A Progressive Second Price Mechanism with a Second Round for Excluded Players*

Patrick Maillé[1] and Bruno Tuffin[2]

[1] ENST Bretagne, BP 78, 2 rue de la Châtaigneraie
35512 Cesson Sévigné Cedex, France
pmaille@rennes.enst-bretagne.fr
[2] IRISA-INRIA, Campus universitaire de Beaulieu
35042 Rennes Cedex, France
btuffin@irisa.fr
WWW home page:
http://www.irisa.fr/armor/lesmembres/Tuffin/Tuffin_en.htm

Abstract. In the large number of papers published on pricing schemes for telecommunication networks in the past few years, auctionning for bandwidth has been proved to be one of the main streams. We deal here with a method called *Progressive Second Price (PSP) Auction*, we show two drawbacks for this method: first, the initial bidder has no interest in giving his true valuation of the bandwidth as stated, and second switching the order of bid between players can provide different Nash equilibria resulting in different seller revenue. We then design an adaptation of PSP allowing to solve these problems by asking to the players excluded from the game how much they would have valued the service otherwise.

1 Introduction

Designing charging schemes for telecommunication networks has become a challenging task in the networking community. Indeed, even if the capacity keeps increasing, it is commonly admitted that demand will still be ahead. Also, since different applications with different quality of service requirements are concerned, differentiating the services is becoming mandatory. For these reasons, switching from the usual flat-rate pricing like in the current Internet to a usage-based scheme seems relevant. Many different ways to charge for the service have been devised (see for instance [1] and the references therein). In this paper, we focus on the schemes where users compete for bandwidth by means of auctions, and more specifically on the Progressive Second Price (PSP) Auction [2, 4–6] where the players (i.e., the users) sequentially bid for bandwidth until an equilibrium is reached.

In this paper, we show that the PSP presents two major drawbacks. First, we show that the first bidding player has an incentive to overestimate his bandwidth

* This work has been partially supported by the INRIA project PRIXNET (see http://www.irisa.fr/armor/Armor-Ext/RA/prixnet/ARC.htm)

unit price, so that the following players will be detered from entering the game, giving him the maximum bandwidth at a minimum price. Second, we show that switching the order of the bids among players leads to different Nash equilibria, and then different revenues for the seller, which is an unpleasant (since unpredictable) behavior of the scheme. We have then designed an extension of the PSP mechanism that solves these problems. Basically, the players excluded from the game are asked to submit, at no cost, a bid maximizing their utility of no other player were present in the game. We prove that both of the above problems are then solved. It requires a slight modification of the best-reply strategy that will be described.

The layout of the paper is the following. In Section 2 we present the PSP mechanism and its properties. Section 3 highlights some drawbacks of the schemes, that are solved in Section 4 by requiring that the players excluded from the game bid as if they were alone. Finally, we conclude in Section 5.

2 The PSP Mechanism

The PSP mechanism has been first to allocate bandwidth among users for a single resource first [2], and then to a e general network [5]. Because of lack of space, and for the sake of understanding, we limit ourselves here to the case of a single resource. Assume that its capacity is Q and and that there are I players competing for it in an auction process, where palyers bid sequentially. Player i's bid is $s_i = (q_i, p_i)$ where q_i is the capacity player i is asking and p_i is the unit price he is proposing. A bid profile is $s = (s_1, \cdots, s_I)$. Let $s_{-i} = (s_1, \cdots, s_{i-1}, s_{i+1}, \cdots, s_I)$ be the profile where player i's bid is excluded from the game. We write s as $s = (s_i; s_{-i})$ in order to emphasize player i's bid. For $y \geq 0$ define

$$\underline{Q}_i(y; s_{-i}) = \left[Q - \sum_{k \neq i \ : p_k \geq y} q_k \right]^+ .$$

The progressive second price allocation rule [2, 4] gives to player i a bandwidth

$$a_i(s) = \min(q_i, \underline{Q}_i(p_i; s_{-i})) \tag{1}$$

and set the total cost to

$$c_i(s) = \sum_{j \neq i} p_j [a_j(0; s_{-i}) - a_j(s_i; s_{-i})]. \tag{2}$$

Thus the highest bids are allocated the desired quantity and the cost is given by the declared willingness to pay (bids) of the users who are excluded by i's presence.

In [6], the allocation rule given by Equation (1) has been modified to make sure that the bandwidth is optimally allocated. If we define

$$Q_i(y, s_{-i}) = \left[Q - \sum_{k \neq i \ : p_k > y} q_k \right]^+ \tag{3}$$

as the remaining bandwidth at a given unit price, we allocate

$$a_i(s) = q_i \wedge \frac{q_i}{\sum_{k\ :p_k=p_i} q_k} Q_i(p_i, s_{-i}) \tag{4}$$

to player i.

It is assumed that there is a bid fee ε each time a player submit a bid and that player i has a budget constraint b_i such that $c_i(s_i, s_{-i}) \leq b_i$. Let $\mathcal{S}_i(s_{-i})$ be the set of player i's bids verifying this constraint. Also, a bid $s_0 = (Q, p_0)$ is introduced, meaning that the seller will give qllocate bandwidth at a minimum unit price p_0, which is called the reserve price. The seller can thus be seen as a player (not in $\mathcal{I}$) with a valuation function $\theta_i(q) = p_0 q$.

Assume that player i attemps to maximize his utility $u_i(s) = \theta_i(a_i(s)) - c_i(s)$ where θ_i is the valuation function that player i gives to his allocation. We use the following smoothness assumptions for function θ_i :

- $\theta_i(0) = 0$,
- θ_i is differentiable with $\theta_i'(0) < +\infty$,
- $\theta_i' \geq 0$, is non-increasing and continuous,
- $\exists \gamma_i > 0$, $\forall z \geq 0$, $\theta_i'(z) > 0 \Rightarrow \forall \eta < z, \theta_i'(z) < \theta_i'(\eta) - \gamma_i(z - \eta)$.
- $\exists \kappa > 0, \forall i \in \mathcal{I}$, $\forall z, z', z > z' \geq 0, \theta_i'(z) - \theta_i'(z') > -\kappa(z - z')$.

As a consequence, we have that

$$\gamma_i(b - a) \leq \theta_i'(a) - \theta_i'(b) < \kappa(b - a), \tag{5}$$

which will be used later on.

The following results are then showed:

- **(Incentive Compatibility)**. Let

  $$G_i(s_{-i}) = \sup \left\{ z : z \leq Q_i(\theta_i'(z), s_{-i}) \text{ and } c_i(z) \leq b_i \right\}.$$

  Under the above assumptions on θ_i and $\forall 1 \leq i \leq I, \forall s_{-i}$ such that $Q_i(0, s_{-i}) = 0$, $\forall \varepsilon > 0$, there exists a truthful ε-best reply

  $$t_i(s_{-i}) = \left(v_i = \left[G_i(s_{-i}) - \varepsilon/\theta_i'(0) \right]^+, \omega_i = \theta_i'(v_i) \right)$$

  where a truthful bid as a bid $s_i = (q_i, p_i)$ such that $p_i = \theta_i'(q_i)$.
- **(Convergence)**. If all the players bid like described above, the game converges to a 2ϵ-Nash equilibrium, where an ϵ-Nash equilibrium is a bid profile s such that $\forall i$ $\mathcal{S}_i^\epsilon(s_{-i}) = \{ s_i \in \mathcal{S}_i\ : u_i(s_i\ ; s_{-i}) \geq u_i(s_i'\ ; s_{-i}) - \epsilon, \forall s_i' \in \mathcal{S}_i \}$, meaning that s is a fixed-point of $\mathcal{S}^\epsilon = \prod_{i \in \mathcal{I}} \mathcal{S}_i^\epsilon(s_{-i})$.
- **(Optimality)**. For the previous ϵ-Nash equilibrium, the resulting overall utility $\sum_{i \in \mathcal{I} \cup \{0\}} \theta_i(a_i)$ is maximized.

3 Drawbacks

As previously said in the introduction, the first player has no incentive to reveal truthful best-reply. Indeed, denote by player 1 this initial player, and assume that he knows the maximal valuation $p_{max} = \max_{i \in \mathcal{I}} \theta_i'(0)$ and that $\theta_1'(Q) > p_0$. Then, if player 1 submits the bid

$$s_1 = (Q, p_{max})$$

then he will be allocated the total monut of bandwidth at unit price p_0. Then, the next players will be detered from entering the game (i.e., they will ask for 0 units of bandwidth) since whatever they would ask, their utility will be negative, i.e., $\forall i \geq 2$,

$$u_i = \theta_i(a_i) - a_i p_{max} \leq (\theta_i'(0) - p_{max}) a_i \leq 0.$$

We then get a Nash equilibrium that is undesirable.

As a second undesirable effect, even when using the truthful best-reply as the initial bid, switching the order of bids among players leads to different Nash equilibria, at least in terms of total costs. Consider the following example to illustrate the problem with $I = 2$ players, with $\theta_1(x) = AM \ln(1 + x/M)$ and $\theta_2(x) = \frac{AM}{1+1/M} \ln(1 + x/M)$ for $A, M > 0$, with a reserve price of $p_0 = 0$ and no bid fee, i.e., $\epsilon = 0$. Table 3 displays the different outcomes depending on the bid order, which especially makes the seller revenue unpredictable.

	Player 1 bids first	Player 2 bids first
bid s_1	$\left(1, \frac{A}{1+\frac{1}{M}}\right)$	$\left(1, \frac{A}{1+\frac{1}{M}}\right)$
bid s_2	$(0, 0)$	$\left(1, \frac{A}{(1+\frac{1}{M})^2}\right)$
Allocation a_1	1	1
Allocation a_2	0	0
Cost c_1	p_0	$\frac{A}{(1+\frac{1}{M})^2}$
Cost c_2	0	0
Seller revenue	p_0	$\frac{A}{(1+\frac{1}{M})^2}$

Table 1. Different outcomes depending on the bid order

4 Second round for excluded players

We present here a way to tackle the problems highlighted in the previous section. The players that do not get any bandwidth are required to submit a bid, at no cost, corresponding to the case where the player were alone (i.e., the bid profile consists only of the seller bid (Q, p_0)). As described Figure 1, the idea is to maximize the corresponding utility.

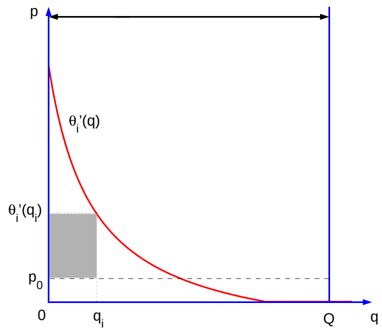

Fig. 1. Second round auction for an excluded player

4.1 Solving the initial bid problem

Using this strategy, the first player bid drawback is then circumvented. Indeed, assuming that the first player has bet (Q, p_{max}), the second player is then excluded from the game, but submits $(q_2, \theta_2'(q_2))$, so that player 1 will pay $p_0 Q + q_2(\theta_2'(q_2) - p_0)$, which can be greater than $\theta_1'(q_1)$, showing that player 1 has no interest in choosing the high initial bid strategy.

4.2 Uniqueness of the Nash equilibrium

In this subsection, we prove the uniqueness of the resulting Nash equlibrium, provided that the truthfull best-reply strategy is slightly modified as follows, meaning basically that the equal-bid cases are taken into account.

Proposition 1. *Under the smoothness assumptions over θ_i, $\forall i \in \mathcal{I}, \forall s_{-i} \in S_{-i}$ such that $Q_i(0, s_{-i}) = 0$, $\forall \epsilon > 0$, there exists a ϵ-best reply $\tilde{t}_i(s_{-i}) \in \mathcal{T}_i \cap S_i^\epsilon(s_{-i})$ providing a larger utility than the one described in the Incentive Compatibility property of Sectionsec-semret. If we note*

$$\underline{G}_i(s_{-i}) = \left\{ z \in [0, Q] : z \leq \underline{Q}_i(\theta_i'(z), s_{-i}) = \left[Q - \sum_{p_k \geq \theta_i'(z), k \neq i} q_k \right]^+ \right\}, \quad (6)$$

$x_i = \sup \underline{G}_i(s_{-i})$ *and*

$$\tilde{v}_i = \begin{cases} x_i & \text{if } x_i \in \underline{G}_i(s_{-i}) \\ \max([x_i - \epsilon/\theta_i'(0)]^+, \underline{Q}_i(\theta_i'(x_i), s_{-i})) & \text{if } x_i \notin \underline{G}_i(s_{-i}) \end{cases}$$

we the have that $\tilde{t}_i = (\tilde{v}_i, \theta_i'(\tilde{v}_i)) \in \mathcal{T} \cap S_i^\epsilon(s_{-i})$, et $\tilde{t}_i$ gives a larger utility than if submitting $t_i = (v_i, \theta_i'(v_i))$ like in the Incentive Compatibility property of Sectionsec-semret..

Before proving this result, let us show the following lemma

Lemma 1. *For a given bid s_i, denote by $u_i(s_i, s_{-i})$ the utility of player i when facing the bid-profile s_{-i}. We have that $\forall \tilde{v}_i, v_i, \tilde{v}_i \geq v_i \geq 0 | \tilde{v}_i \in \underline{G}_i(s_{-i})$ (where $\underline{G}_i(s_{-i})$ is defined by Equation (6)), if $\begin{cases} t_i = (v_i, \theta_i'(v_i) \\ \tilde{t}_i = (\tilde{v}_i, \theta_i'(\tilde{v}_i)) \end{cases}$, then*

$$u_i(\tilde{t}_i, s_{-i}) \geq u_i(t_i, s_{-i}).$$

Proof. Since $\tilde{v}_i \in \underline{G}_i(s_{-i})$, then $\tilde{v}_i \leq \underline{Q}_i(\theta_i'(\tilde{v}_i), s_{-i})$, which gives $P_i(\tilde{v}_i) \leq \theta_i'(\tilde{v}_i)$ and $a_i(\tilde{t}_i, s_{-i}) = \tilde{v}_i$. Moreovern $a_i(t_i, s_{-i}) = v_i$ since v_i is also in $\underline{G}_i(s_{-i})$.

Thus we have

$$\begin{aligned} u_i(\tilde{t}_i, s_{-i}) - u_i(t_i, s_{-i}) &= \theta_i(a_i(\tilde{t}_i, s_{-i})) - \theta_i(a_i(t_i, s_{-i})) - \int_{a_i(t_i,s_{-i})}^{a_i(\tilde{t}_i,s_{-i})} P_i(z, s_{-i}) dz \\ &= \theta_i(\tilde{v}_i) - \theta_i(v_i) - \int_{v_i}^{\tilde{v}_i} P_i(z, s_{-i}) dz \\ &\geq \theta_i'(\tilde{v}_i)(\tilde{v}_i - v_i) - \int_{v_i}^{\tilde{v}_i} \theta_i'(\tilde{v}_i) dz \\ &\geq 0 \end{aligned}$$

from which the lemma is obtained (we have used the concavity of function θ_i, and the property that function $P_i(., s_{-i})$ is non-decreasing).

Proof of Proposition 1. The proof is then straightforward: we just need to prove that, in any case, $\tilde{v}_i \in \underline{G}_i(s_{-i})$ and that $\tilde{v}_i \geq v_i$. By applying Lemma 1, we then obtain the result.

Let us show first that we always have $\tilde{v}_i \geq v_i$. In order to do that, it can be shown that $\sup G_i(s_{-i}) = \sup \underline{G}_i(s_{-i})$ ($= x_i$ by definition). Indeed, if this equality is not verified, it then means that there exists $r > 0$ such that $\sup G_i(s_{-i}) = x_i + r$, and then $x_i + r/2 \in G_i(s_{-i})$. We would then have

$$x_i + r/4 \leq x_i + r/2 \leq Q_i(\theta_i'(x_i + r/2), s_{-i}) \leq \underline{Q}_i(\theta_i'(x_i + r/4), s_{-i}).$$

The second inequality stems from the assumption that $x_i + r/2 \in G_i(s_{-i})$, and the last one from the equation $Q_i(y, s_{-i}) = \lim_{z \searrow y} \underline{Q}_i(z, s_{-i})$ and from the property that $\underline{Q}_i(., s_{-i})$ is non-decreasing. We thus obtain $x_i + r/4 \in \underline{G}_i(s_{-i})$, which ccontradicts the assumption. Finally, $\sup G_i(s_{-i}) = \sup \underline{G}_i(s_{-i})$, which shows that $\tilde{v}_i \geq v_i$ in every case mentioned by Proposition 1.

It remains now to show that $\tilde{v}_i \in \underline{G}_i(s_{-i})$. Two different cases have to be studied:

- Either $x_i \in \underline{G}_i(s_{-i})$, in which case $\tilde{v}_i = x_i \in \underline{G}_i(s_{-i})$ eis obvious (since it corresponds to one of the situations displayed in Figure 2).
- Or $x_i \notin \underline{G}_i(s_{-i})$, meaning that $\underline{G}_i(s_{-i}) = [0, x_i[$ (indeed, $x_i > 0$ since otherwise $\underline{G}_i(s_{-i}) = \{0\}$ which would give $x_i = 0 \in \underline{G}_i(s_{-i})$). The case where $[x_i - \epsilon/\theta_i'(0)]^+ \geq \underline{Q}_i(\theta_i'(x_i), s_{-i})$ est straightforward and implies that $\tilde{v}_i < x_i$ and then that $\tilde{v}_i \in \underline{G}_i(s_{-i})$ (this is the case displayed on the left hand side of Figure 3).

It remains to show that $[x_i - \epsilon/\theta_i'(0)]^+ < \underline{Q}_i(\theta_i'(x_i), s_{-i})$, as described on the right hand side of Figure 3. We then have $\tilde{v}_i = \underline{Q}_i(\theta_i'(x_i), s_{-i}) < x_i$, since otherwise $x_i \in \underline{G}_i(s_{-i})$. Thus in this case $\tilde{v}_i \in \underline{G}_i(s_{-i})$.

On Figures 2 and 3, all the possible situations are depicted, and the hatched area corresponds to the difference $u_i(\tilde{v}_i, s_{-i}) - u_i(v_i, s_{-i})$.

Proposition 1 is then proved.

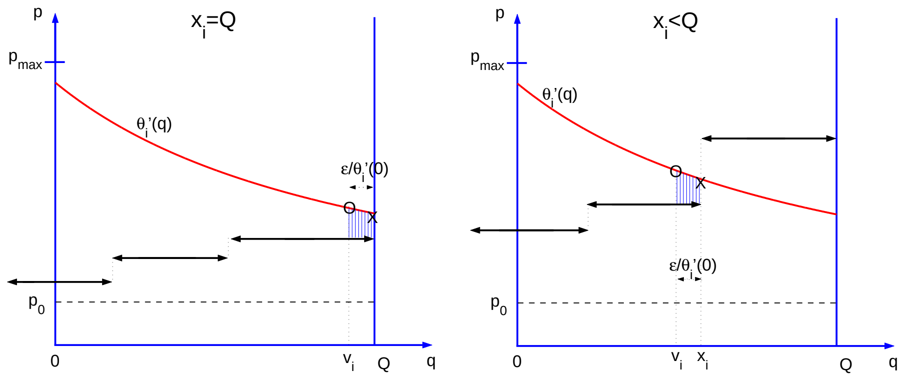

Fig. 2. Situations where $\underline{G}_i(s_{-i}) = [0, x_i]$ (i. e. $\tilde{v}_i = x_i$)

We now show a last lemma about the modified strategy.

Lemma 2. *Assume that the smoothness assumptions over* θ_i $\forall i \in \mathcal{I}$ *, that* $p_0 > 0$*, and that a player* $i \in \mathcal{I}$ *has followed the modified best-reply strategy and bids* (q_i, p_i) *(following Proposition 1). Then, considering a player* j *entering the game or which had bidden* (p_j, q_j) *before, with* $p_j < p_i - \epsilon\gamma_i/\theta_i'(0)$*, we have*

$$\underline{Q}_j(p_i - \epsilon\gamma_i/\theta_i'(0), s_{-j}) = 0 \tag{7}$$

Proof. A consequence of the assumption about player j is that for $p \leq p_i - \epsilon\gamma_i/\theta_i'(0)$, we have $\underline{Q}_j(p, s_{-j}) = \underline{Q}_k(p, s)$ for every player k who would be entering the game.

Showing Lemma 2 is reduced to show that $\sum_{l \in I | p_l \geq p_i - \epsilon\gamma_i/\theta_i'(0)} q_l \geq Q$. Using the modified best-reply strategy, we can prove this inequality following the values of $\tilde{v}_i$:

- if $x_i \in \underline{G}_i(s_{-i})$, the we consider two cases. First if $x_i = Q$ the lemma is verified (all the capacity is for player i, thus $\underline{Q}_k(p_i, s) = 0$). Second $x_i < Q$ means that $x_i = \underline{Q}_i(\theta_i'(x_i), s_{-i})$, and the lemma is still verified, with $(q_i, p_i) = (x_i, \theta_i'(x_i))$ (we again check that $\underline{Q}_k(p_i, s) = 0$).
- If $x_i \notin \underline{G}_i(s_{-i})$.

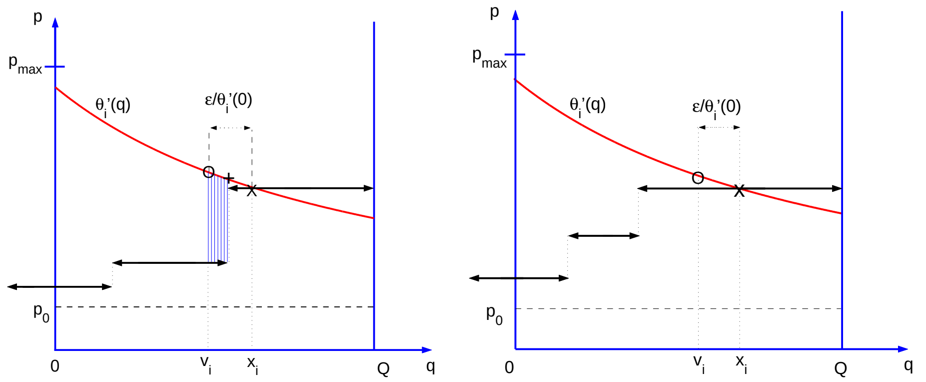

Fig. 3. Situations where $\underline{G}_i(s_{-i}) = [0, x_i[$:
on the left $\tilde{v}_i = \underline{Q}_i(\theta_i'(x_i), s_{-i})$ (case where $\underline{Q}_i(\theta_i'(x_i), s_{-i}) > [x_i - \epsilon/\theta_i'(0)]^+$),
on the right $\tilde{v}_i = v_i$ (case where $\underline{Q}_i(\theta_i'(x_i), s_{-i}) < [x_i - \epsilon/\theta_i'(0)]^+$).

- If $[x_i - \epsilon/\theta_i'(0)]^+ > \underline{Q}_i(\theta_i'(x_i), s_{-i})$ (necessarily $x_i > \epsilon/\theta_i'(0)$), player i asks for $\tilde{v}_i = x_i - \epsilon/\theta_i'(0) > 0$ at a unit price $p_i = \theta_i'(\tilde{v}_i) \geq \theta_i'(x_i) + \epsilon\gamma_i/\theta_i'(0)$ (from Inequality (5)), thus $\theta_i'(x_i) \leq p_i - \epsilon\gamma_i/\theta_i'(0)$. Once player i has submitted his bid, we have $\underline{Q}_k(\theta_i'(x_i), s) = 0$.
- If $[x_i - \epsilon/\theta_i'(0)]^+ \leq \underline{Q}_i(\theta_i'(x_i), s_{-i})$, then player i requires $\tilde{v}_i = \underline{Q}_i(\theta_i'(x_i), s_{-i})$ at a nuit price larger than $\theta_i'(x_i)$. Since $x_i - \tilde{v}_i \leq \epsilon/\theta_i'(0)$, the inequality $\theta_i'(x_i) \leq p_i - \epsilon\gamma_i/\theta_i'(0)$ remains true, as well as the result of the lemma.

Using the strategy defined above, we can now prove the following result showing the uniqueness of the resulting Nash equilibrium. In order to do that, define the *market price* as the unique price such that

$$\sum_{i \in \mathcal{I}} d_i(u) = Q. \tag{8}$$

Define also $\mathcal{I}^+ := \{i \in \mathcal{I} | d_i(u) > 0\}$ as the subset of players reuquiring some bannwidth at this market price. It can then be proved [**?**] that, if nobody asks for a quantity greater than Q,

$$\max_{i \in \mathcal{I}^+} |u - p_i| \leq C_1\sqrt{\epsilon} + C_2\epsilon \tag{9}$$

with

$$C_1 = \sqrt{2}\kappa \max_{i \in \mathcal{I}^+} \left\{ \frac{1}{\sqrt{\gamma_i}} \right\} \tag{10}$$

$$C_2 = \frac{1}{2} \max_{i \in \mathcal{I}^+} \left\{ \frac{1}{d_i(u)} \right\}. \tag{11}$$

Proposition 2. *Under the smoothness assumptions over* θ_i $\forall i \in \mathcal{I}$*,*

- *the players* i *excluded from the game bid* $s_i = (\bar{q}_i, \theta_i'(\bar{q}_i))$ *with*

$$\bar{q}_i = \arg\max_{q\in[0,Q]} \{q(\theta_i'(q) - p_0)\} \tag{12}$$

- *the players* j *obtaining some bandwidth follow the modified best-reply strategy (defined in Proposition 1),*

then if the market price u *exists and is larger than* p_0*, and if* $\sup_{i\in\mathcal{I}\setminus\mathcal{I}^+}\theta_i'(0) < u$*, then the bid-profile converges to a unique Nash* ϵ*-equilibreium* s^* *for* ϵ *small enough, where uniqueness is porvided upto* ϵ*. More precisely:*

$$\forall i \in \mathcal{I}^+ \ , \begin{cases} |u - p_i^*| \leq C_1\sqrt{2\epsilon} + 2C_2\epsilon \\ q_i^* = d_i(p_i^*) \end{cases} \tag{13}$$

$$\forall i \in \mathcal{I}\setminus\mathcal{I}^+ \ , \begin{cases} q_i^* = \bar{q}_i \\ p_i = \theta_i'(q_i^*) \end{cases} \tag{14}$$

where C_1 *et* C_2 *are the constants in Equations (10) and (11), and* $\bar{q}_i$ *is defined by Equation (12).*

Proof. First, it can be shown that the algorithm converges (in $O(\sum_{i\in\mathcal{I}}\theta_i(Q)/\epsilon)$): the proof strictly flows the one given in Proposition 4 of [4] for the initial best-reply strategy. The obtained convergence point is then a 2ϵ-Nash equilibrium. We can then apply Relations (10) and (11), leading to Equation (13)).

We choose ϵ small enough to make sure that there exists r such that $u - (C_1\sqrt{2\epsilon} + 2C_2\epsilon) > r > \sup_{i\in\mathcal{I}\setminus\mathcal{I}^+}\theta_i'(0) + \epsilon\sup_{i\in\mathcal{I}^+}\{\gamma_i/\theta_i'(0)\}$. This means that each player of $\mathcal{I}^+$ has bet a unit price strictly greater than r. Lemma 2 shows that for each player j in $\mathcal{I}\setminus\mathcal{I}^+$, we have $\underline{Q}_j(\theta_j'(0), s_{-j}) = 0$, which is equivalent to saying that the players of $\mathcal{I}\setminus\mathcal{I}^+$ are excluded from the game at equilibrium. Equation (14) is then verified.

5 Conclusions

In this paper, we have illustrated two drawbacks of the PSP mechanism: the initial player has an incentive to cheat about his bandwidth valuation and to declare a high unit-price in order to get all the bandwidth at a minimal cost given by the reserve price. Also, changing the bidding order among players results in different outcomes in terms of seller revenue, even if the overall utility is still maximized. By requiring that the excluded players reveal how much they would have valued the bandwidth if they would have been alone, those two problems are proved to be solved. Variants of PSP have been designed in [**?**,6] in order to solve some drawbacks.

As a direction for future research, we plan to look at the behavior of this mechanism in a stochastic environment where players enter or leave the game like we have done for the initial PSP mechanism in [3].

References