Joint Equalization and Decoding: why choose the iterative solution?

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Abstract

This paper deals with turbo-equalization as a joint equalization and decoding algorithm. The performance analysis shows that there is a trigger point in this iterative process, followed by a breakdown effect. Actually after a given point (the trigger one), the BER decreases steeply as a function of the decoding step p. We compare the performance of the turbo-equalizer with that of the optimal joint receiver and show that it can match the bound over the optimal disjoint receiver.

1 Introduction

Since we consider high rate communication over band-limited channels, the transmitted signal is subject to intersymbol interference (ISI) due to multipath propagation inherent in wireless systems (PCS, WLANs, ...). In order to reduce the distortion, we use equalization to compensate for, or reduce, the ISI and channel coding to correct any remaining errors (see Figure 1).

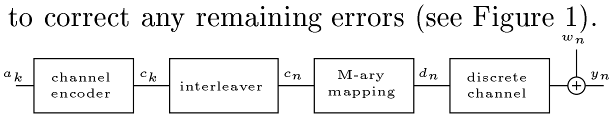

Figure 1: Communication system

where
- a_k : information bits,
- c_k : coded bits,
- c_n : coded, interleaved bits,
- d_n : coded, interleaved symbols,
- w_n : Additive White Gaussian Noise,
- y_n : received signal vector.

In the classical receiver scheme, the equalizer does not use the redundancy introduced by the channel coding: the equalizer and decoder are disjoint which is suboptimal. Moreover the channel decoder is fed by the decided equalizer outputs and hard decision-decoding is used. There is therefore a large loss in performance, because only a very small part of the information is used. Joint algorithms should outperform the classical ones because they use all of the received information. The optimal joint receiver is achieved through a MAP (Maximum a Posteriori) algorithm including {coder+interleaver+channel}. This receiver is clearly infeasible because of the large number of states in the trellis due to the presence of the deinterleaver between the equalizer and the decoder. The so called "turbo" principle first discussed in [2] for iterative decoding of parallel concatened codes proposes an interesting alternative: maintaining both good enough performance and low enough complexity. There are several possible methods for finding a joint receiver using the turbo-principle. The first one [3], that we will call "turbo-detection" in the following, is based on an iterated MAP detector for equalization and decoding. Because the complexity of this detector is still high, we will consider in the rest of the paper the so called "turbo-equalizer" [4], that is based on an interference canceler (IC) equalizer (see section 2).

Our goal is to study the best achievable performance in terms of BER for the turbo-equalizer and to compute some lower bounds on it.

2 Turbo-equalizer: principle

The turbo-equalizer is implemented in a modular pipelined structure with p identical iterations, where each module consists of (see Figure 2) the concatenation of a Soft Input/Soft Output equalizer, a deinterleaver, a Soft Input/Soft Output MAP decoder and an interleaver. The output of the p^{th} iteration is used together with the channel output to feed the next iteration, and so on.

Decoding is carried out by using the MAP algorithm [1]. The decoder computes information about the coded bits in the form of log-likelihood ratios: $\Lambda(c_k) = \frac{\mathcal{P}(c_k=1|r_k)}{\mathcal{P}(c_k=0|r_k)}$

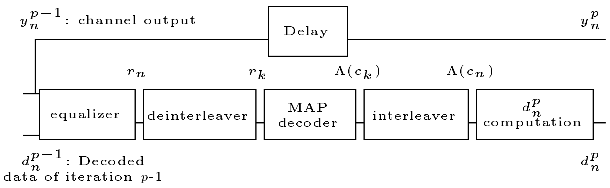

Figure 2: Module p of the turbo-equalizer

The decoder also gives estimates of the information bits $\hat{a}_k$ and the corresponding BER. After deinterleaving an estimate $\bar{d}_n^p$ of the mean value of the estimated symbols d_n is calculated [5]. In the case of binary modulation (BPSK):

$$\bar{d}_n^p = \mathcal{P}(c_k = 1 \mid r_k) - \mathcal{P}(c_k = 0 \mid r_k)$$

$\bar{d}_n^p$ feeds the equalizer of the p^{th} iteration.

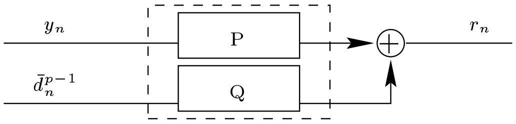

Figure 3: Equalizer in the module $p > 1$.

The equalizer used here is the IC. It consists of two filters and is fed by both the output of the equivalent discrete channel (y_n) and the output of the previous module ($\bar{d}_n^{p-1}$). It therefore takes into account the redundancy of the coded symbols transmitted over the time dispersive channel thanks to $\bar{d}_n^{p-1}$. Note that the equalizer of the very first iteration is a DFE that just processes the output of the channel because $\bar{d}_n^0$ is not accessible. Let us denote the transfer function of the discrete equivalent channel as $H(z)$. The minimization of the mean square error MSE $= E[\mid r_n - d_n \mid^2]$ over the coefficients of the filters P and Q under the constraint $q_0 = 0$, when Q is fed by the d_n, yields the optimal filters in the form [5]:

$$P(z) = \alpha H^*(z^{-1}),\ Q(z) = \alpha(H(z)H^*(z^{-1}) - 1)$$

- $H(z) = \sum_i h_i z^{-i}$, $H^*(z) = \sum_i h_i^* z^{-i}$,

where - $\alpha = \frac{\sigma_d^2}{\sigma_d^2 + \sigma_w^2} \approx 1$,

- σ_d^2 symbol power, σ_w^2 noise power.

P is the filter matched to the channel H and $Q + 1$ its auto-correlation. Q is used to remove the ISI from the present estimate caused by previous and future detected symbols. It results in *complete* elimination of ISI, provided that the previous and future decisions are correct. Therefore the following experience can be carried out (see Figure 4): the IC is now fed by both y_n and d_n, the correct coded and interleaved symbols. Because elimination of the ISI is complete, the performance after the IC and after the decoder does not depend on the channel and is similar to the performance over the AWGN channel. Let us expand the expression for the output of the IC: $r_n = \alpha d_n + \alpha h_{-n}^* * w_n$

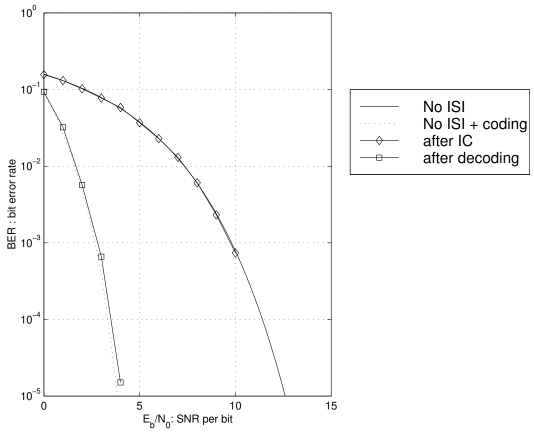

Figure 4: Performance of the IC fed by the correct symbols.

Because the coefficients h_i of the channel are normalized, the final signal to noise ratio (SNR) is unchanged. This explains the performance at the output of the IC. The only difference between the AWGN model and the output of the IC is that the noise is colored. But, thanks to the deinterleaver, the data feeding the decoder are whitened, which explains why the performance at the output of the decoder is the same as the performance of the AWGN coded channel(see Figure 4).

3 Simulation settings

In the following, simulation results were obtained by using:
- recursive systematic convolutional code with constraint length 7 and generators $[1, \frac{171}{133}]$,
- modulation: BPSK,
- MAP decoding [1],
- simulation over 10^6 information bits,
- channel : Proakis B and C [6, page 616],

Set 1:
- interleaver: 59×59 uniform matrix,
- equalizer: DFE in the 1^{st} iteration, then IC,

Set 2:
- interleaver: 64×64 non-uniform matrix,
- equalizer: MAP in the 1^{st} iteration, then IC.

We have shown earlier [7] that there is a trigger point followed by a breakdown effect i.e. the performance gets better as the iteration step grows if the probability of error of the first iteration is lower than about $2 \cdot 10^{-1}$ (trigger point

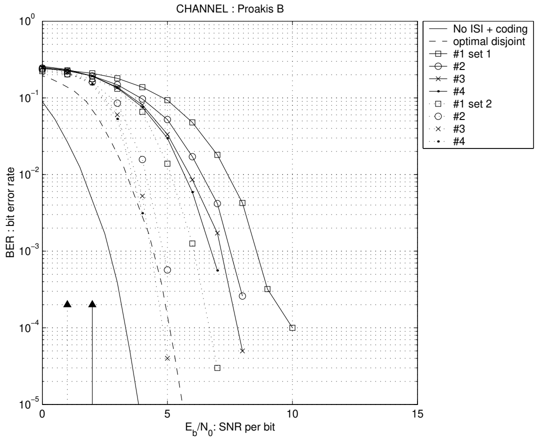

Figure 5: Performance of the turbo-equalizer after decoding over the channel Proakis B.

at 2 dB (set 1) or 1 dB (set 2) over the channel Proakis B and at 10 dB (set 1) or 6 dB (set 2) over the channel Proakis C, see Figures 5 and 6).

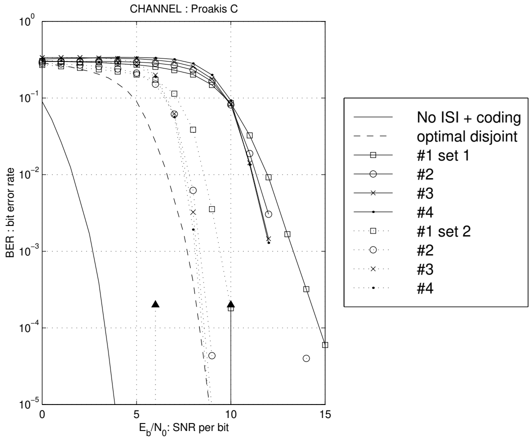

Figure 6: Performance of the turbo-equalizer after decoding over the channel Proakis C.

Next we will study different performance bounds and compare them to the performance of the turbo-equalizer.

4 Bounds for turbo-equalization

4.1 The AWGN coded bound

The AWGN coded bound is a lower bound on the probability of error for soft-decision decoding where the communication system is made of a coder/decoder over the AWGN channel. It is computed from the probability of error for binary (uncoded) modulation [6, page 258]:

$$P_{e,AWGN} = Q\left(\sqrt{\frac{2\mathcal{E}_b}{N_0}}\right), Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty} e^{-\frac{x^2}{2}}\,dx$$

Then we compute the probability of error for soft-decision decoding and a lower bound on it in the form [6, page 457]:

$$\begin{aligned} P_{e,AWGN\_coded} &= \sum_{d=d_{free}}^{\infty} a_d Q\left(\sqrt{\frac{2\mathcal{E}_b R_c d}{N_0}}\right) \\ &> \underbrace{Q\left(\sqrt{\frac{2\mathcal{E}_b R_c d_{free}}{N_0}}\right)}_{\text{AWGN coded bound}} \end{aligned}$$

where d_{free} denotes the free distance of the code, a_d denotes the number of sequences of distance d from the all-zero sequence, $a_d \geq 1$ and R_c is the code rate.

From the AWGN coded bound $P_{e,AWGN\_coded}$, it is apparent that the coding gain (in dB) achieved by a convolutional code over an uncoded binary modulation is:

$$\text{coding gain} = 10\,\log_{10}(R_c d_{free})$$

Therefore the AWGN coded bound is computed through the coding gain as (see Figure 7):

$$P_{e,AWGN\_coded} \geq \underbrace{P_{e,AWGN}(SNR + coding\ gain)}_{P_{e,AWGN\_coded\_bound}}$$

Note that the AWGN coded bound is a lower bound for any coded communication system and depends on the code characteristics only.

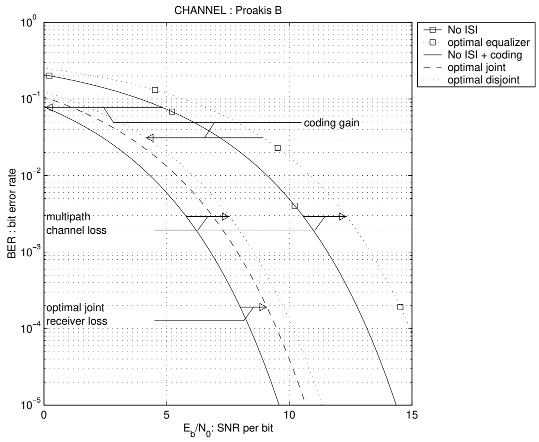

Figure 7: Computation of the bounds for channel Proakis B.

4.2 The optimal disjoint receiver

Next we compute a lower bound for the optimal disjoint receiver that is constituted of an optimal soft input/soft output equalizer, a deinterleaver and a soft input decoder. First we compute a lower bound on the optimal equalizer. The probability of error for optimal equalization can be computed in the same way as the probability of error for soft-decision decoding. That's why we calculate the minimum distance of the multipath channel denoted by d_{min}, i.e. the minimum euclidean distance between all sequences convoluted by the channel, where each sequence merges with a defined state at the beginning and at the end of the sequence. Note that d_{min}

for the channel is similar to d_{free} for the code. Hence the loss in SNR due to intersymbol interference is [6, page 598]:

$$\text{channel loss} = 10\ \log_{10}(\frac{d^2_{AWGN}}{d^2_{min}})$$

where d_{AWGN} is the minimum distance over the AWGN channel. Therefore

$$P_{e,optimal\_equalizer} \geq P_{e,AWGN}(SNR + channel\ loss)$$

After equalization, soft decision decoding is carried out and the performance of the optimal disjoint receiver is given by (see Figure 7):

$$P_{e,optimal\_disjoint} \geq \underbrace{P_{e,AWGN}(SNR + channel\ loss + coding\ gain)}_{optimal\ disjoint\ bound}$$

The optimal disjoint bound is thus a lower bound for the optimal disjoint receiver and depends on the code and the channel characteristics.

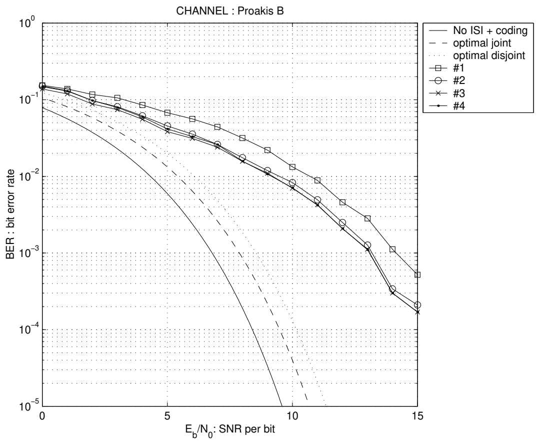

Figure 8: Performance of the turbo-equalizer (set 1) after decoding over Proakis B.

4.3 The optimal joint receiver

In this section, we compute a lower bound for the optimal joint receiver. Such a receiver computes the MAP probability of the transmitted signal and is based on a trellis including code+interleaver+channel. This receiver is clearly unfeasible because of the large number of states in the trellis due to the presence of the interleaver. However, it is possible to predict its performance. Then we compute the global minimum distance of the transmitter (denoted by d_{global_min}) as the minimum distance over all possible sequences coded, interleaved and convoluted by the channel. So, similar to the bound for the optimal equalizer (compared with the AWGN channel), we get the bound for the optimal joint receiver (compared with the AWGN coded channel) by using the optimal disjoint loss (see Figure 7):

$$\text{optimal joint loss} = 10\ \log_{10}(\frac{d^2_{AWGN\_coded}}{d^2_{min\_global}})$$

where d_{AWGN} is the minimum distance over the AWGN coded channel. And we obtain:

$$P_{e,optimal\_joint} \geq \underbrace{P_{e,AWGN}(SNR + joint\ loss + coding\ gain)}_{optimal\ joint\ bound}$$

Note that the optimal joint bound is a lower bound for the optimal joint receiver and depends on the code, the interleaver and the channel characteristics.

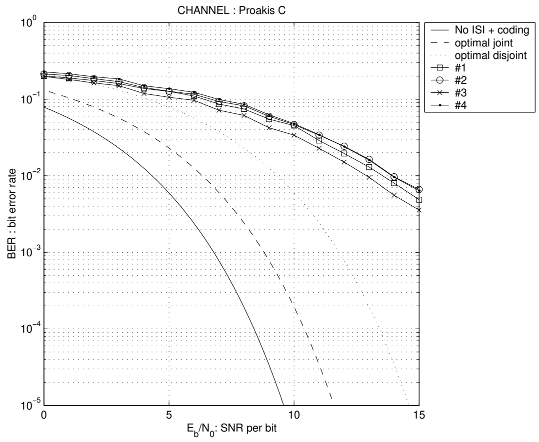

Figure 9: Performance of the turbo-equalizer (set 1) after decoding over Proakis C.

4.4 Simulation settings and results

For the simulation conditions specified in section 3, $2^{\frac{59\times 59}{2}}$ different sequences are generated and about $2^{59\times 59}$ distances are computed to calculate d_{min_global}. Thus the number of computation required is prohibitive. In order to be able to compute d_{min_global}, we choose a simple example and new simulation settings (in Figures 8, 9, 10, 11, 12) concerning the code and the interleaver:
- repeating code with constraint length 0 and generators [111],
- interleaver: 12-length non uniform matrix,

channel	joint loss	disjoint loss
Proakis B	-1.09 dB	-1.75 dB
Proakis C	-2.01 dB	-5.01 dB

Loss over the AWGN coded bound

Note that in our example the coding gain is 0 dB. Because we plot the performance of the equalizer versus the coded SNR (i.e. SNR of the coded bits) and that of the decoder versus the useful SNR (i.e. SNR of the information bits), the coding gain plotted in Figure 7 is 10 $\log_{10}(R_c)$ and worth about 4.77 dB.

4.5 Analysis of the turbo-equalizer

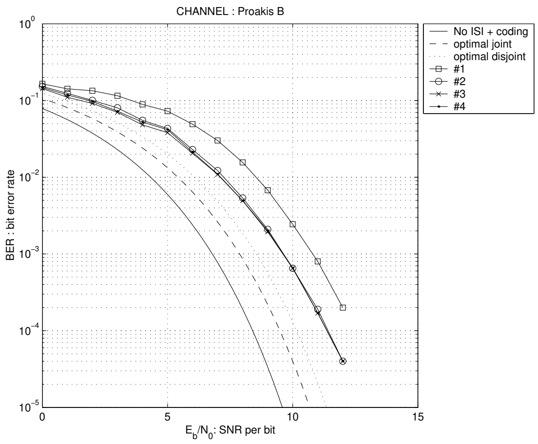

Figure 10: Performance of the turbo-equalizer (set 2) after decoding over Proakis B.

At first sight, the AWGN bound seems to be the best one for the turbo-equalizer, because the simulation made in section 2 shows that the turbo-equalizer fed by the correct symbols gives the same performance as the AWGN coded channel (see Figure 4). This bound does not depend on the channel. So, for an easy channel (i.e. channel with a minimum distance close to that of the AWGN channel) this bound seems to be the most suitable one. But it does not seem to be achievable for channels with severe distortion such as the Proakis B or C.

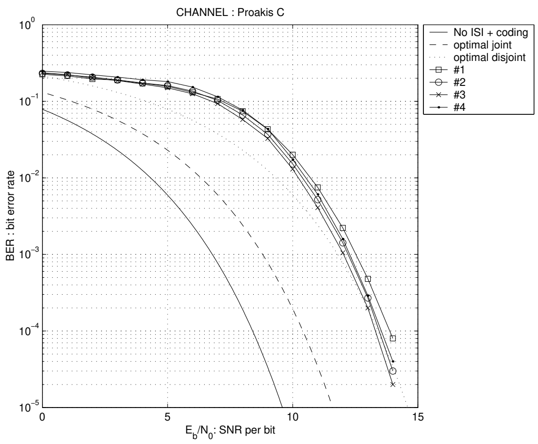

Figure 11: Performance of the turbo-equalizer (set 2) after decoding over Proakis C.

The best lower bound is the one given by the optimal joint receiver but simulation shows that in our example the turbo-equalizer never achieves this bound (see Figures 8, 9, 10, 11). However it achieves asymptotically the bound of the optimal disjoint receiver over the channel Proakis C when the equalizer of the first iteration is the optimal MAP (see Figure 11).

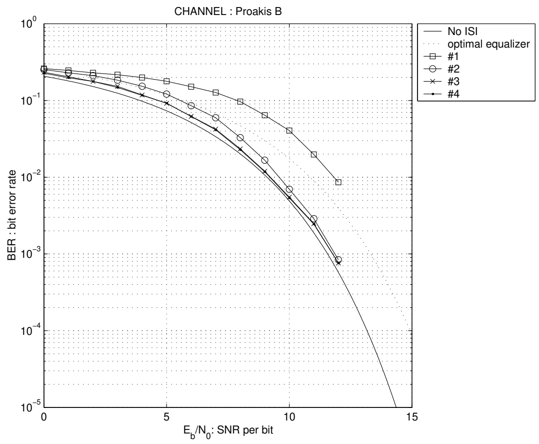

Figure 12: Performance of the turbo-equalizer (set 2) after equalization over Proakis B.

Figure 12 shows the performance of the turbo-equalizer after the equalizer. Note that the IC output outperforms the optimal equalizer. This is explained because the IC uses information about the coded bits and is not a classical equalizer. However, it is clear that the decoder doesn't achieve the coding gain that would be expected. This may be explained by the fact that the decoder assumes that the model of the data feeding it is the AWGN model, which is incorrect. Better turbo-equalizer may be found.

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