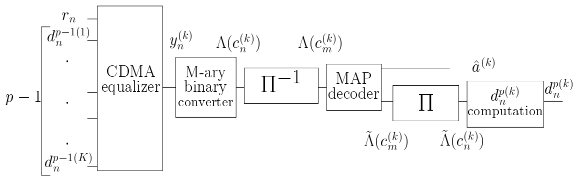

Figure 2: Module p of the Turbo-CDMA for user (k). (Π: interleaver, Π^{-1}: deinterleaver)

A way to improve this scheme is the work by [14]. The multiuser IC used does not assume that the fed back symbols are correct and takes into account theirs statistics. This may yield better performance however, the method needs one matrix inversion per symbol and per user, leading to a prohibitive complexity. In contrast, we propose here to improve our method without growing the complexity.

4. Full Turbo-CDMA principle

Note that all the previous Turbo-CDMA schemes don't fully use the decoded information. The CDMA equalizers of the pth iteration are fed by the output of iteration $p-1$. For user k, we propose to feed the equalizer of the pth iteration with the output of iteration p for already decoded users (i.e. for users 1 to $k-1$) and with the output of iteration $p-1$ for the others as in Figure 3. Note that since the data of $k-1$ inputs of iteration p have been decoded once more than the data of the previous iteration, they are more reliable and so is the output of the equalizer.

Since the equalizer of the kth user processes already decoded data of previous users $(1\cdots k-1)$, the structure is said to be successive, i.e. the users have to be processed one after each other. The successive scheme is more reliable than the parallel one but it requires an extra delay of time.

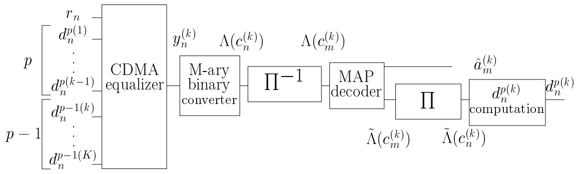

Figure 3: Module p of the Full Turbo for user (k).

Then, each module of this Full Turbo-CDMA receiver consists of (see Figure 2) the concatenation of a Soft Input/Soft Output CDMA equalizer, a deinterleaver, a Soft Input/Soft Output decoder and an interleaver.
Decoding is carried out by using the MAP algorithm [2]. The decoder is fed by soft information about the coded bits in the form of log-likelihood ratios (LLR):

$$\Lambda(c_n^{(k)}) = \log \frac{\mathcal{P}(c_n^{(k)} = 1 \mid \mathbf{y}^{(k)})}{\mathcal{P}(c_n^{(k)} = 0 \mid \mathbf{y}^{(k)})}.$$

Under the assumptions that there is no ISI left at the output of the equalizer $y_n^{(k)} = \beta_n^{(k)} c_n^{(k)} + \xi_n^{(k)}$, that the modulation is binary and that the noise $\xi_n^{(k)}$ is independent of $c_n^{(k)}$, the LLR can be rewritten as:

$$\Lambda(c_n^{(k)}) = \frac{4}{2\sigma_\xi^2} \mathcal{R}e\{y_n^{(k)} \beta_n^{(k)*}\}.$$

The decoder gives estimates of the information bits $\hat{a}_m^{(k)}$ and computes the LLR $\tilde{\Lambda}(c_m^{(k)})$ about the coded bits. Thanks to the decoder, the bits have been corrected and $\tilde{\Lambda}(c_m^{(k)})$ is hopefully more reliable than $\Lambda(c_n^{(k)})$. After deinterleaving, it is used to estimate $d_n^{p\,(k)}$ that feeds the equalizer of the following iteration [5]. In the case of a binary modulation (BPSK):

$$d_n^{p\,(k)} = \mathcal{P}(c_n^{(k)} = 1 \mid \mathbf{y}^{(k)}) - \mathcal{P}(c_n^{(k)} = 0 \mid \mathbf{y}^{(k)}).$$

5. Full DFE

The equalizer of the first iteration doesn't have a feedback from previous iterations. We consider therefore a DFE that we are going to improve using the Full structure. Let k refers to the user of interest.

The DFE should first cancel the contribution of the already decoded users of the current iteration:

$$\mathbf{z}^{(k)} = \mathbf{r} - \sum_{j=1}^{k-1} \mathbf{H}^{(j)} \mathbf{d}^{1(j)}.$$

Then, the DFE is aimed to minimize the **user block** Mean Square Error (MSE) $E[\|\mathbf{y}^{(k)} - \mathbf{d}^{(k)}\|^2]$,

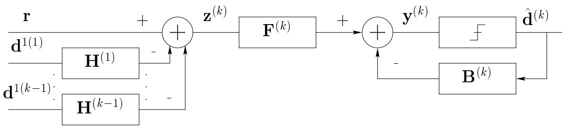

Figure 4: Full DFE for user (k).

where $\mathbf{y}^{(k)}$ is the output of the DFE:

$$\mathbf{y}^{(k)} = \mathbf{F}^{(k)}\mathbf{z}^{(k)} - \mathbf{B}^{(k)}\hat{\mathbf{d}}^{(k)}$$

If correct decision are considered relatively to the already decoded users (1 to $k-1$),

$$\mathbf{z}^{(k)} = \mathbf{H}^{(k)}\mathbf{d}^{(k)} + \sum_{j=k+1}^{K} \mathbf{H}^{(j)}\mathbf{d}^{(j)} + \mathbf{w} = \mathbf{H}^{(k)}\mathbf{d}^{(k)} + \mathbf{n}.$$

The next users are considered as "noise" $\mathbf{n}$, with covariance matrix

$$\mathbf{R}_n = E[\mathbf{n}\mathbf{n}^\dagger] = \sigma_w^2\mathcal{I} + \sigma_d^2 \sum_{j=k+1}^{K} \mathbf{H}^{(j)}\mathbf{H}^{(j)\dagger}.$$

The minimization of the **user block** MSE under the assumption that fed back symbols of the kth user are correct is derived in a similar way to [13, page 377] and leads to:

$$\left\{ \begin{array}{lcl} \mathbf{F}^{(k)} & = & \mathbf{\Sigma}^{-2}\mathbf{L}^{-1\dagger}\mathbf{H}^{(k)\dagger}\mathbf{R}_n^{-1} \\ \mathbf{B}^{(k)} & = & \mathbf{L} - \mathcal{I} \end{array} \right.$$

where $\mathbf{H}^{(k)\dagger}\mathbf{R}_n^{-1}\mathbf{H}^{(k)} + \sigma_d^{-2}\mathcal{I} = (\mathbf{\Sigma}\mathbf{L})^\dagger\mathbf{\Sigma}\mathbf{L}$ is the Cholesky decomposition.

Moreover, the users are demodulated in the order of decreasing level that we proposed in [12]. This criterion takes into account all kind of interferences (Intersymbol- and Multiuser) in order to reduce error propagation. Now, that we derived the equalizer of the first iteration, we present the equalizer of the next iterations.

6. Full IC

We propose a Multiuser /Inter-symbol interference canceler (CDMA-IC) as for equalizer for the next iterations. The output of this IC, $\mathbf{y}^{p(k)} = [y_1^{p(k)} \cdots y_n^{p(k)} \cdots y_N^{p(k)}]$ can be expressed as:

$$\mathbf{y}^{p\,(k)} = \mathbf{F}^{(k)}\mathbf{r} - \sum_{j=1}^{k-1} \mathbf{B}_j^{(k)}\mathbf{d}^{p\,(j)} - \sum_{j=k}^{K} \mathbf{B}_j^{(k)}\mathbf{d}^{p-1\,(j)}.$$

This leads to the structure shown in Figure 5. It consists of a "feedforward" matrix $\mathbf{F}^{(k)}$ and a bank of "feedback" matrices $(\mathbf{B}_1^{(k)} \cdots \mathbf{B}_j^{(k)} \cdots \mathbf{B}_K^{(k)})$. These matrices are chosen so to minimize the symbol criterion: $E[|y_n^{(k)} - d_n^{(k)}\|^2]$, under the assumption that

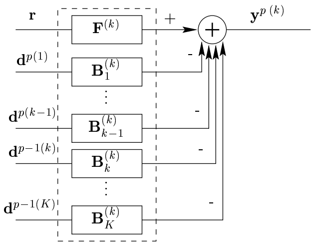

Figure 5: Proposed multiuser IC

all fed back inputs are correct. Because of this assumption, whatever the scheme is, Full or not, the expressions are the same ([11] and equation (1)):

$$\begin{array}{rcl} \mathbf{F}^{(k)} & = & \mathbf{D}^{(k)}\mathbf{H}^{(k)\dagger}, \\ \mathbf{B}_j^{(k)} & = & \mathbf{D}^{(k)}\left(\mathbf{H}^{(k)\dagger}\mathbf{H}^{(j)} - diag(\mathbf{H}^{(k)\dagger}\mathbf{H}^{(j)})\right), \\ \mathbf{D}^{(k)} & = & Diag(\alpha_1^{(k)} \cdots \alpha_n^{(k)} \cdots \alpha_N^{(k)}), \\ \alpha_n^{(k)} & = & \dfrac{\sigma_d^2}{\sigma_d^2\|\mathbf{h}_n^{(k)}\|^2 + \sigma_w^2}, \end{array} \tag{1}$$

where $Diag(x_1 \cdots x_K)$ denotes a diagonal matrix whose diagonal elements are $x_1 \cdots x_K$, whereas $diag(\mathbf{A})$ is a diagonal matrix whose diagonal elements are the diagonal elements of the matrix $\mathbf{A}$.

Note that the CDMA-IC processes the entire block of data at once. Therefore, there is no problem of causality (as in regular DFE) and the set of matrices $\{\mathbf{B}_j^{(k)}\}_j$ can remove the interference caused by the previous but also by the future symbols of any user. This allows $\mathbf{F}^{(k)}$ to be the matched matrix to $\mathbf{H}^{(k)}$ which maximizes the output signal-to-noise ratio (SNR) and which leads to better performance than the DFE.

7. Simulation settings & Performance

In the following simulations, we use for each user a rate $R_c = \frac{1}{2}$ systematic convolutional code with 64 states and generator polynomials (133,171) [8]. The interleaver is a (64,64) matrix read non-uniformly. The modulation is a BPSK. The spreading code sequences are orthogonal variable spreading factor codes (OVSF), as in UMTS. Spreading is followed by scrambling. The number of users is $K = 4$ and the spreading factor is $Q = 16$. The channel chip-length W is set to 14. Moreover the users' transmissions are asynchronous. All users have equal received powers.

The behavior of the Full Turbo-CDMA receiver is similar to that of the Turbo-equalizer (single user case) we have studied in [10]. Indeed, there is a trigger point followed by a breakdown effect, i.e. performance improves as the iteration step grows.

In the following, we compare the performance of the Turbo-CDMA receiver [11] (see Figure 6) and of the Full Turbo-CDMA receiver derived above (see Figure 7). In both cases, the users are demodulated

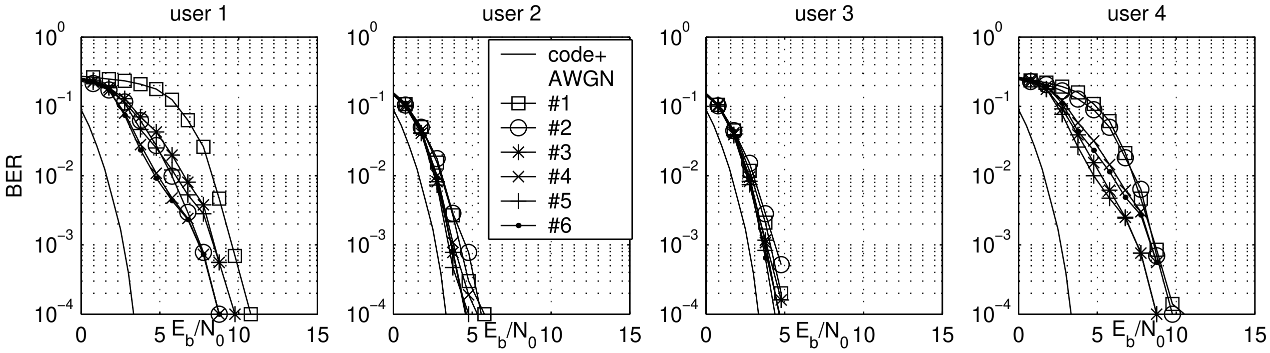

Figure 6: Turbo-CDMA performance, with multiuser **user block** MMSE DFE and best order of demodulation (2/3/4/1).

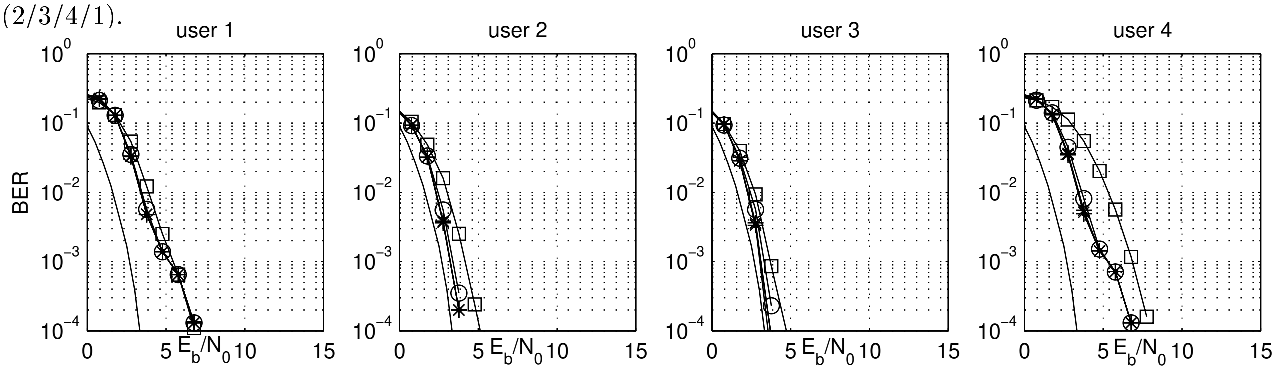

Figure 7: Full Turbo-CDMA performance, with multiuser **user block** MMSE DFE and best order of demodulation (2/3/4/1). See legend in Figure 6.

in the order of decreasing levels defined in [12]. Since $k-1$ users have been decoded once more in the Full receiver than in the regular one, fed back are more reliable, which should leads to better performance for the Full structure.

In Figure 7, the performance of users 2 (as 3) for the Full structure is only slightly better than the regular one, since it was already (see Figure 6) very close to the performance of a one user transmission over an AWGN. This bound is called code+AWGN in the Figures. In contrast, the influence of the Full scheme is so important that for user 1, the 1st iteration of the Full receiver is already better than the 6th iteration of the regular receiver. In this case, the Full Turbo-CDMA, with users demodulated in the order of decreasing level [12], leads to much better asymptotic performance than the regular structure.

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