Low-Cost Turbo Receiver for Coded Asynchronous DS-CDMA in Frequency Selective Channels

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This paper was presented in part at the 2nd IEEE International Symposium on Turbo-codes-2000 and at the IEEE Sixth International Symposium on Spread Spectrum Techniques & Applications (ISSSTA'00).

Abstract

When channel coding is used on a CDMA transmission, the most appropriate approach for reception is joint Multiuser detection and decoding. This joint problem can be successfully processed by the Turbo-techniques. Therefore, we consider a low-complexity iterative receiver for the CDMA uplink transmission, called Turbo-CDMA, and we propose to further improve it. First, we derive a new DFE to be used at the first iteration. Then, we propose to fully use the decoded information in a new structure called Full Turbo-CDMA. Finally, we derive a new demodulation order that is well suited to the Turbo-CDMA receiver. This improved receiver, called Full Turbo-CDMA, aims to eliminate MUI and ISI almost completely in the presence of frequency selective channels. Improved performance is illustrated by simulations.

Keywords

Turbo equalization, multiuser receiver, Code division multiaccess (CDMA), interference suppression.

I. Introduction

Direct Sequence Code division multiaccess (DS-CDMA) features prominently today in cellular and personal communications today and is already involved in several standards (IS-95, IMT-2000 and UMTS). In orthogonal DS-CDMA, all users access the same frequency bandwidth by modulating orthogonal spreading sequences. In the uplink transmission of mobile radio channels, synchronization between each mobile station and the base station cannot be maintained. Hence, orthogonality is lost and multiuser interference (MUI) occurs. Moreover, frequency selective channels due mainly to multipath propagation yield intersymbol interference (ISI) and also affect the orthogonal property. In this case the most appropriate approach for the base station is multiuser detection, where the receiver must acquire and demodulate the signals of all users, as opposed to demodulating them independently (single user receiver).

Much of the literature on multiuser CDMA receiver focuses on uncoded systems [1]. However, because most wireless systems use channel coding, the interest in the problem of Multiuser Detection for coded data has increased. In this context, [2] derived the optimal receiver for the transmission of convolutionally coded data over an asynchronous non-dispersive additive white gaussian noise (AWGN) channel, without interleaving. This "Super-trellis" includes both the code and asynchronous CDMA channel and minimizes the error-rate of the information-bit sequences of the whole users. This receiver is clearly infeasible due to its complexity that grows exponentially with the number of users and the code constraint length. As such, [3] designed sub-optimal solutions and used their equalizer soft output to decode the first user and suppress its contribution in a multistage DFE. Unfortunately, their approach cannot suppress future ISI by feedback.

The "Turbo" techniques, first discussed in [4] for iterative decoding of parallel concatenated codes, have recently received increased attention as a trade-off algorithm: this technique maintains both good enough performance and low enough complexity. The iterated approach of Turbo-techniques should allow to further suppress ISI and MUI as soon as the turbo process is triggered. It can also be successfully applied to our problem. In [5], the Turbo-principle solves the joint equalization (ISI suppression for one source) and decoding problem. This receiver is called "Turbo-detector" because it is based on an iterated Maximum *a posteriori* (MAP) detector for equalization and decoding. This detector can also be applied to the CDMA case: in [6] for synchronous channels, in [7] for asynchronous flat fading channels, in [8] for multipath channels.

Unfortunately the complexity of the detector grows exponentially with the number of users and is all the more complex when multipath occurs, which forbids its practical use. Several suboptimal solutions have been proposed for the equalizer, among them the zero forcing and minimum mean square error receivers [9]. In the Turbo context, [10] proposed a suboptimal solution based on filtering and well suited to the coded context. It is based on an interference canceler (IC) that generalizes the single user Turbo-equalizer [11] to the CDMA case. The IC derivation is based on the multiplication by matrices (no inversion) which leads to a very low complexity algorithm. Then, [8] proceeded to improve this structure by taking into account the statistics of the fed back data. However, this improvement grows the complexity. On the contrary, we propose in this study to improve the structure of [10] without increasing the complexity. First, we propose a new structure called Full-Turbo CDMA receiver, since it fully uses the Turbo process. Then, we propose to demodulate the users in an ordering determined by a new criterion that takes into account both ISI and MUI, to trigger the Turbo-CDMA earlier in terms of SNR.

Organization and Notations

In the following sections, we present the Turbo-CDMA receiver. In Section II we introduce the CDMA model and provide the precise definition of the Multiuser Detection problem for coded data. We consider one solution to this problem, i.e. the Turbo-CDMA receiver introduced in Section III. The proposed improvements are explained in Section IV. After presenting the new Full structure, we derive a new DFE and propose a new demodulation order. We close by analyzing the simulation results in section V.

Throughout this paper, scalars are in lower case whereas vectors are also in lower case but in boldface and matrices are in upper case and in boldface. The symbols $(.)^T$, $(.)^\dagger$ and $\|.\|$ stand respectively for transposition, trans-conjugation and L_2 norm. Variables have subscripts that refer to the time increment, superscripts that refer to the iteration and superscripts in brackets to the user.

II. Problem setting

A. System Model

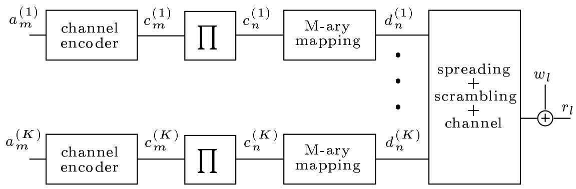

Fig. 1. Received signal model ($\prod$: interleaver)

We model here the uplink of a coded DS-CDMA communication system where K asynchronous users transmit information over K different frequency selective channels (see Figure 1). It will be assumed that each user employs binary convolutional coding on its link with code rate $R_c^{(k)}$. Without loss of generality, we assume that all users employ the same convolutional code. The coded bits $\{c_m^{(k)}\}$ of the kth user are interleaved to split the error bursts that may occur at the equalizer output [12, page 468] and also to decorrelate the noise at the decoder input [12, page 484]. All users employ the same interleaver. The bit to symbol mapper yields symbols $\{d_n^{(k)}\}$ at the rate $1/T_s$ that are taken from a complex alphabet of size M.

Then the DS-CDMA system is involved where K users transmit over frequency selective channels with additive Gaussian noise. The transmitted signal due to the kth user can be written as:

$$\sum_n d_n^{(k)} s^{(k)}(t - nT_s), \text{ where } s^{(k)}(t) = \sum_{q=0}^{Q-1} s_q^{(k)} \psi(t - qT_c),$$

where $1/T_s$ denotes the symbol rate, $1/T_c$ the chip rate, and Q the spreading factor. $\{d_n^{(k)}\}$ and $\{s_q^{(k)}\}$ denote, respectively, the symbol stream and the spreading sequence. ψ is a normalized chip waveform of duration T_c. The baseband received signal contains the contribution of all users in:

$$\begin{aligned} r(t) &= \sum_{k=1}^{K} \sum_n \tilde{g}^{(k)} \star \left(d_n^{(k)} s^{(k)}(t - nT_s) \right) + w(t) \\ &= \sum_{k=1}^{K} \sum_n \sum_{q=0}^{Q-1} d_n^{(k)} s_q^{(k)} \left(\tilde{g}^{(k)} \star \psi \right)(t - qT_c - nT_s) + w(t), \end{aligned}$$

where the channel of the kth user is characterized by its impulse response, $g^{(k)}(t)$:

$$g^{(k)}(t) = \tilde{g}^{(k)} \star \psi(t),$$

that includes equipment filtering (i.e. transmitter power, chip pulse waveform, transmitter filter and its matched filter in the receiver) and propagation effects (multipath, time delay). Note that we get a convolutive mixture. Sampling this received signal at the chip rate $Q/T_s (= 1/T_c)$ leads to a chip-rate discrete-time model that can be written as:

$$r_l = r(lT_c) = \sum_{k=1}^{K} \sum_n \sum_{q=0}^{Q-1} d_n^{(k)} s_q^{(k)} \left(g^{(k)} \star \psi \right)((l - q)T_c - nT_s) + w_l, \tag{1}$$

which is a cyclostationary signal of period Q. Spreading and convolution by the channel are put together in the global channel function:

$$b_l^{(k)} = \sum_{q=0}^{Q-1} s_q^{(k)} g^{(k)}((l - q)T_c) = s^{(k)} \star g^{(k)}(lT_c). \tag{2}$$

It follows from (1) and (2) that $r_l = \sum_{k=1}^{K} \sum_n d_n^{(k)} b_{l-nQ}^{(k)} + w_l$.

Throughout the paper we employ the block transmission CDMA model similar to that described in [9]. The blocks of N symbols of each user are put in:

$$\mathbf{d} = [\mathbf{d}^{(1)T}, \mathbf{d}^{(2)T} \cdots \mathbf{d}^{(k)T} \cdots \mathbf{d}^{(K)T}]^T \text{ where } \mathbf{d}^{(k)} = [d_1^{(k)}, d_2^{(k)} \cdots d_n^{(k)}]^T.$$

The global channel impulse response $b_l^{(k)}$ is put in the vector $\mathbf{b}^{(k)}$ of size $(Q+W-1)$, where W is the chip-length of the discrete global channels:

$$\mathbf{b}^{(k)} = [b_1^{(k)}, b_2^{(k)} \cdots b_l^{(k)} \cdots b_{Q+W-1}^{(k)}]^T.$$

The transmission of the data sequence $\mathbf{d}^{(k)}$ over the CDMA channel can be expressed by the received sequence $\mathbf{r}$:

$$\mathbf{r} = [r_1 \cdots r_{QN+W-1}]^T = \sum_{k=1}^{K} \mathbf{H}^{(k)} \mathbf{d}^{(k)} + \mathbf{w}, \tag{3}$$

where the matrix $\mathbf{H}^{(k)}$ is of size $(QN+W-1, N)$. Equation (3) can be summarized as

$$\mathbf{r} = {\scriptstyle QN+W-1} \updownarrow \underbrace{\left[\begin{array}{c|c|c} \overset{\xleftrightarrow{N}}{\mathbf{H}^{(1)}} & \cdots & \overset{\xleftrightarrow{N}}{\mathbf{H}^{(K)}} \end{array} \right]}_{\mathbf{H}} \underbrace{\begin{bmatrix} \mathbf{d}^{(1)} \\ \hline \vdots \\ \hline \mathbf{d}^{(K)} \end{bmatrix} \updownarrow {\scriptstyle KN}}_{\mathbf{d}} + \mathbf{w}, \tag{4}$$

$$\text{where } \mathbf{H}^{(k)} = [\mathbf{h}_1^{(k)} \cdots \mathbf{h}_N^{(k)}] \ ,$$

$$\mathbf{h}_n^{(k)} = [\ 0 \cdots 0 \quad \underset{\underset{(n-1)Q+1}{\uparrow}}{b_1^{(k)}} \quad \cdots \quad \underset{\underset{nQ+W}{\uparrow}}{b_{Q+W-1}^{(k)}} \quad 0 \cdots 0\ ]^T. \tag{5}$$

Provided that $N > K$ in the previous model, we get more observations than transmitted symbols, causing the system to have sufficient statistics [13].

We assume that the channel adds zero-mean complex gaussian noise $\{w_l\}_{(l=1 \cdots QN+W-1)}$ to the covariance matrix $\mathbf{R}_w$. For the sake of simplicity, the propagation channels are assumed to be time invariant during the transmission of a whole block and we also assume that the interferences due to symbols before and after the data block $\mathbf{d}$ can be completely canceled. This is possible when those interfering symbols are known by the receiver (training sequence). Under these conditions all columns of the matrix $\mathbf{H}^{(k)}$ differ only by a time shift. Each column vector $\mathbf{h}_n^{(k)}$ of the kth user consists of the same non-zero coefficients $b_l^{(k)}$ (time invariant) and because of the complete elimination of the known symbols, these symbols can be set to zero so that the first and the last vectors are not truncated. However (4) is more general and may include differences in the columns of $\mathbf{H}$, in particular scrambling, as in the UMTS standard.

B. Coded multiuser detection problem and MAP solution

The Maximum *a posteriori* (MAP) solution to the problem of multiuser detection, when information bits are coded, can be stated as follows: given the received sequence $\mathbf{r}$ defined in Equation (3), find the more likely information data streams $\{a_m^{(k)}\}$, i.e. the data streams $\{\hat{a}_m^{(k)}\}$ of each user that maximizes the *a posteriori* probability:

$$\mathcal{P}(\{a_m^{(k)}\}_{m,k} \mid \mathbf{r}) = \frac{\mathcal{P}(\{a_m^{(k)}\}_{m,k})}{\mathcal{P}(\mathbf{r})}\mathcal{P}(\mathbf{r} \mid \{a_m^{(k)}\}_{m,k}).$$

When multiuser detection is made (without coding), a popular approach is to assume that the symbols $d_n^{(k)}$ are iid. However, this fails to consider that the symbols are coded. Here, we only need to assume that the information bits $a_m^{(k)}$ are iid and equiprobable, which holds true in general. Under this assumption on the $a_m^{(k)}$, and because $\mathcal{P}(\mathbf{r})$ is independent of which data stream is transmitted, the decision rule based on the MAP criterion is equivalent to the criterion that maximizes the likelihood:

$$\mathcal{P}(\mathbf{r} \mid \{a_m^{(k)}\}_{m,k}). \tag{6}$$

[2] derives the optimal receiver for criterion (6) on a non-dispersive AWGN channel and without interleaving. Yet the complexity of this algorithm is still high and grows exponentially with the number of users and the constraint length of the code. Because this method is not feasible when multipath occurs and when interleaving is used, we propose in the sequel suboptimal low complexity methods that use the "Turbo" principle as explained in Section III.

C. Simulation settings

Since simulations illustrate the different steps of the paper, we next introduce the simulation settings. As usual a log-log plot of the Bit error rate (BER) versus the signal to noise ratio E_b/N_0 will show the performance of the Turbo-CDMA receiver. For the purpose of presenting and interpreting these results, we shall define a generalized E_b/N_0 that holds true for all channels and all channel codes and for deterministic spreading sequences. We show in the following that spreading with a deterministic sequence nay incur an energy loss and we take into account this loss in the definition of the E_b/N_0 computed at the receiver. Therefore, we figure out the Signal to noise ratio (SNR) of the CDMA transmission as a function of the SNR of a one-user equivalent transmission.

In the case of the CDMA transmission, the SNR can be written as:

$$SNR_{\text{CDMA}} = \frac{E_c}{N_0},$$

where E_c is the energy per chip and N_0 the power spectral density, so that the noise variance is:

$$\sigma_w^2 = \frac{N_0}{T_c}. \tag{7}$$

The one-user equivalent transmission can be defined as a transmission with SNR

$$SNR_{\text{equivalent}} = \frac{E_d}{N_0'},$$

where the symbol variance is:

$$\sigma_d^2 = \frac{E_d}{T_s}. \tag{8}$$

Both systems are equivalent iff E_c and N_0 are figured out after spreading the symbols data $d_n^{(k)}$. Since spreading does not affect the noise level:

$$N_0' = N_0.$$

Assuming independence of the symbols $d_n^{(k)}$, the spread and non-spread power are therefore related by:

$$\frac{E_c}{T_c} = \|s^{(k)} \star g^{(k)}(t)\|^2 \frac{E_d}{T_s} = \|\mathbf{h}_n^{(k)}\|^2 \frac{E_d}{T_s},$$

which follows from (2) and (5). So,

$$E_c = E_d \frac{\|\mathbf{h}_n^{(k)}\|^2}{Q}. \tag{9}$$

With a non-dispersive channel we get $\|\mathbf{h}_n^{(k)}\|^2 = Q$ and $E_c = E_d$. More generally, a determistic spreading sequence (like the OVSF sequences, we use here) leads to

$$\|\mathbf{h}_n^{(k)}\|^2 = \|s^{(k)} \star g^{(k)}(t)\|^2 \leq \|s^{(k)}\|^2 \|g^{(k)}\|^2 = Q\|g^{(k)}(t)\|^2,$$

so that for the normalized channel g, we get $E_c \leq E_d$. Therefore, spreading may incur an energy loss. Since we are interested in the efficiency of the equalizer to cancel interference, we may take into account this loss. Consequently, the simulation over the CDMA channel will be performed with:

$$SNR_{\text{CDMA}} = \frac{E_c}{N_0} = \frac{\|\mathbf{h}_n^{(k)}\|^2}{Q}\frac{E_d}{N_0} = \frac{\|\mathbf{h}_n^{(k)}\|^2\sigma_d^2}{\sigma_w^2}, \tag{10}$$

whereas the performance will be plotted versus the SNR of the equivalent one-user transmission over the AWGN channel:

$$SNR_{\text{equivalent}} = \frac{E_d}{N_0}. \tag{11}$$

Moreover, we may take also into account coding and plot the performance of the Turbo-CDMA receiver versus the useful SNR (i.e. SNR of the information bits). Let us denote E_b the energy per information bits. Under the assumption that the symbols $d_n^{(k)}$ are taken from an alphabet of size M and that the rate of the convolutional code is $R_c^{(k)}$, the energies are related by $E_d = \log_2(M)R_c^{(k)}E_b$ (see [12, pages 396, 487]). It yields:

$$SNR_{\text{useful}} = \frac{E_b}{N_0} = SNR_{\text{equivalent}}\frac{1}{\log_2(M)R_c^{(k)}}. \tag{12}$$

In the following simulations, decoder performance is plotted versus $\frac{E_b}{N_0}$ as defined in (12), whereas equalizer performance is plotted versus $\frac{E_d}{N_0}$ (11). We use for each user a rate $R_c^{(k)} = \frac{1}{2}$ systematic convolutional code with 64 states and generator polynomials (133,171) [12]. The interleaver is the (64,64) non-uniform matrix [11]. The modulation is a BPSK. The spreading code sequences are orthogonal variable spreading factor codes (OVSF), as in UMTS. Spreading is followed by scrambling over 16 symbols. We choose the same scrambling sequence for all users. The number of users is $K = 4$ and the spreading factor is $Q = 16$. The channel chip-length W is set to 14, so that ISI occurs over 2 symbols. Moreover the users' transmissions are asynchronous (see Table 1). This leads to a crosscorrelation of 0.71 between users and to a crosscorrelation of 0.19 between the symbols of one user. Simulations were performed with 10^5 information bits.

III. Turbo-CDMA

The so-called "Turbo" principle first discussed in [4] for iterative decoding of parallel concatenated codes proposes an interesting decoder maintaining both good enough performance and low enough complexity. According to Berrou's definition of a Turbo-code

Table 1. Channel impulse response.

user	impulse response at chip rate
1	[0.084 0.225 0.407 0.603 0.754 0.815 0.754 0.603 0.407 0.225 0.084 0 0 0]
2	[0 0.084 0.225 0.407 0.603 0.754 0.815 0.754 0.603 0.407 0.225 0.084 0 0]
3	[0 0 0.084 0.225 0.407 0.603 0.754 0.815 0.754 0.603 0.407 0.225 0.084 0]
4	[0 0 0 0.084 0.225 0.407 0.603 0.754 0.815 0.754 0.603 0.407 0.225 0.084]

(see [14, footnote 1]) we define the Turbo-principle as an iterative technique, where data are soft processed. Moreover, it can be applied to many joint problems as proposed in [15] such as the joint multiuser detection and decoding problem.

Take as an example the Turbo-detector. [6] (as [15]) proposed to use the Turbo-detector in a chip- and symbol-synchronous CDMA context and designed an algorithm devoted to the asynchronous CDMA channel [7]. Owing to the complexity of the detector, both papers proposed a method to reduce it using the M-algorithm.

The purpose of our paper is to consider multiuser asynchronous transmissions with frequency selective channels and to constrain ourselves to the class of low-complexity Turbo algorithms. Because the complexity of the detectors used in Turbo-detection is still high, we propose in the sequel to use the IC proposed in [10]. It can be seen as a generalization of the single user Turbo-equalizer [11] to the multiuser case.

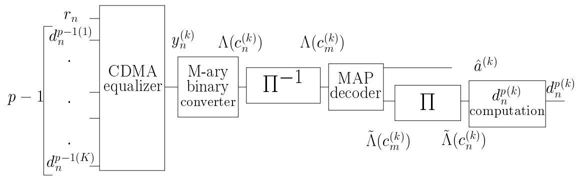

Fig. 2. Turbo-CDMA: receiver for user k at iteration p.

($\prod$: interleaver, $\prod^{-1}$: deinterleaver)

The Turbo-CDMA receiver is a modular pipelined structure that consists of P identical iterations. Each iteration contains K parallel receivers. Figure 2 shows user k receiver at iteration p, where k refer to the user of interest. Each module of this Turbo-CDMA receiver consists of (see Figure 2) the concatenation of a Soft Input/Soft Output CDMA equalizer, a soft de-mapper (symbol to bit), a deinterleaver, a Soft Input/Soft Output decoder, an interleaver and a soft re-mapper (bit to symbol). The output of iteration p is used together with the observation $\mathbf{r}$ to feed the next iteration, and so on. Since we consider here the uplink transmission, the receiver is the base station that has to demodulate every user. Thus, the multiuser approach, that should lead to better performance than the single user one, is developed.

The soft de-mapper figures out a soft information about the equalizer output in the form of a log-likelihood ratio. Under the assumptions that there is no interference left at the output of the equalizer (which corresponds to the Gaussian approximation in [8]) $y_n^{(k)} = \alpha_n^{(k)} d_n^{(k)} + \xi_n^{(k)}$, that the modulation is binary and that the noise $\xi_n^{(k)}$ is independent of $d_n^{(k)}$, the log-likelihood ratio can be rewritten as:

$$\Lambda(c_n^{(k)}) = \log \frac{\mathcal{P}(y^{(k)} \mid c_n^{(k)} = 1)}{\mathcal{P}(y^{(k)} \mid c_n^{(k)} = 0)} = \frac{4}{2\sigma_\xi^2} \mathcal{R}e\{y_n^{(k)} \alpha_n^{(k)*}\}.$$

For each user, the MAP decoder [16] gives estimates of the information bits $\hat{a}_m^{(k)}$ and computes soft information about the coded bits in the form of the log-ratio of the APP:

$$\tilde{\Lambda}(c_m^{(k)}) = \log \frac{\mathcal{P}(c_m^{(k)} = 1 \mid \mathbf{y}^{(k)})}{\mathcal{P}(c_m^{(k)} = 0 \mid \mathbf{y}^{(k)})}.$$

Due to the decoder, the bits have been decoded and $\tilde{\Lambda}(c_m^{(k)})$ is more reliable than $\Lambda(c_n^{(k)})$. After deinterleaving, an estimate $d_n^{p\,(k)}$ of the conditional mean value of the symbols $d_n^{(k)}$ is calculated as in [11]. In the case of a binary modulation (BPSK):

$$d_n^{p\,(k)} = \mathcal{P}(c_n^{(k)} = 1 \mid \mathbf{y}^{(k)}) - \mathcal{P}(c_n^{(k)} = 0 \mid \mathbf{y}^{(k)}).$$

[17] analyzed this Turbo-CDMA receiver, where the equalizer was made of filters, so that its per-bit complexity increased only linearly. They showed, that for channels with short dispersion, the interference can be almost completely eliminated, even for an over-loaded system ($K > N$). When severe distorsion occurs due to the dispersive channel, simulations show that all interference may not be cancelled. Thus, in the following we attempt to further improve the structure. One way to improve this scheme is the work by [8]. The multiuser IC they proposed does not assume that the fed back symbols are correct. So, it takes into account their statistics and yields better performance. However,

the method needs one matrix inversion per symbol and per user estimation. Even the recursive procedure for computing this matrix inversion increases the complexity. On the contrary, we propose in the next section to improve the Turbo-CDMA receiver without increasing the complexity.

IV. Full Turbo-CDMA

Note that the Turbo-CDMA scheme in Figure 2 does not use all of the decoded information available for user k at iteration step p. Here, we propose to fully use the decoded information and refer to it as the Full Turbo-CDMA scheme. In the original scheme (Fig. 2), the equalizer for user k at iteration p is fed by the output of iteration $p-1$. When user k is processed, the previous users (1 to $k-1$) have already been equalized and they can also be decoded. Therefore for these users, the output at iteration p is already available. In the Full scheme, we propose to use these outputs at iteration p as soon as they are available rather than still using the output at iteration $p-1$. In other words, we

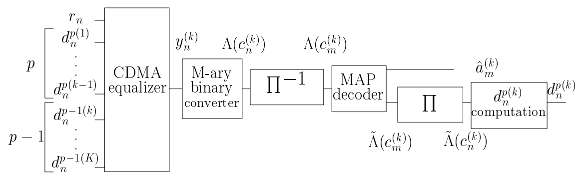

Fig. 3. Module p of the Full Turbo for user (k).

propose in the Full structure to feed the equalizer for user k at iteration p with the output of iteration p for the already decoded users (i.e. for users 1 to $k-1$, rather than iteration $p-1$) and with the output of iteration $p-1$ for the others as in Figure 3. Note that since the data of $k-1$ inputs have been decoded once more than in the previous scheme, they are more reliable and so is the output of the equalizer.

Since the equalizer of the kth user already processes decoded data of the previous users $(1 \cdots k-1)$, the structure is said to be successive, i.e. the users have to be processed (equalizer + decoder) one after each other. The successive scheme is more reliable than the parallel one and it does not increase the complexity but it requires an extra delay of time. Next, we discuss in more detail the equalizer used in this new structure. First we present the equalizer of the first iteration, a DFE, and then the equalizer used in subsequent iterations, an interference canceler (IC).

A. Full Minimum user-by-user block MSE DFE

Let us consider first the equalizer of the first iteration. Note that $d_n^{0\,(k)}$ is not accessible at the first iteration, we therefore need a conventional equalizer that just processes the output of the channel. We derive here a DFE rather than linear equalizers (as in [15], [17], [8]) for the first iteration. We have actually shown earlier [18] that the better the first equalizer (of the first iteration) is, the better is the asymptotic performance: the "Turbo" effect is triggered earlier in terms of E_b/N_0. Therefore, using a DFE rather than a linear equalizer helps to get better performance [19].

Let k refer to the user of interest. The DFE cancels first the contribution of the previous users (say 1 to $k-1$). Here we propose a Full DFE that uses the decoded information as soon as it is available. Therefore, the DFE is calculated for each user, given all the information obtained by the soft decoding of the previous users. For users k, the DFE input is the data estimated (as in [20]) from the decoder output of users $j = 1, ..., k-1$, $\mathbf{d}^{1(j)}$:

$$\mathbf{z}^{(k)} = \mathbf{r} - \sum_{j=1}^{k-1} \mathbf{H}^{(j)} \mathbf{d}^{1(j)}.$$

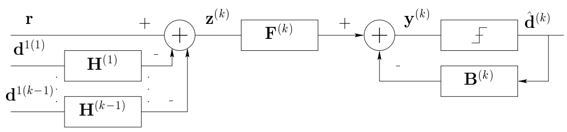

Fig. 4. Full DFE for user (k).

Then the DFE is aimed to minimize the user block Mean Square Error (MSE) $E[\|\mathbf{y}^{(k)} - \mathbf{d}^{(k)}\|^2]$, where $\mathbf{y}^{(k)}$ is the output of the DFE:

$$\mathbf{y}^{(k)} = \mathbf{F}^{(k)} \mathbf{z}^{(k)} - \mathbf{B}^{(k)} \hat{\mathbf{d}}^{(k)}.$$

If correct past decisions are considered relative to the already decoded users (1 to $k-1$), then:

$$\mathbf{z}^{(k)} = \mathbf{H}^{(k)} \mathbf{d}^{(k)} + \sum_{j=k+1}^{K} \mathbf{H}^{(j)} \mathbf{d}^{(j)} + \mathbf{w} = \mathbf{H}^{(k)} \mathbf{d}^{(k)} + \mathbf{n}.$$

The next users $j = k+1, ..., K$ are considered as "noise" $\mathbf{n}$, with covariance matrix

$$\mathbf{R}_n = E[\mathbf{n}\mathbf{n}^\dagger] = \mathbf{R}_w + \sigma_d^2 \sum_{j=k+1}^{K} \mathbf{H}^{(j)}\mathbf{H}^{(j)\dagger}.$$

The minimization of the **user block** MSE under the assumption that fed back symbols of the kth user are correct is derived in a similar way to [1, page 377] and leads to:

$$\begin{cases} \mathbf{F}^{(k)} & = & \mathbf{\Sigma}^{-2}\mathbf{L}^{-1\dagger}\mathbf{H}^{(k)\dagger}\mathbf{R}_n^{-1} \\ \mathbf{B}^{(k)} & = & \mathbf{L} - \mathcal{I} \end{cases}$$

where $\mathbf{H}^{(k)\dagger}\mathbf{R}_n^{-1}\mathbf{H}^{(k)} + \sigma_d^{-2}\mathcal{I} = (\mathbf{\Sigma}\mathbf{L})^\dagger\mathbf{\Sigma}\mathbf{L}$ is the Cholesky decomposition, $\mathbf{\Sigma}$ is diagonal and $\mathbf{L}$ strictly triangular.

In the case of a white gaussian noise, $\mathbf{R}_w = \sigma_w^2\mathcal{I}$. Afterwards we feed the decoder with the DFE soft output i.e. $y_n^{(k)}$.

Note that our DFE is slightly different from the MMSE DFE proposed in [9], which minimizes the block MSE $E[||\mathbf{y} - \mathbf{d}||^2]$. [9] used one Cholesky decomposition and one matrix inversion each of size (KN,KN), whereas our DFE uses K decompositions and K matrix inversions each of size (N,N). Therefore, our method is less complex ($O(KN^3)$ against $O(K^3N^3)$). Moreover, since our DFE achieves the minimum MSE for each user, it should yield better performance than the block DFE that performs minimum MSE in average for all users. Yet unless the SNR is very low, both methods achieve a BER very close to each other (as mentioned in [1, page 155] about Minimum probability of error method). Indeed simulation results have shown that all 2 DFE's behave similarly. This leads us to choose our method for DFE in the Turbo-CDMA simulations, due to its reduced complexity.

Figure 5 shows the performance of the DFE when data from the previous users are either correct (case Correct), decided at the DFE output (Decided) or decoded thanks to the Full structure (Full). In other words, Decided stands for the conventional DFE, Correct for the best achievable performance of the DFE (i.e. without error propagation) and Full for the improvement due to the Full structure.

Since the DFE is derived under the assumption of correct feedback, it fails to take into account this effect of decision error. This leads to the well known problem of error propagation. Hence, there is a gap between the best achievable performance of the DFE (correct)

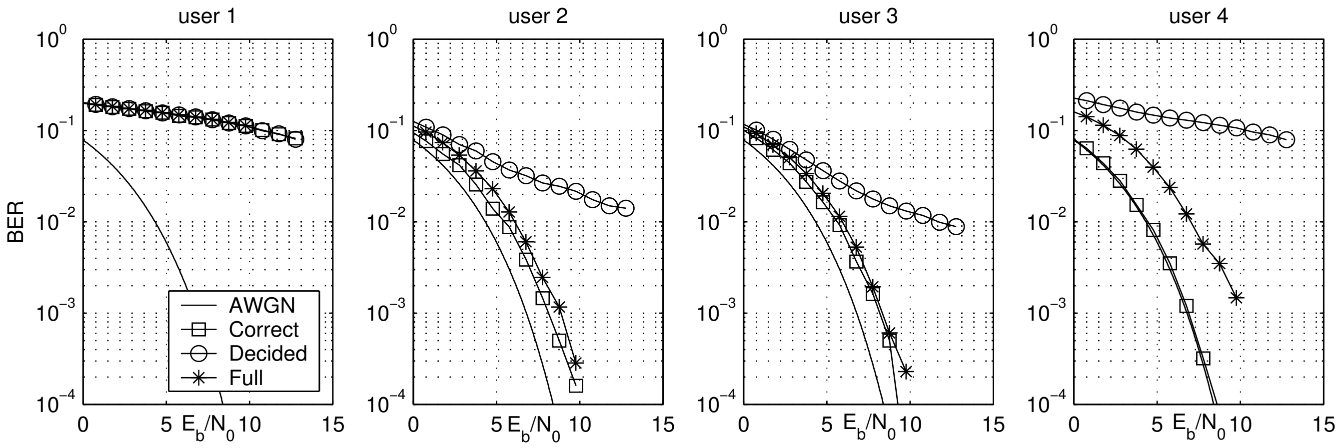

Fig. 5. Performance of the **user block** MMSE DFE, with order of demodulation (1/2/3/4).

and the conventional DFE (decided). Note that the Full structure is an improvement over the conventional DFE, since it helps to get closer to the best achievable performance. But, there is still a remaining gap.

B. Full IC

In the following, we use the IC we proposed in [21] and turn it into a "Full" IC: in order to use all of the decoded information available at iteration p for user k, we change the inputs of the IC by feeding it by $\mathbf{d}^{p\,(j)}, j = 1, ..., k-1$ (iteration p for the previous users) and $\mathbf{d}^{p-1\,(j)}$ for $j = k, ..., K$ (iteration $p-1$ for the current user and the future users). The matrices (13) remain unchanged. The IC output, $\mathbf{y}^{p(k)} = [y_1^{p(k)} \cdots y_n^{p(k)} \cdots y_N^{p(k)}]$ can be expressed as:

$$\mathbf{y}^{p\,(k)} = \mathbf{F}^{(k)}\mathbf{r} - \sum_{j=1}^{k-1} \mathbf{B}_j^{(k)} \mathbf{d}^{p\,(j)} - \sum_{j=k}^{K} \mathbf{B}_j^{(k)} \mathbf{d}^{p-1\,(j)}.$$

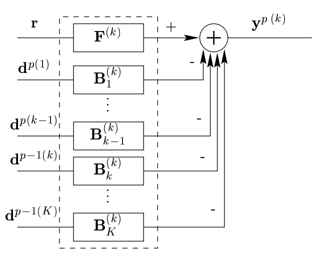

Fig. 6. Full multiuser IC

This leads to the structure shown in Figure 6. It consists of a "feedforward" matrix

$\mathbf{F}^{(k)}$ and a bank of "feedback" matrices $(\mathbf{B}_1^{(k)} \cdots \mathbf{B}_j^{(k)} \cdots \mathbf{B}_K^{(k)})$. These matrices are chosen so as to minimize the symbol criterion: $E[|y_n^{(k)} - d_n^{(k)}|^2]$, under the assumption that all fed back inputs are correct. Because of this assumption, whatever the scheme is, Full or not, the expressions are the same:

$$\begin{aligned}
\mathbf{F}^{(k)} &= \mathbf{D}^{(k)}\mathbf{H}^{(k)\dagger}\mathbf{R}_w^{-1}, \\
\mathbf{B}_j^{(k)} &= \mathbf{D}^{(k)}\left(\mathbf{H}^{(k)\dagger}\mathbf{R}_w^{-1}\mathbf{H}^{(j)} - \delta_{k-j}diag(\mathbf{H}^{(k)\dagger}\mathbf{R}_w^{-1}\mathbf{H}^{(j)})\right), \\
\mathbf{D}^{(k)} &= Diag(\alpha_1^{(k)} \cdots \alpha_n^{(k)} \cdots \alpha_N^{(k)}), \\
\alpha_n^{(k)} &= \frac{1}{\sigma_d^{-2} + \|\mathbf{h}_n^{(k)}\|^2_{\mathbf{R}_w}}, \\
\|\mathbf{h}_n^{(k)}\|^2_{\mathbf{R}_w} &= \mathbf{h}_n^{(k)\dagger}\mathbf{R}_w^{-1}\mathbf{h}_n^{(k)}
\end{aligned} \tag{13}$$

where $Diag(x_1 \cdots x_K)$ denotes a diagonal matrix whose diagonal elements are $x_1 \cdots x_K$, whereas $diag(\mathbf{A})$ is a diagonal matrix whose diagonal elements are the diagonal elements of the matrix $\mathbf{A}$.

Note that our IC is proved to be a scaled solution proposed in [10] (the scaling factor is the matrix $\mathbf{D}^{(k)}$, see (13)).

Note that the CDMA-IC processes the entire block of data at once. Therefore, there is no problem of causality (as in regular DFE) and the set of matrices $\{\mathbf{B}_j^{(k)}\}_j$ can remove the interference caused by the previous symbols but also by the future symbols of any user. This allows $\mathbf{F}^{(k)}$ to be the matched matrix to $\mathbf{H}^{(k)}$ which maximizes the output signal-to-noise ratio (SNR) and leads to better performance than the DFE.

C. Demodulation order

Note that a usual approach consists in demodulating the users in the order of decreasing received power. However, it fails to take into account all interferences and error propagation as shown in Fig. 5. Thus, we propose in the following a more appropriate demodulation order that is well suited to the Turbo-CDMA receiver in the frequency selective channel case.

We consider now the case of the IC. The feedforward matrix $\mathbf{F}^{(k)}$ output leads to a signal that contains the weighted contribution of the user of interest and of the other users. The weighted coefficient for user k is its received power. For all other users the coefficients are the ones of the feedback matrices. If the feedback symbols do not correspond to the correct ones, the greater the coefficients of the matrices $\mathbf{B}_j^{(k)}$ are, the more error propagation occurs.

In order to reduce error propagation, we should therefore first demodulate the user with maximum received power and minimum feedback coefficients. Consequently, we propose to demodulate the users in decreasing order of:

$$\sum_{n=1}^{N}\left(\mathbf{h}_n^{(k)\dagger}\mathbf{R}_w^{-1}\mathbf{h}_n^{(k)} - \sum_{m,m\neq n}|\mathbf{h}_n^{(k)\dagger}\mathbf{R}_w^{-1}\mathbf{h}_m^{(k)}| - \sum_{j,j\neq k}\sum_{m}|\mathbf{h}_n^{(k)\dagger}\mathbf{R}_w^{-1}\mathbf{h}_m^{(j)}|\right). \quad (14)$$

Note that this can be interpreted as the distance between the 2 nearest states at the output of the Rake receiver relative to user k, when all interferences are taken as destructive.

Take now the case of the DFE. A well suited demodulation order should take into account the coefficient of the feedback matrix as well. The problem lies in the fact that the derivation of this matrix requires the users to be ordered. Hence, the problem cannot be solved. We therefore propose to use the demodulation order (14) despite it being well suited to the IC and not to the DFE.

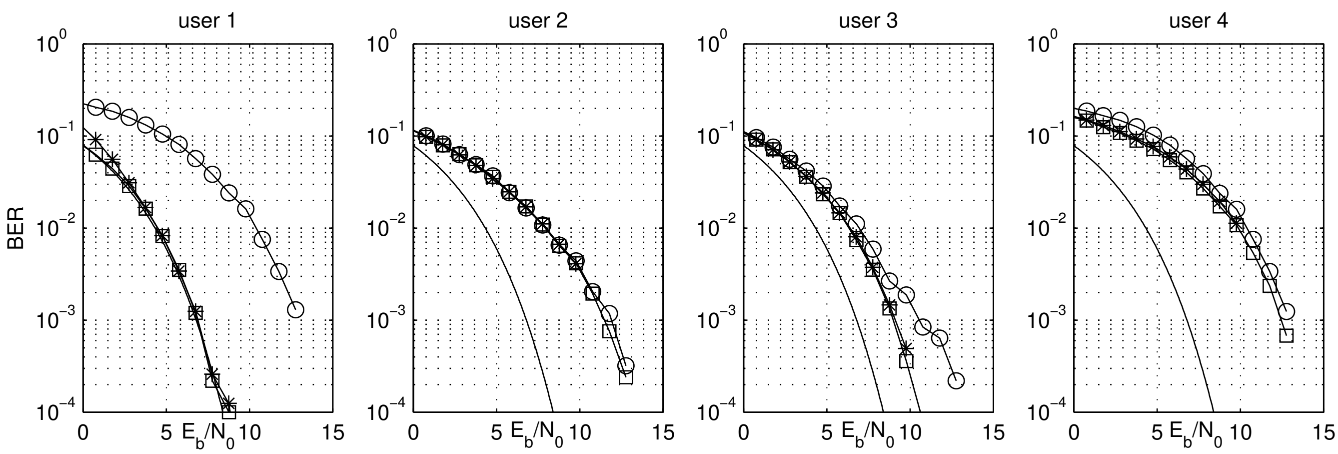

Fig. 7. Performance of the **user block** MMSE DFE, with best order of demodulation (2/3/4/1). See legend in Figure 5.

We now examine the influence of this new demodulation order on the DFE's performance (see Fig. 7). As before (see Fig. 5 and Subsection IV-A) Correct stands for correct feedback, Decided for feedback data, that have been decided at the DFE output and Full for feedback data, that have been decided at the decoder output. Note that, with this new demodulation order, error propagation is very few (little distance between correct and decided) and it can be almost completely eliminated with the Full structure. So, both improvements (Full structure + demodulation order) help to get almost the best achievable DFE's performance.

V. Performance of the Full Turbo-CDMA

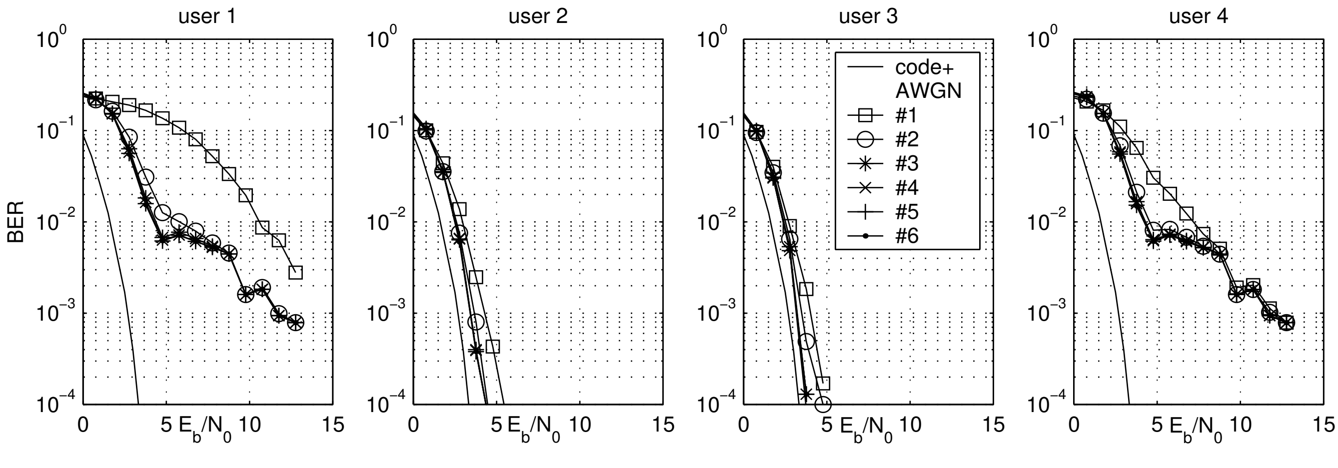

Fig. 8. Full Turbo-CDMA performance, with multiuser **user block** MMSE DFE and order of demodulation (1/2/3/4).

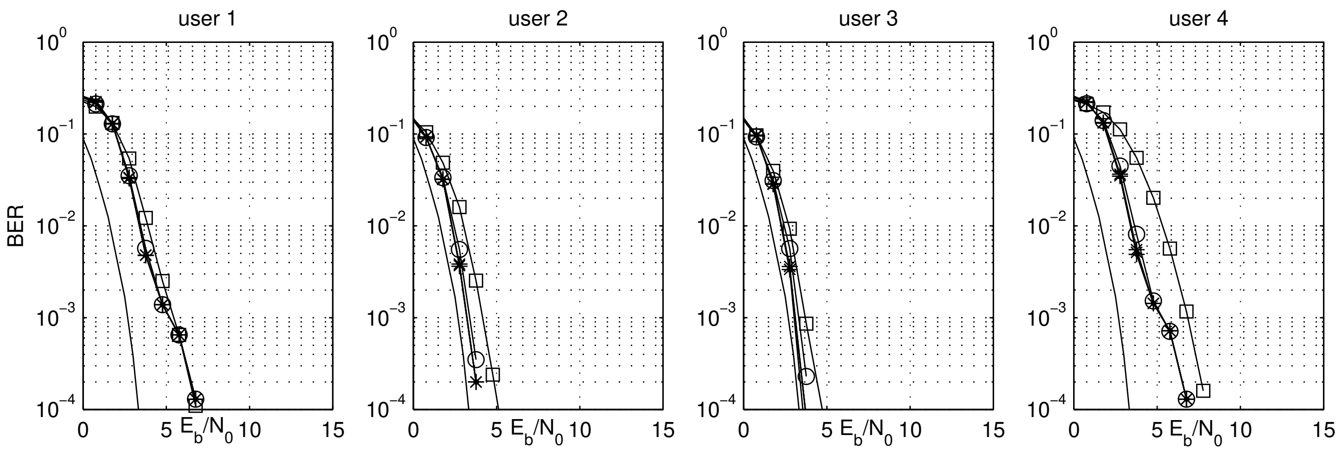

Fig. 9. Full Turbo-CDMA performance, with multiuser **user block** MMSE DFE and best order of demodulation (2/3/4/1). See legend in Figure 8.

The behavior of the Full Turbo-CDMA receiver is similar to that of the Turbo-equalizer (single user case) we have studied in [18]. Indeed, there is a trigger point followed by a breakdown effect, i.e. performance improves as the number of iteration steps grows.

Figure 8 shows the performance of this Full Turbo-CDMA receiver where the users were demodulated according to decreasing received power. Figure 9 shows the performance of the same Full Turbo-CDMA receiver where the users were demodulated in the order of decreasing levels defined in (14). The performance of users 2 (as 3) is only slightly better since it was already (see Figure 8) very close to the performance of a single user transmission over an AWGN channel. This bound is called code+AWGN in the Figures. In contrast, users 4 and 1 take advantage of this good demodulation order. Here the influence of the order of demodulation is more important. The amelioration is so important that for user 1, the 1st iteration of the well ordered Full receiver is already better than the 6th iteration of the arbitrary ordered one. In this case, the Full Turbo-CDMA leads, with users demodulated in the order of decreasing level (14), to much better asymptotic performance.

However, simulations show that even this Turbo receiver never achieves the bound derived with the multiuser IC fed by correct data. We analyzed earlier in [18] this gap between simulation performance and the AWGN+code bound. First, AWGN+code is not the most tight bound since it is not achievable when severe distortion occurs. Second, the gap is also due to the suboptimal Turbo process.

VI. Conclusion

The contributions of this paper are the derivation of two improvements for a low complexity multiuser Turbo-CDMA receiver when multipath propagation induce significant frequency selective channels. First, we proposed a new structure called Full Turbo-CDMA receiver. In this receiver, we turn the usual parallel structure into a serial one and so we fully use the decoded information. Furthermore, we provide a criterion to choose the demodulation order between users and show its accuracy in simulations.

Simulations show that the proposed Full Turbo-CDMA receiver with the user-block DFE approaches the performance of AWGN channel even for severe dispersive channels. The gain in number of iterations required to achieve a given performance is all the more important that the user is difficult to equalize.

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