Numerical simulations of perforated plate liners: Analysis of the visco-thermal dissipation mechanisms

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ABSTRACT:
In the linear regime and in the absence of mean flow, the impendance of perforated liners is driven by visco-thermal effects. In this paper, two numerical models are employed for predicting these visco-thermal losses. The first model is the linearized compressible Navier–Stokes equations (LNSE), solved in the frequency domain. The second model is the Helmholtz equation with a visco-thermal boundary condition, accounting for the influence of the acoustic boundary layers. These models are compared and validated against measurements. The quantitative analysis of the dissipation rate due to viscosity, computed from the LNSE solutions of four perforated plates, highlights significant differences between the edge effects of a macro- and a micro-perforated plate. In the latter case, a jet is present at the entrances of the perforation. In contrast, the proposed numerical method to calculate the impedance of perforated liners, based on the Helmholtz equation and a visco-thermal boundary condition, is found to be computationally cheaper and to provide reliable predictions. © 2021 Acoustical Society of America.

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I. INTRODUCTION

Perforated plate liners are widely used to reduce noise emissions from turbofans. These liners are the source of aerodynamic losses due to their surface roughness. Such friction losses can be minimized by reducing the surface roughness of the liners, which, as shown experimentally by Roberts (1977), is a function of the plate porosity and the holes diameter. Therefore, using micro-perforated plates (MPPs), with a porosity $\sigma$ below 5% and a neck radius $R_{\text{neck}}$ below 0.5 mm, has developed interest to lower the flow drag of the liners.

In the linear regime without mean flow, the main dissipation mechanisms responsible for the acoustic dissipation are the visco-thermal losses. Various models have been proposed for this impedance such as those by Guess (1975) and Maa (1998). In addition, mathematical foundations are proposed by Laurens et al. (2013) for the Rayleigh conductivity of the cylindrical perforation and unconventional apertures. In a similar modeling effort, Honzik et al. (2013) propose a transfer function, derived from the theory of Zwikker and Kosten (1949) to model the viscous and thermal boundary layers in small horns. Joly et al. (2006) introduced a formulation of two coupled equations, accounting for viscous and thermal effects in a fluid. This formulation is shown to be efficient in solving linear acoustics problems using standard numerical methods.

In the present study, two numerical models are presented and validated with a view to investigate the acoustic losses occurring at a perforation. The first model solves the Helmholtz equation with a boundary condition, accounting for the viscous and thermal acoustics boundary layers from Berggren et al. (2018). This boundary condition is obtained by assuming that the rigid wall is smooth and its radius of curvature is much greater than the viscous and thermal boundary layers thicknesses. This so-called “Helmholtz with losses” model is compared against a more detailed but more costly alternative model based on the linearized compressible Navier–Stokes equations (LNSE). Both models are solved in the frequency domain using finite elements. Their results are compared with impedance tube measurements. This allows us to assess the validity of the Helmholtz with losses model. Additionally, the LNSE model allows for a detailed investigation of the losses phenomenon of the micro-perforated and the standard macro-perforated liners ($\sigma > 5\%$, $R_{\text{neck}} > 0.5$ mm, defined in Sec. II). The dissipation rate per unit mass due to viscous effects is computed and the difference between micro- and macro-perforated liners is investigated.

The remainder of this paper is as follows. We first present the theoretical models in Sec. II. In Sec. III, a brief description of the numerical methods is given. A comparison between the numerical models and measurements follows in Sec. IV, and the dissipation rate is analyzed in...
Sec. V. In Sec. VI, the convergence of the numerical models is assessed. Finally, we conclude on the relevance of the Helmholtz with losses model in Sec. VII.

II. NUMERICAL MODELS

In this section, two numerical models are described to predict the acoustic impedance of a perforated plate.

The geometry of the perforated plate, which is composed of a periodic arrangement of cylindrical holes, is simplified to a single hole. The computational domain is composed of three cylindrical ducts, corresponding to the exterior, the neck, and the cavity of the liner. The geometry is further simplified to the two-dimensional (2D) axisymmetric domain $\Omega$ shown in Fig. 1.

To ensure that this non-pavable single-hole model is representative of a periodic arrangement of perforations, the percentage open area (POA), also known as porosity $\sigma$, must correspond to that of the actual perforated plate. The cavity radius $R_{cav}$, which is equal to the exterior radius, is chosen to achieve the target plate porosity $\sigma$ such that

$$\sigma = \left( \frac{R_{neck}}{R_{cav}} \right)^2,$$

where $R_{neck}$ is the radius of the neck.

In Fig. 1, the $\Gamma_s$ boundary corresponds to sliding surfaces, whereas $\Gamma_{bc}$ and $\Gamma_{neck}$ are no-slip walls. $\Gamma_{axis}$ is the symmetry axis of domain $\Omega$. An implicit time dependence $e^{i\omega t}$ is used, where $\omega$ is the angular frequency. An incoming plane wave is imposed as a boundary condition on $\Gamma_{in}$. The LNSE and Helmholtz equation are solved in the domain $\Omega$ to calculate the reflected waves and, hence, the reflection coefficient and the effective impedance of the liner.

A. Linearized Navier–Stokes equations

The problem is made nondimensional by using the following quantities: the sound speed $c_0^s$, the fluid density $\rho_0^s$, and the reference length $L_{ref} = R_{neck}$ (recall that $R_{cav} = R_{neck}/\sqrt{\sigma}$). The symbol “$*$” denotes dimensional quantities.

It follows that variables are made nondimensional as follows:

$$x = \frac{x^*}{L_{ref}}, \quad u = \frac{u^*}{c_0^s}, \quad \rho = \frac{\rho^*}{\rho_0^s}, \quad p = \frac{p^*}{\rho_0^s c_0^s}, \quad T = \frac{T^* c_0^s}{e_0^s}, \quad e = \frac{e^*}{e_0^s}.$$

$u = (u_x, u_z)^T$ is the fluid velocity, $c$ is the sound speed, $\rho$ is the density, $p$ is the pressure, $T$ is the temperature, $c_0^s$ is the specific heat capacity at constant pressure, and $e$ is the specific internal energy.

The acoustic Reynolds number, based on the sound speed, and the Prandtl number are defined as

$$Re_a = \frac{\rho_0^s c_0^s L_{ref}}{\mu^*}, \quad \text{and} \quad Pr = \frac{c_0^s \mu^*}{k^*},$$

where $\mu^*$ is the dynamic viscosity and $k^*$ is the thermal conductivity. Both are assumed to be independent of temperature.

A perfect gas is assumed, which leads to the following relations between thermodynamic quantities:

$$c^2 = \gamma r T, \quad e = \frac{T}{\gamma} \quad \text{and} \quad p = \rho r T \quad \text{with} \quad r = \frac{\gamma - 1}{\gamma}.$$

$\gamma = c^p/c_v^s$ is the heat capacity ratio and $c_v^s$ is the specific heat capacity at constant volume.

The equations stating the conservation of mass, momentum, and energy are as follows:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho u) = 0,$$

$$\frac{\partial \rho u}{\partial t} + \rho (u \cdot \nabla) u = -\nabla p + \nabla \cdot \tau,$$

$$\frac{\rho}{\gamma} \left( \frac{\partial T}{\partial t} + u \cdot \nabla T \right) = -p \nabla \cdot u + \tau : \nabla u + \frac{1}{Re_a Pr} \nabla^2 T.$$

The viscous stress tensor $\tau$ is given by

$$\tau = \frac{1}{Re_a} \left[ \nabla u + (\nabla u)^T + \left( \frac{\mu^*}{2} - \frac{2}{3} \right) (\nabla \cdot u) \mathbb{I} \right],$$

where $\mathbb{I}$ is the identity matrix and $\mu_B = \mu_B^*/\mu^*$ is the normalized bulk viscosity. We follow the Stokes hypothesis by setting $\mu_B = 2/3$. This choice, which implies that the effect of dilatation on the viscous stress tensor is removed, is discussed in more detail in Schlichting (1979).

The mass conservation equation (3), the momentum conservation equation (4), and the energy conservation
equation (5) are linearized around a steady state, defined by \( \rho_0, u_0, p_0, \) and \( T_0 \),

\[
\begin{aligned}
\rho &= \rho_0 + \rho', \quad u = u_0 + u', \quad p = p_0 + p', \\
T &= T_0 + T',
\end{aligned}
\]

in which \( \rho' \) is the perturbed density, \( u' \) is the perturbed velocity, \( p' \) is the perturbed pressure, and \( T' \) is the perturbed temperature. We consider a uniform quiescent medium, hence, \( u_0 = 0, \rho_0 = 1, \) \( c_0 = 1, \rho_0 = 1/\gamma, \) and \( T_0 = 1 / (\gamma - 1) \). In addition, we solve these equations in the frequency domain with a \( e^{i\omega t} \) time dependence. To discuss the results, we introduce the thickness \( \delta_v \) of the acoustic viscous boundary layer as well as the shear number \( Sh \),

\[
\delta_v = \sqrt{\frac{2\mu^2}{\rho_0 \omega}}, \quad Sh = \frac{R_{\text{neck}}}{\delta_v}.
\]

\( Sh \) relates the viscous boundary layer thickness to the radius of the perforation. It is useful to distinguish between the macro- and micro-perforated regimes and to assess the range of validity of the Helmholtz equation with losses. When \( Sh \) is high, the viscous boundary layer thickness is small compared to the perforation radius.

Therefore, the linearized Navier–Stokes equations reduce to

\[
\begin{aligned}
\quad &i\omega \rho' + \nabla \cdot u' = 0, \quad (8) \\
\quad &i\omega u' = -\nabla p' + \frac{\omega}{2Sh^2} \nabla \cdot (\nabla \cdot u' + (\nabla u')^T), \quad (9) \\
\quad &i\omega T' = -\nabla \cdot u' + \frac{\omega^2}{2Sh^2} \nabla^2 T'. \quad (10)
\end{aligned}
\]

Equations (8)–(10) are solved together with the following boundary conditions. This set of equations, for instance, implemented in Malinen et al. (2004) in the finite element method (FEM) software Elmer to model the thermo-viscous effects in acoustics. Other alternatives are proposed in Kampinga and Wijnant (2010, 2011) in which different formulations of the LNSE are implemented in COMSOL. (Pryor, 2009). In the present study, we rely on the GetFEM library (Renard and Pommier, 2017). An incident plane wave is defined on the upper boundary of the domain \( \Omega \),

\[
\nabla p' \cdot n + ikp' = 2ikW e^{i\omega L_z} \quad \text{on} \quad \Gamma_{\text{in}},
\]

where \( L_z = L + h + L_T \) and \( W \) is the amplitude of the incoming plane wave. \( k \) is the acoustic wave number, accounting for the viscosity of the fluid and neglecting thermal effects, and we obtain

\[
k = \omega \left( 1 + i\omega^2 \frac{\mu B + \frac{4}{3}}{2Sh^2} \right)^{-1/2}. \quad (12)
\]

On the perforated plate and the back cover, a no-slip condition is implemented together with an isothermal condition,

\[
\begin{aligned}
u' &= 0, \quad T' = 0 \quad \text{on} \quad \Gamma_{\text{neck}} \quad \text{and} \quad \Gamma_{\text{bc}}.
\end{aligned}
\]

Alternatively, we will also consider the case where an adiabatic condition is imposed by setting \( \nabla T' \cdot n = 0 \) on these surfaces. In Sec. IV, we will compare numerical results obtained with the isothermal and adiabatic boundary conditions. We will observe that the results are very similar and the choice of this boundary condition is insignificant. In practical applications, an isothermal boundary condition is considered as a good approximation, given the thermal conductivities of the wall materials.

On the sides and the axis of the domain, a free-slip boundary condition is enforced with an adiabatic condition,

\[
\begin{aligned}
u' \cdot n &= 0, \quad \nabla T' \cdot n = 0 \quad \text{on} \quad \Gamma_{\text{axis}} \quad \text{and} \quad \Gamma_{\gamma}.
\end{aligned}
\]

This free-slip condition is indeed representative of the interaction between perforations for a normal plane wave because of the symmetry of the configuration.

The reflection coefficient \( R \) on the surface of the perforated plate is defined by

\[
R = \left( \frac{\bar{p}}{W} - e^{i\omega L_z} \right) e^{i\omega L_z},
\]

where \( W = 1 \) and \( \bar{p} \) is the averaged pressure over the boundary \( \Gamma_{\text{in}} \).

Thus, the normalized impedance at the surface of the perforated plate is determined using the following expression:

\[
\frac{Z}{\rho_0 c_0} = \frac{R + 1}{R - 1}. \quad (16)
\]

**B. Helmholtz equation with losses model**

Inside the computational domain, we solve the Helmholtz equation written for pressure,

\[
\nabla^2 p + \omega^2 p = 0 \quad \text{in} \quad \Omega.
\]

As for the LNSE, an incident plane wave is defined on the upper boundary of the domain by writing

\[
\nabla p \cdot n + i\omega p = 2W e^{i\omega L_z} \quad \text{on} \quad \Gamma_{\text{in}}.
\]

On the axis and sides of the domain, we impose a free-slip boundary condition,

\[
\nabla p \cdot n = 0 \quad \text{on} \quad \Gamma_{\text{axis}} \quad \text{and} \quad \Gamma_{\gamma}.
\]

Even though a lossless Helmholtz equation is used in the computational domain, it is still possible to account for some visco-thermal losses through the use of a boundary condition that accounts for these effects within the acoustic...
boundary layers over the solid surfaces. This approach was first proposed and developed by Morse and Ingard (1968) and recently further refined by Berggren et al. (2018). These equivalent boundary conditions are derived by performing an asymptotic expansion with respect to the viscous boundary layer thickness. The boundary condition derived by Berggren et al. (2018) reads

$$\nabla \cdot \mathbf{n} = \delta_v \nabla_i \gamma - \frac{1}{2} \nabla^2 \rho + \delta_T \omega^2 \frac{(\gamma - 1)(i - 1)}{2} \rho \text{ on } \Gamma_{\text{neck}} \text{ and } \Gamma_{\text{bc}}, \quad (20)$$

in which $\nabla^2_i$ is the tangential Laplacian, defined as

$$\nabla^2 i = \nabla_i^2 p + \frac{\partial^2 p}{\partial n^2} + (\nabla \cdot \mathbf{n}) \frac{\partial p}{\partial n}. \quad (21)$$

The dimensionless viscous and thermal boundary layer thicknesses are defined as

$$\delta_v = \frac{1}{Sh} \quad \text{and} \quad \delta_T = \frac{1}{Sh \sqrt{Pr}}.$$  

One objective of this paper is to assess the applicability of this equivalent boundary condition to predict the acoustic impedance of perforated plate liners. Equation (20) is derived from the LNSE. A no-slip condition and an isothermal condition are applied on the rigid walls. The radius of curvature of the surface should also be large compared to the viscous boundary layer thickness. Therefore, in the case of a perforate plate, this boundary condition is valid when the radius of the neck is much larger than $\delta_v$ and $\delta_T$. This limitation is stated by Mbailassem et al. (2019) in a similar approach. Since the wall model (20) is not suitable for strongly curved surfaces, this model is not expected to be valid at the corners of the hole. The corners of the perforation and their sharpness can play an important role in the acoustic dissipation as discussed by Morse and Ingard (1968). Another limitation is that the viscous boundary layer should not interact. For a perforate plate, this occurs when $\delta_v \geq R_{\text{neck}}$. Berggren et al. (2018) compared their solutions to those of Keefe (1984) for cylindrical wave guides of radius 0.1 mm and obtained good correspondence for $Sh > 2$. In order to perform consistent comparisons between the numerical models and measurements, we will not consider $Sh < 2$ in the following.

Equation (20) is derived using an isothermal boundary condition. If an adiabatic boundary condition is used instead, the term $\delta_T \omega^2 (\gamma - 1)(i - 1) \rho / 2$ in Eq. (20) is removed because it represents the heat flux through the thermal boundary layer.

This model based on the classical Helmholtz equation and the equivalent boundary condition (20) allows us to perform rapid predictions of the acoustic impedance of a perforated plate. This is because it involves only a single variable compared to the LNSE, which involve density, velocity, and temperature. In addition, the LNSE modelled require a very fine mesh to resolve the thermal and viscous boundary layers while the net effect of the acoustic boundary layers are directly modelled by the equivalent boundary condition (20). This is especially true for high frequencies when the boundary layers are very thin. The Helmholtz with losses model is implemented using GetFEM, but other implementations could be considered. This is the case in Kampinga and Wijnant (2011) in which a similar formulation, implemented in COMSOL, based on a reduced set of the full linearized Navier–Stokes equations is proposed, to obtain a more efficient FEM model for visco-thermal acoustics.

### III. NUMERICAL METHOD

The LNSE model and Helmholtz model are both solved using the FEM. The variational formulations are detailed in Appendixes B and C. These formulations are implemented using the GetFEM++ package (Renard and Pommier, 2017), and the meshes are generated using Gmsh (Geuzaine and Remacle, 2009). Unstructured, triangular meshes are used. In the LNSE model, first-order polynomials are used to approximate the density and temperature, whereas second-order polynomials are used for the velocity. In the Helmholtz model, the pressure field is approximated with second-order polynomials.

Figure 2 shows an example of the finite element mesh used for the LNSE model. This mesh corresponds to configuration 3, detailed in Table I. The mesh is refined near the neck to properly resolve the boundary layers. In addition, the corners of the neck are rounded with a radius $R_c = R_{\text{neck}} / 100$ to avoid geometrical singularities. No measurable difference is observed on the predicted impedance when the computations are performed using sharp corners, but rounded corners lead to a faster mesh convergence of the numerical model. This method is also used by Temiz et al. (2015) to perform numerical simulations based on the linearized incompressible Navier-Stokes equations.
TABLE I. Perforated plate configurations.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>$\sigma$ (%)</th>
<th>$R_{neck}$ (mm)</th>
<th>$R_{ave}$ (mm)</th>
<th>$h$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (Macro)</td>
<td>6</td>
<td>0.8</td>
<td>3.27</td>
<td>1.5</td>
</tr>
<tr>
<td>2 (Macro)</td>
<td>10</td>
<td>0.8</td>
<td>2.53</td>
<td>1.5</td>
</tr>
<tr>
<td>3 (Micro)</td>
<td>1.4</td>
<td>0.15</td>
<td>1.27</td>
<td>0.6</td>
</tr>
<tr>
<td>4 (Micro)</td>
<td>4.2</td>
<td>0.15</td>
<td>0.73</td>
<td>0.6</td>
</tr>
</tbody>
</table>

IV. COMPARISONS WITH MEASUREMENTS

The results obtained from the numerical models are now compared with impedance tube measurements. These measurements are carried out in accordance with the NF EN ISO 10534–2 (ISO, 1998) standard method. The diameter of the tube is 29 mm. Two 1/4" microphones with a 20 mm spacing are used to determine the surface impedance of the liners. The distance between the sample and the closest microphone is 45.2 mm. The comparison between the calculation and the measurements is carried out between 850 and 3000 Hz. The cavity depth is $L = 29$ mm. Four configurations are treated in this study to investigate the impact of the liner’s geometry on the impedance. The chosen parameters are summarized in Table I.

Mechanical drilling is used to manufacture the perforated plates. Figure 3 shows a photograph of a perforated plate cut to assess qualitatively the sharpness of the corners. We can observe that the perforations have a clear cylindrical shape and the corners appear to be sharp. The diameter of the perforations is known to be within ±0.01 mm, which provides good accuracy on the overall porosity of the plate. In the macro-perforated case, the maximum relative error on the porosity is 1.3%, whereas for the micro-perforated case, it is 6.8%. More details on the accuracy of this manufacturing process can be found in Drevon (2004).

Configurations 1 and 2 correspond to macro-perforated plates with a low and high porosity, respectively. Configurations 3 and 4 are micro-perforated plates with a low and high porosity, respectively. The neck radius is kept constant in the macro- and micro-perforated cases. The sound pressure level (SPL) is set to 115 dB at the plate surface with a white noise source. We introduce the Strouhal number

$$St = \frac{2\omega R_{neck}}{|u_p|}, \quad (22)$$

in which $|u_p|$ is the acoustic velocity through one perforation. In order to remain in the linear regime, $St$ must be larger than one, according to Temiz et al. (2016). Because we are using a white noise source, this velocity $|u_p|$ can be defined in two ways. First, we can consider that the perforation is submitted to the acoustic velocity associated with each frequency independently. Second, we can use the root mean square velocity calculated over the complete frequency range. This is defined as

$$u_p = \sqrt{\sum_i u_{pi}^2} \quad (23)$$

in which $u_{pi}$ is the acoustic velocity at each frequency. It is necessary to check that $St > 1$ using both definitions of the velocity $|u_p|$. Figure 4 shows the Strouhal number based on the acoustic velocity at each frequency. The lowest value observed between 850 and 3000 Hz is $St = 36$ for configuration 3. When using the root mean square velocity calculated between 400 and 6400 Hz, we see that its lowest value is $St = 2.9$.

In the following the normalized resistance, $Re(Z)/(\rho c_0)$ and the normalized plate reactance, $Im(Z)/(\rho c_0) + \cot(k_0 L)$ with $k_0 = \omega / c_0$, are plotted as functions of the shear number.

Figure 5 shows the impedances obtained for the macro-perforated plates for which $10.6 < Sh < 20$ approximately. The LNSE model appears to underestimate the resistance of the macro-perforated liners. It is also the case for the Helmholtz model but to a lesser extent. However, the significance of these differences is limited as we are looking at low resistance values. As a result, the corresponding absolute error is low. The computed plate reactances present a good correspondence with the measurements.

Figure 6 shows the impedances for the micro-perforated configurations for which $2 < Sh < 3.7$, which corresponds to the validity limit of the Helmholtz model. In the micro-perforated case, according to Figs. 6(a) and 6(b), the resistance is accurately predicted by the linearized
Navier–Stokes model. The Helmholtz model also provides correct predictions of the resistance despite the fact that it is less accurate close to its validity limit [Fig. 6(a)], i.e., when \( \text{Sh} \approx 2 \). Good correspondence between the measured and the modelled plate reactances is visible in Figs. 6(c) and 6(d).

As mentioned in Sec. III, the LNSE predictions are based on the isothermal condition on the plate and the backing plate. It is interesting to assess whether the use of an adiabatic condition would significantly change these predictions. This is shown in Fig. 6(a) where the results with adiabatic and isothermal conditions are presented side-by-side for configuration 3. Changing the nature of the thermal boundary condition only has a very limited influence on the predicted impedance. This is expected because the dissipation is dominated by shear effects rather than by thermal effects in this particular case.

V. ANALYSIS OF THE RATE OF DISSIPATION

In order to gain more insight into the dissipation mechanisms influencing the impedance of a perforated plate, we calculate the viscous dissipation rate (Lighthill, 1978). From Eq. (6), we derive its expression, separating the dissipation due to the shear and bulk effects. The dissipation rate per unit mass resulting from the shear stresses is

\[
\Phi_{\text{shear}} = \frac{1}{\text{Re}_a} \left\{ \frac{1}{2} \left[ \nabla \mathbf{u} + (\nabla \mathbf{u})^T \right] : \left[ \nabla \mathbf{u} + (\nabla \mathbf{u})^T \right] - \frac{2}{3} (\nabla \cdot \mathbf{u})^2 \right\},
\]

and the dissipation rate per unit mass due to the bulk viscosity is

\[
\Phi_{\text{bulk}} = \frac{1}{\text{Re}_a} \mu_B (\nabla \cdot \mathbf{u})^2.
\]

The expressions of \( \Phi_{\text{shear}} \) and \( \Phi_{\text{bulk}} \) are consistent with Batchelor (1967). In the results presented below, the integral over the whole FEM domain of the dissipation due to the bulk effect \( \Phi_{\text{bulk}} \) was found to be negligible compared to the overall value of \( \Phi_{\text{shear}} \) for all four configurations. Indeed, the dissipation due to dilatation is expected to be negligible in our range of shear number, i.e., \( 2 < \text{Sh} < 20 \). For this reason, \( \Phi_{\text{bulk}} \) is not discussed further.

The dissipation rates \( \Phi_{\text{shear}} \) for the macro-perforated configurations 1 and 2 and the micro-perforated configurations 3 and 4 are shown in Fig. 7 at their respective resonance frequencies for \( rL_\text{ref} \in [0; R_{\text{cv}}] \) and \( zL_\text{ref} \in [L - 2R_{\text{neck}}; L + h + 2R_{\text{neck}}] \). In both cases, an important part of the dissipation is localized in the neck. This well-known
dissipation mechanism is modelled by the theory from Zwikker and Kosten (1949). In the case of a MPP, the viscous boundary layer thickness is large compared to the neck radius, thus, the dissipation is spread across the neck. In the macro-perforated configuration, for which the ratio $Sh$ is high, the dissipation is concentrated close to the wall of the neck.

In addition to the viscous losses in the neck, significant losses can also be seen just outside of the neck and at its corners. However, the relative magnitude of these losses varies markedly between the macro- and micro-perforated cases. In the first case, the losses are localized mainly at the corners of the neck, whereas in the second case, they are found outside the neck at its entrances. The net effects on the acoustic impedance of the viscous losses occurring outside the neck are generally modelled using end correction terms. For instance, Guess (1975) introduces a corrected length $h' = h + 2R_{\text{neck}}$ determined empirically. More recently, it has been possible to use numerical simulations to calculate these end correction terms more systematically. For instance, Temiz et al. (2015) solved the linearized incompressible Navier–Stokes equations to determine the end correction for a wide range of perforates ($1 < Sh < 35$) covering both the micro- and macro-perforates. Although the determination of an end correction is useful to build a simple formula for the acoustic impedance, the detailed analysis of the dissipation rate presented here helps us to understand where the viscous losses are located.

In order to quantitatively assess the differences in the distribution of the dissipation rate between the micro- and macro-perforated cases, the dissipation rate due to shear effects is integrated over four domains as shown in Fig. 8. The domains correspond to the exterior close to the hole entrances, the neck, the wall of the neck, and the corners of the neck. The contributions from these zones are computed for the four previous configurations and compared to each other.

Table II summarizes the contributions from each zone to the overall dissipation rate for each configuration at their respective resonance frequencies. The remaining losses in the domain do not exceed 0.6% for each configuration.

The relative distribution of losses is significantly different between the macro-perforated cases 1 and 2 and the micro-perforated cases 3 and 4. In the first cases, the viscous boundary layer thickness is small compared to the neck radius, and the dissipation is localized mainly near the corners and along the neck wall. The important contribution from the corners is partly due to the fact that the ratio

![FIG. 6. (Color online) [(a) and (c)] Normalized resistance and normalized plate reactance for configuration 3: a micro-perforated case with a low porosity ($Re = 3413$). [(b) and (d)] Normalized resistance and normalized plate reactance for configuration 4: a micro-perforated case with a high porosity ($Re = 3413$).](https://doi.org/10.1121/10.0002973)
\( h/(2R_{\text{neck}}) \) is close to one in the macro-perforated case, meaning that the corner integration zone extends far in the wall of the neck. Therefore, most of the losses occur close to the wall. This explains why, for macro-perforated configurations, the Helmholtz model with the equivalent boundary condition (20) is able to provide results similar to those of the LNSE model. In the micro-perforated cases, the shear number \( Sh \) is low and a significant contribution from the neck and the wall regions is visible. In addition, about a quarter of the losses occur in the exterior domain near the entrance of the holes. This highlights the presence of a jet in the linear regime when considering micro-perforation. Despite the fact that the Helmholtz approach is not modeling this effect, it so happens that for this configuration, it provides correct predictions of the impedance.

VI. CONVERGENCE OF THE NUMERICAL MODELS

The convergence of the numerical models is investigated in this section. In a first stage, we compare the results obtained from impedance tube measurements to computations performed with different numbers of degrees of freedom (DOF) using the macro-perforated configuration 1.

Good convergence of the LNSE model is obtained with 36.8 k DOF [Fig. 9(b)]. This corresponds to elements of sizes 3.34\( \delta V \) on the axis and 0.43\( \delta V \) on the surface of the neck. The size of the elements on the edges of the perforation is the same as the size of the elements on the surface of the neck. In fact, the normalized plate reactance in Fig. 9(d) has already converged for a very coarse model with just 11.4 k DOF. Figures 9(a) and 9(c) show that the Helmholtz

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**TABLE II. Comparison of the contributions for each configuration.**

<table>
<thead>
<tr>
<th>Configuration</th>
<th>( Sh )</th>
<th>Exterior (%)</th>
<th>Neck corners (%)</th>
<th>Neck (%)</th>
<th>Wall (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (Macro)</td>
<td>13.4</td>
<td>12.7</td>
<td>37.8</td>
<td>2.1</td>
<td>46.9</td>
</tr>
<tr>
<td>2 (Macro)</td>
<td>15</td>
<td>11.1</td>
<td>39.4</td>
<td>1.6</td>
<td>47.5</td>
</tr>
<tr>
<td>3 (Micro)</td>
<td>2.2</td>
<td>25.1</td>
<td>7.1</td>
<td>31.3</td>
<td>36.1</td>
</tr>
<tr>
<td>4 (Micro)</td>
<td>2.9</td>
<td>25.3</td>
<td>7.1</td>
<td>31.3</td>
<td>36.9</td>
</tr>
</tbody>
</table>

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**FIG. 7.** (Color online) Absolute value of the dissipation rate per unit mass due to shear effects at the resonance frequency of configuration 1 at \( Sh = 13.4 \) (a), configuration 2 at \( Sh = 15 \) (b), configuration 3 at \( Sh = 2.2 \) (c), and configuration 4 at \( Sh = 2.9 \) (d).

**FIG. 8.** (Color online) Schematic of the integrated surfaces.
model converges for a number of DOF near 6.4 k DOF. This corresponds to elements of sizes $8.16\delta_0$ on the axis and $1.07\delta_0$ on the surface of the neck. This much smaller model size is explained by the facts that (i) the Helmholtz equation is a scalar model while the LNSE involves four variables, and (ii) there is no need to resolve the boundary layers with the Helmholtz model.

In a second stage, the accuracy of the models is investigated by computing the impedance at the resonance frequency of configuration 1. A series of computations is performed with increasingly finer meshes. To assess the accuracy of each of these computations, the error on the predicted impedance is calculated as

$$\epsilon_r = \frac{|Z - Z_f|}{\rho_0 c_0}, \tag{26}$$

where $Z_f$ is the reference value of the impedance calculated for each model using an extremely fine mesh. This error on the impedance is shown in Fig. 10 for configuration 1 at the resonance frequency. It is clear that the Helmholtz model with the boundary condition (20) converges more rapidly than the LNSE model. As a consequence, a converged prediction of the impedance ($\epsilon_r \approx 10^{-4}$) is obtained with a much smaller problem size with the Helmholtz model (around 5000 DOF compared to around $10^5$ DOF for the LNSE model). This provides quantitative evidence of the computational benefits of this Helmholtz model.

VII. CONCLUSION

Two computational models were considered to predict the acoustic impedance of perforated plates in the linear regime. Both macro- and micro-perforated configurations were considered and detailed comparisons with measured data from an impedance tube were used for validation.
The model based on the linearized Navier–Stokes equations is particularly expensive to solve, but it provides a more detailed and complete picture of the absorption mechanisms. It was used to calculate the viscous dissipation rate for a single hole. It was observed that the overall distribution of dissipation is very different between the macro- and micro-perforated cases. For the macro-perforated case, there is a significant amount of dissipation taking place at the corners of the perforation. In the micro-perforated case, an important contribution to the viscous dissipation comes from the regions just above and below the neck. These contributions to the acoustic absorption are generally accounted by introducing end correction terms that have been determined empirically (Ingard 1953) or numerically (Temiz et al., 2015). The quantitative analysis of the dissipation rate presented here provides more detailed insight into the location and significance of these dissipation mechanisms.

The second model considered is based on the classical Helmholtz equation combined with an equivalent boundary condition developed by Berggren et al. (2018), which accounts for the visco-thermal losses in the acoustic thermal and viscous boundary layers. This model is much cheaper to solve compared to the LNSE because it is a scalar model and does not require resolving the thermal and viscous boundary layers in the finite element mesh.

Based on comparisons with experimental data, both models are able to predict accurately the impedance. Despite its inherent simplifications (compared to the LNSE model), the approach based on the Helmholtz equation and the equivalent boundary condition appears to provide reliable predictions both for macro-perforated plates and MPPs. Whereas this can be expected for macro-perforates as most of the losses occur along the walls of the perforation, this is more unexpected for micro-perforates because the underlying assumptions of the equivalent boundary condition are not strictly satisfied. This is, however, consistent with the recent results on the use of this model for predicting losses occurring in porous materials.

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APPENDIX A: LNSE DISPERSION RELATION AND PLANE WAVE DERIVATIONS

On the upper boundary \( \Gamma_{in} \) of the computation domain, an incoming plane wave of the form \( e^{i(k_{ext}x+k_{z}z)} \) is generated by using the following Robin boundary condition:

\[
\nabla \rho' \cdot n + ik \rho' = 2ik^2 W e^{iLZ}, \quad \text{on} \quad \Gamma_{in}, \tag{A1}
\]

where \( L_Z = L + h + L_T \) and \( W \) is the amplitude of the plane wave. The numerical implementation is formulated using the acoustic Reynolds number based on the sound speed \( \mathrm{Re}_a \) instead of the shear number \( \mathrm{Sh} \), which is used for the post-processing of the results. The wavenumber \( k \) remains to be determined together with the associated expression for the other variables of the linearized Navier–Stokes equations. To that end, we use the fact that these variables are also of the form \( e^{i(k_{ext}x+k_{z}z)} \) to modify Eqs. (8)–(10) and write:

\[
\rho' = -\frac{k}{\omega} u' \cdot n, \quad p' = \frac{-\omega}{k} u' \cdot n, \quad T' = -\frac{k}{\beta} u' \cdot n \quad \text{with} \quad [\gamma - 1] T' + \rho' = \frac{\beta}{\gamma} \frac{[\gamma - 1]}{\gamma} T', \quad \tag{A2}
\]

These expressions are used below in the variational formulation of the LNSE.

APPENDIX B: LNSE WEAK FORMULATIONS

1. Mass equation

The weak formulation of the mass conservation equation is:

\[
\int_{\Omega} q \frac{\partial \rho'}{\partial t} \, d\Omega = -\int_{\Omega} u' \cdot \nabla q \, d\Omega - \int_{\Gamma_{in}} qu' \cdot n \, d\Gamma, \tag{B1}
\]

where \( q \) is a test function. Incorporating the plane-wave boundary condition (A4) in Eq. (B1) yields:

\[
\int_{\Omega} q \frac{\partial \rho'}{\partial t} \, d\Omega = -\int_{\Omega} u' \cdot \nabla q \, d\Omega + \frac{1}{ik} \int_{\Gamma_{in}} q(n \cdot \nabla) \times (u' \cdot n) \, d\Gamma + \frac{2k}{\alpha} We^{iLZ} \int_{\Gamma_{in}} q \, d\Gamma. \tag{B2}
\]

2. Momentum equation

The boundary, including the axis and the free-slip boundaries, is written as \( \Gamma_s = \Gamma_{axis} \cup \Gamma_{ext.lat} \cup \Gamma_{cav.lat} \). The weak formulation of the momentum equation reads...
\[
\int_{\Omega} \nu \cdot \frac{\partial u}{\partial t} \, d\Omega = \frac{1}{\gamma} \left[ (\nabla \cdot \nu) \left[ (\gamma - 1)T' + \rho' \right] \right] \, d\Omega
- \frac{1}{\text{Re}_a} \left[ \nabla v : \left[ \nabla u' + (\nabla u')^T \right] \right]
+ \left( \mu_B - \frac{2}{3} \right) (\nabla \cdot v)(\nabla \cdot u') \, d\Omega
- \frac{1}{\text{Re}_a} \left[ (\nu \cdot n) \left[ (\gamma - 1)T' + \rho' \right] \right] \, d\Gamma
+ \frac{1}{\text{Re}_a} \left[ \nu \cdot (\nabla u' + (\nabla u')^T) \right] n
+ \left( \mu_B - \frac{2}{3} \right) \nu \cdot n(\nabla \cdot u') \, d\Gamma;
\]  
\tag{B3}

where \( \nu \) is a test function. Using the boundary conditions (A3)–(A5), we find

\[
\rho_0 \int_{\Omega} \nu \cdot \frac{\partial u}{\partial t} \, d\Omega = \frac{1}{\gamma} \left[ (\nabla \cdot \nu) \left[ (\gamma - 1)T' + \rho' \right] \right] \, d\Omega
- \frac{1}{\text{Re}_a} \left[ \nabla v : \left[ \nabla u' + (\nabla u')^T \right] \right]
+ \left( \mu_B - \frac{2}{3} \right) (\nabla \cdot v)(\nabla \cdot u') \, d\Omega
+ \frac{1}{\text{Re}_a} \left[ (\nu \cdot n) \left[ (\gamma - 1)T' + \rho' \right] \right] \, d\Gamma
- \frac{2}{3} \nu \cdot n \left[ (\gamma - 1)T' + \rho' \right] \, d\Gamma
- 2W_i k^{i+\beta} \frac{\mu_B + \frac{4}{3}}{\beta} \right]
\times \int_{\Gamma_m} (\nu \cdot n)(u' \cdot n) \, d\Gamma.
\]  
\tag{B4}

\section*{3. Energy equation}

The weak formulation of the energy equation corresponds to

\[
\int_{\Omega} \varepsilon \frac{\partial T'}{\partial t} \, d\Omega = - \int_{\Omega} (\nabla \cdot u') \varepsilon - \frac{\gamma}{\text{Re}_a} \nabla \varepsilon \cdot \nabla T' \, d\Omega
+ \frac{\gamma}{\text{Re}_a} \int_{\Gamma} \varepsilon' \nabla \cdot n \, d\Gamma,
\]

where \( \varepsilon \) is a test function. Replacing \( \nabla T' \cdot n \) by the plane-wave boundary condition (A5) on \( \Gamma_m \), the weak formulation becomes

\[
\int_{\Omega} \varepsilon \frac{\partial T'}{\partial t} \, d\Omega = - \int_{\Omega} (\nabla \cdot u') \varepsilon - \frac{\gamma}{\text{Re}_a} \nabla \varepsilon \cdot \nabla T' \, d\Omega
- \frac{\gamma}{\text{Re}_a} \int_{\Gamma} \varepsilon' \nabla \cdot n \, d\Gamma
+ \frac{2}{3} \frac{\mu_B + \frac{4}{3}}{\beta} \left[ \frac{\mu_B + \frac{4}{3}}{\beta} \right]
\times \int_{\Gamma_m} (\nu \cdot n)(u' \cdot n) \, d\Gamma.
\]  
\tag{B5}

\section*{APPENDIX C: WEAK FORMULATION OF THE HELMHOLTZ WITH LOSSES MODEL}

The variational formulation for the Helmholtz equation is

\[
- \int_{\Omega} \nabla p \cdot \nabla \eta \, d\Omega + \int_{\Omega} \nabla p \cdot n \eta \, d\Gamma + \omega^2 \int_{\Omega} p \eta \, d\Omega = 0,
\]  
\tag{C1}

where \( \eta \) is a test function.

An incoming plane wave is implemented on the upper boundary of the domain

\[
\frac{\partial p}{\partial n} + i \omega p = 2W_i e^{i\beta z} \text{ on } \Gamma_m.
\]  
\tag{C2}

The boundary condition (20) is implemented on \( \Gamma_w \).

These boundary conditions are introduced in Eq. (C1) to give

\[
- \int_{\Omega} \nabla p \cdot \nabla \eta + \omega^2 p \eta \, d\Omega + \int_{\Gamma_w} \frac{1}{2} \left[ \frac{1}{\beta} \nabla \eta \cdot \nabla \eta \right]
- \sqrt{\delta_T} \frac{1}{2} \left[ (\gamma - 1) - 1 \right] \frac{1}{\beta} p \eta \, d\Gamma
+ \int_{\Gamma_m} - i \omega p \eta + 2W_e e^{i\beta z} \eta \, d\Gamma_m = 0.
\]  
\tag{C3}


