Playing Stochastically in Weighted Timed Games to Emulate Memory

Julie Parreaux
Benjamin Monmege   Pierre-Alain Reynier

Aix–Marseille Université

ISTA
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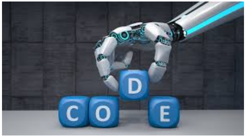
Motivation: game theory for synthesis

Game theory

Interaction between two antagonistic agents: environment and controller

Classical approach

Check the correctness of a system

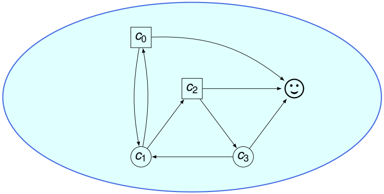


Code synthesis

Correct by construction: synthesis of controller

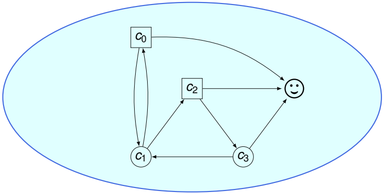
Different classes of games

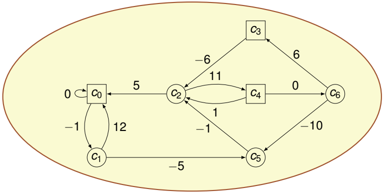
Qualitative games



Different classes of games

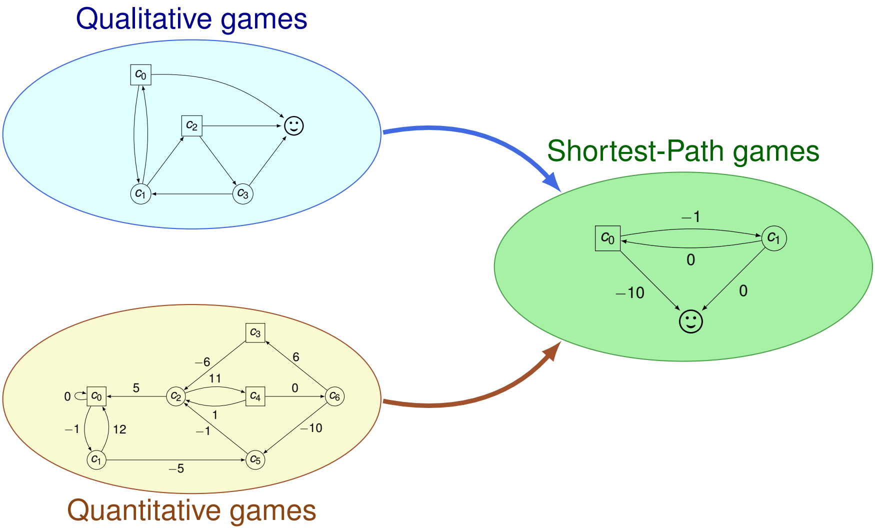
Qualitative games



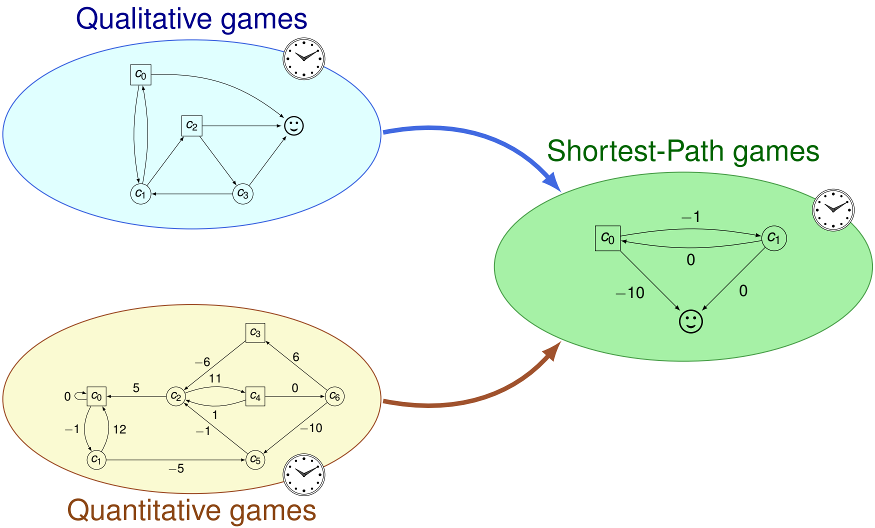


Quantitative games

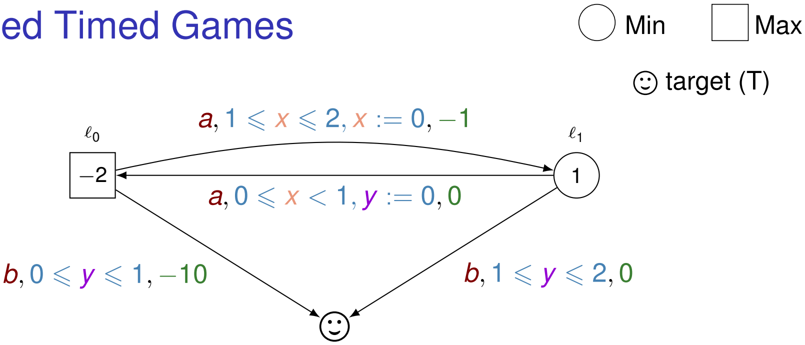
Different classes of games



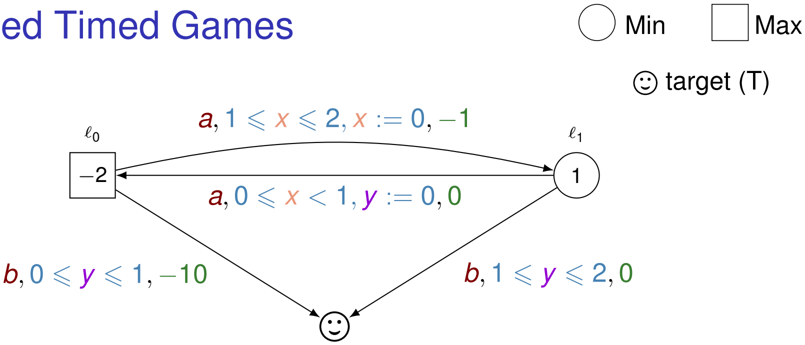
Different classes of games



Weighted Timed Games



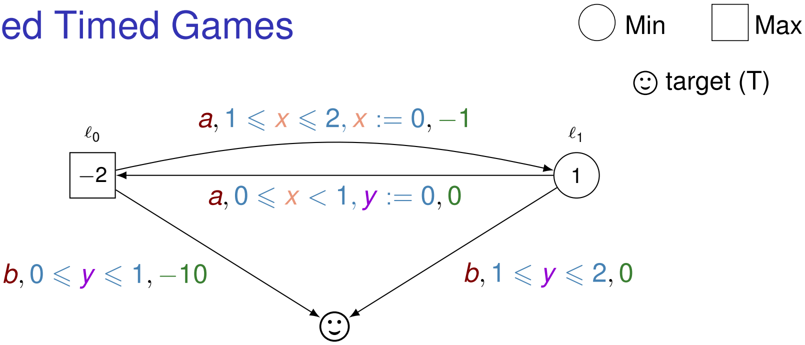
Weighted Timed Games



Play ρ

$$(\ell_1, \begin{bmatrix} 0 \\ 0 \end{bmatrix})$$

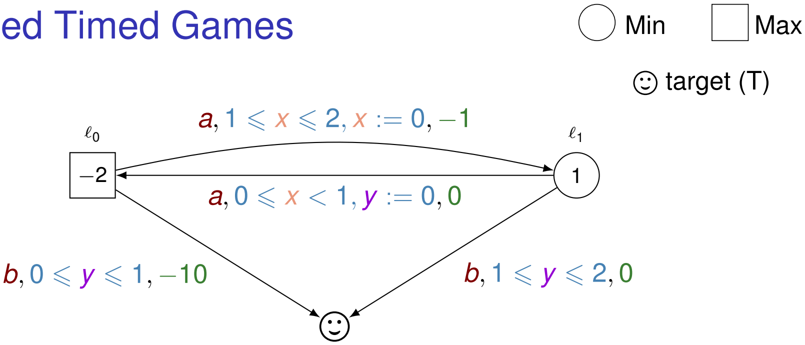
Weighted Timed Games



Play ρ

$$(\ell_1, \begin{bmatrix} 0 \\ 0 \end{bmatrix}) \xrightarrow{0.5,\ a}$$

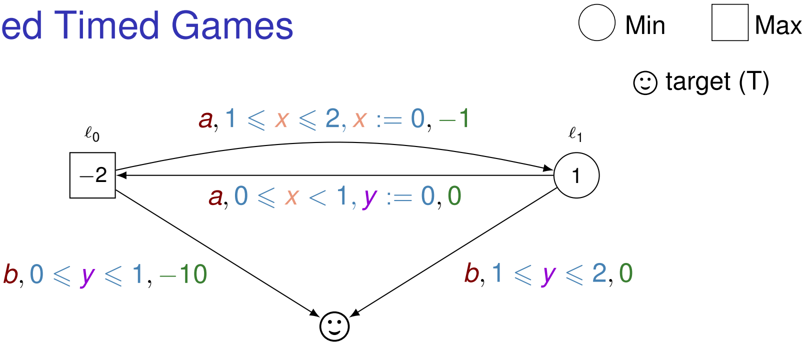
Weighted Timed Games



Play ρ

$$(\ell_1, \begin{bmatrix} 0 \\ 0 \end{bmatrix}) \xrightarrow{0.5,\ a} (\ell_0, \begin{bmatrix} 0.5 \\ 0 \end{bmatrix})$$

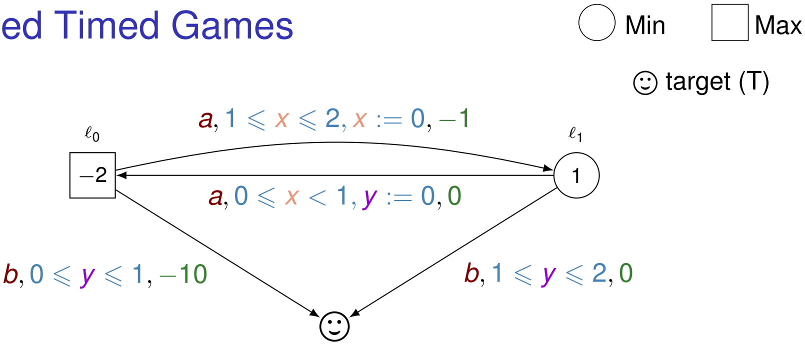
Weighted Timed Games



Play ρ

$$(\ell_1, \begin{bmatrix} 0 \\ 0 \end{bmatrix}) \xrightarrow{0.5,\ a} (\ell_0, \begin{bmatrix} 0.5 \\ 0 \end{bmatrix}) \xrightarrow{1.25,\ a} (\ell_1, \begin{bmatrix} 0 \\ 1.25 \end{bmatrix}) \xrightarrow{1/3,\ b} (☺, \begin{bmatrix} 1/3 \\ 19/12 \end{bmatrix})$$

Weighted Timed Games



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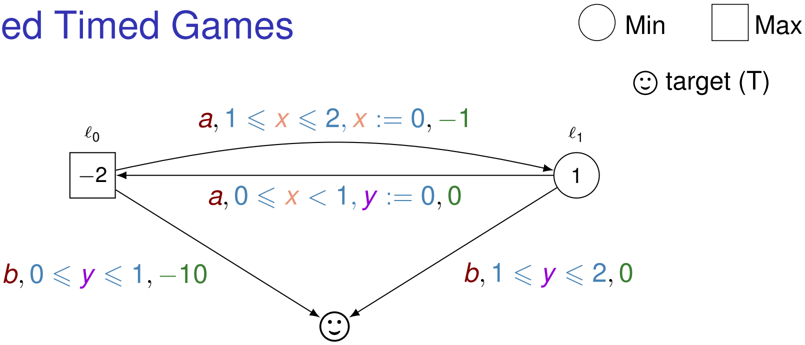
$$+ \qquad\qquad\qquad + $$

Weighted Timed Games



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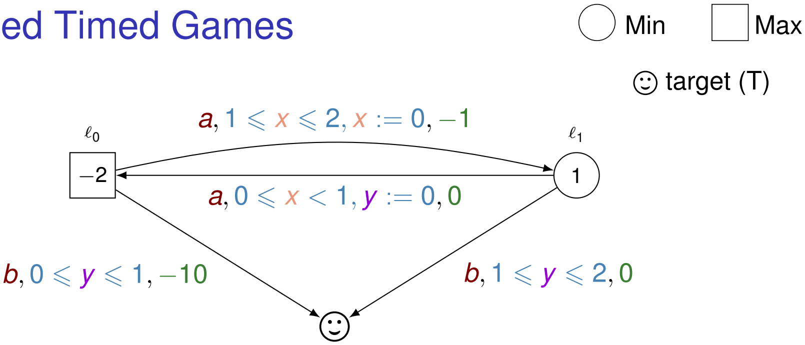
$$1 \times 0.5 + 0 \quad + \qquad\qquad + $$

Weighted Timed Games



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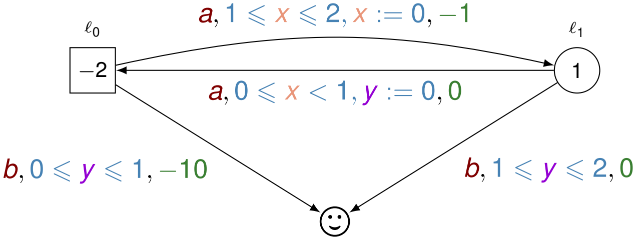
$$1 \times 0.5 + 0 \quad + \quad -2 \times 1.25 - 1 \quad + \quad 1 \times \frac{1}{3} + 0$$

Weighted Timed Games

○ Min □ Max

☺ target (T)



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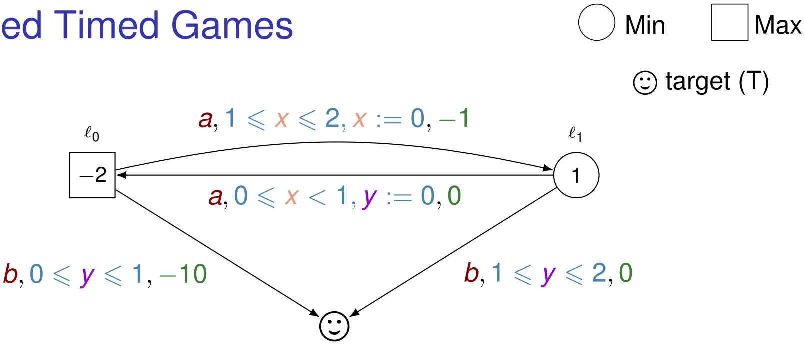
Deterministic strategy

Choose an edge and a delay

Weighted Timed Games



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Deterministic strategy

Choose an edge and a delay

In $(\ell_1, (0, 0))$

Choose a with $t = \frac{1}{3}$

Deterministic strategies: Min needs memory

σ Min

τ Max

Pseudopolynomial iterative algorithm to solve total-payoff games and min-cost reachability games, T. Brihaye, G. Geeraerts, A. Haddad and B. Monmege, 2016, Acta Informatica

Deterministic strategies: Min needs memory

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Deterministic value

$$\mathsf{dVal}(c) = \inf_{\sigma} \sup_{\tau} \mathbf{SP}(\mathsf{Play}(c, \sigma, \tau))$$

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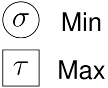

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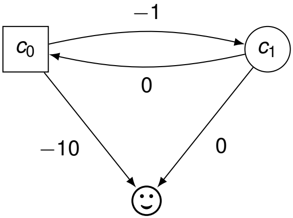

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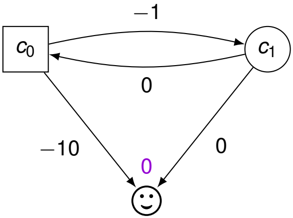

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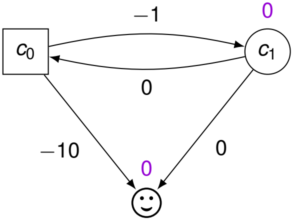

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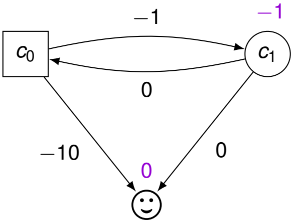

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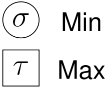

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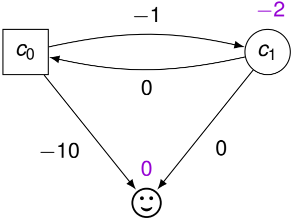

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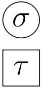

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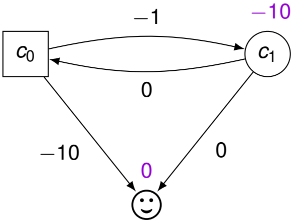

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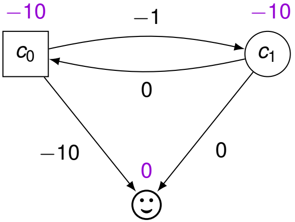

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Deterministic strategies: Min needs memory

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τ Max

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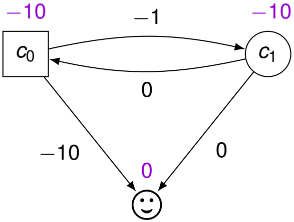
Optimal strategy

$$\mathsf{dVal}^\sigma(c) \leqslant \mathsf{dVal}(c)$$



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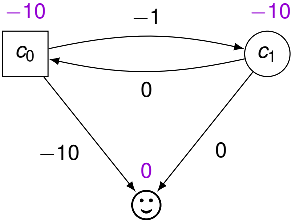

Optimal strategy for Min

An optimal strategy for Min may require finite memory.

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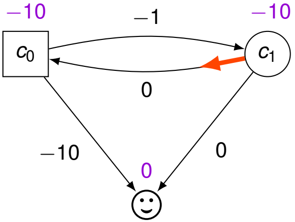

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$\bigcirc\!\!\!\!\sigma$ Min

$\boxed{\tau}$ Max

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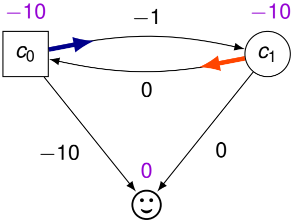

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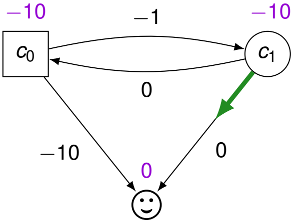

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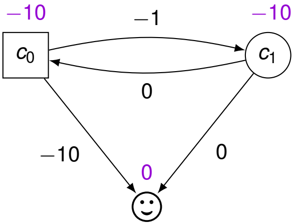

Optimal strategy for Min

Switching strategy:

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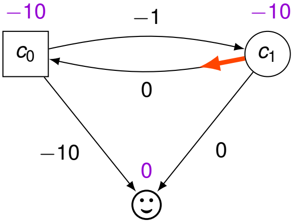

Optimal strategy for Min

Switching strategy:

- σ_1: reach cycle with a weight $\leqslant -1$

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Optimal strategy

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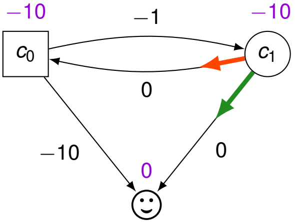

Optimal strategy for Min

Switching strategy:

- σ_1: reach cycle with a weight $\leqslant -1$
- σ_2: reach ☺

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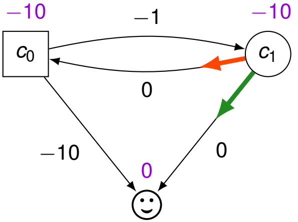

Optimal strategy for Min

Switching strategy:

- σ_1: reach cycle with a weight $\leqslant -1$
- σ_2: reach ☺
- K: number of turns before switch

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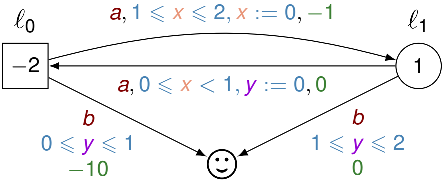
Stochastic strategies



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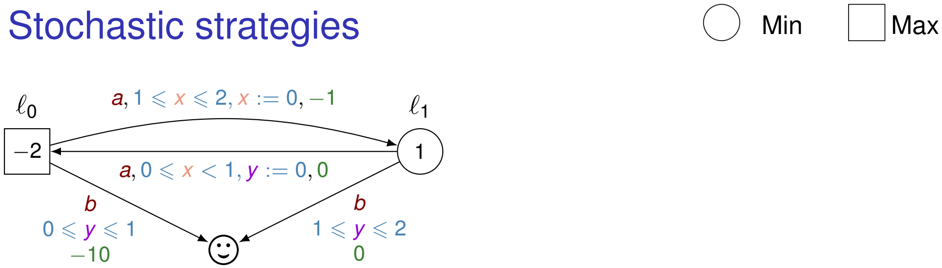
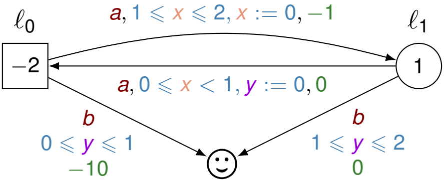


Stochastic strategy

Distribution over possible choices

Stochastic Timed Automata, N. Bertrand, P. Bouyer, T. Brihaye, Q. Menet, C. Baier, M. Grosser, and M. Jurdzinzki, 2014, Logical Methods in Computer Science

Stochastic strategies



Stochastic strategy

Distribution over possible choices

1. Edge a: finite distribution

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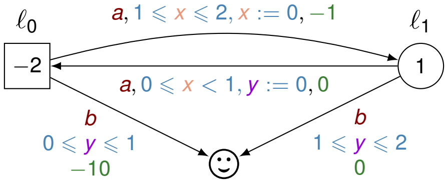

Stochastic strategy

Distribution over possible choices

1. Edge a: finite distribution
2. Delay for a: infinite distribution

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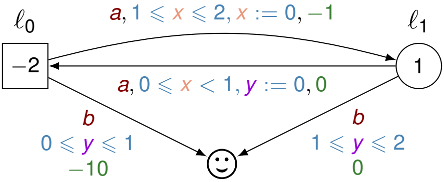
Stochastic strategies

Min   Max



In $(\ell_1, [0, 0])$

Choose between a or b with $\mathcal{B}(\frac{1}{2})$

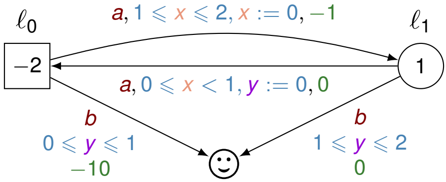
Stochastic strategy

Distribution over possible choices

1. Edge a: finite distribution
2. Delay for a: infinite distribution

Stochastic Timed Automata, N. Bertrand, P. Bouyer, T. Brihaye, Q. Menet, C. Baier, M. Grosser, and M. Jurdzinzki, 2014, Logical Methods in Computer Science

Stochastic strategies



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- a: choose t with $\mathcal{U}([0, 1))$

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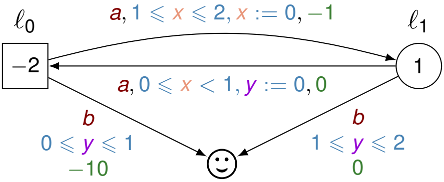
Stochastic strategies

○ Min □ Max



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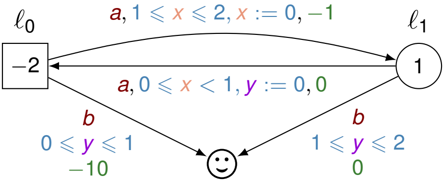
Stochastic strategies

η Min $\quad$ θ Max



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Stochastic strategy

Distribution over possible choices

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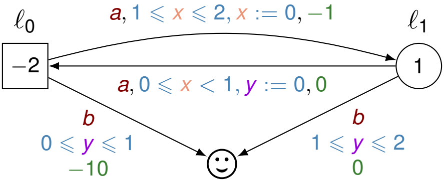
2. Delay for a: infinite distribution

When we fix two strategies

Stochastic strategies

η Min θ Max



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Stochastic strategy

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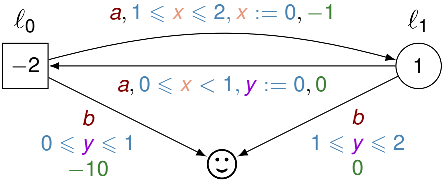
When we fix two strategies

- Infinite Markov Chain

Stochastic strategies

η Min θ Max



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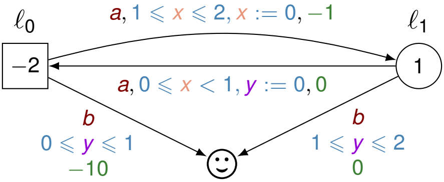
When we fix two strategies

- Infinite Markov Chain
- Replace **SP**(Play(c, η, θ)) by $\mathbb{E}_c^{\eta,\theta}$(**SP**)

Stochastic strategies

η Min θ Max



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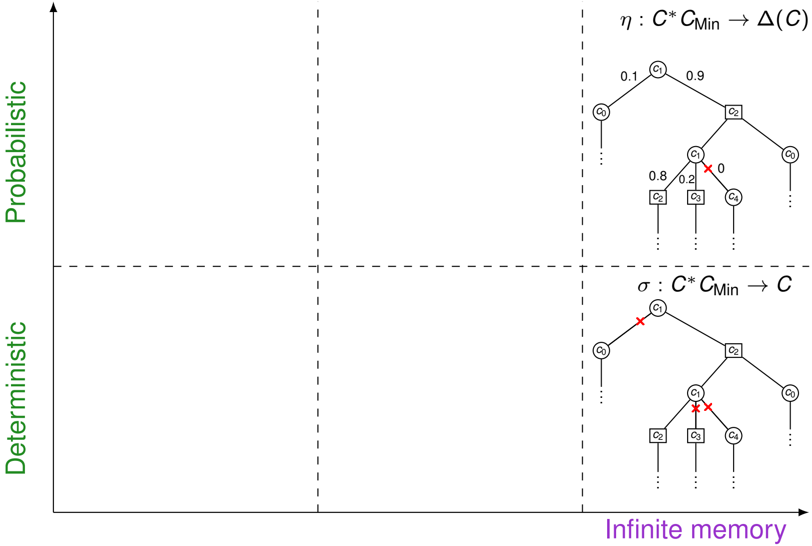
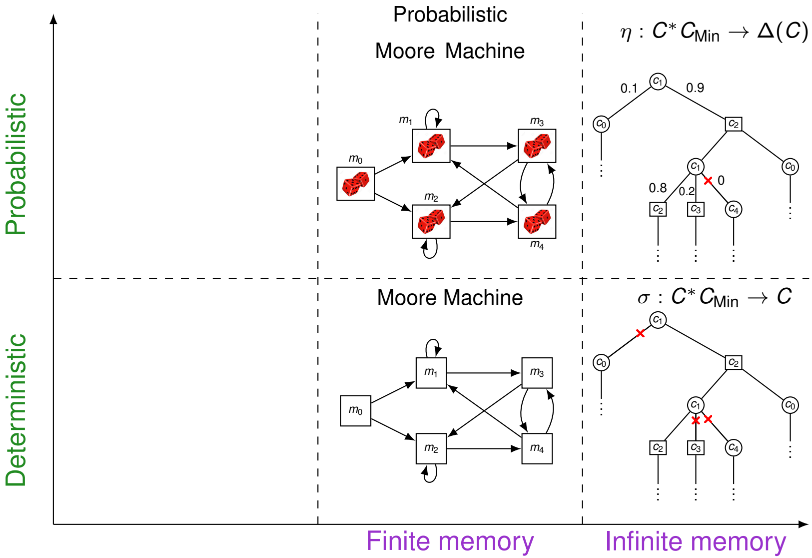
⚠ Measurability conditions on η and θ

Zoology of strategies



Zoology of strategies



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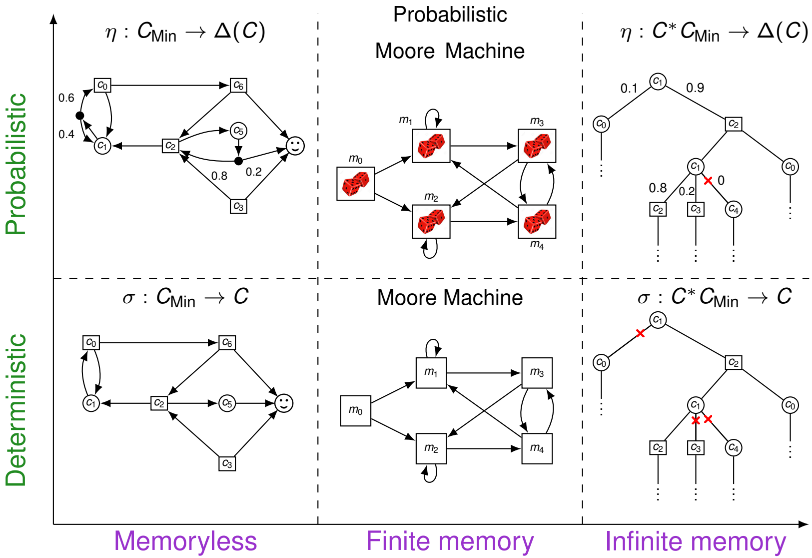
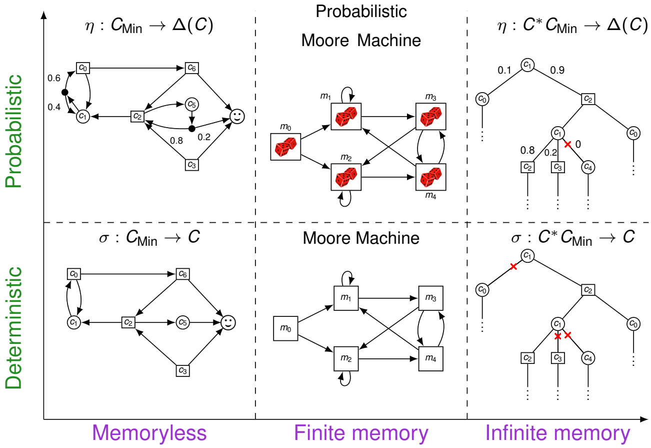
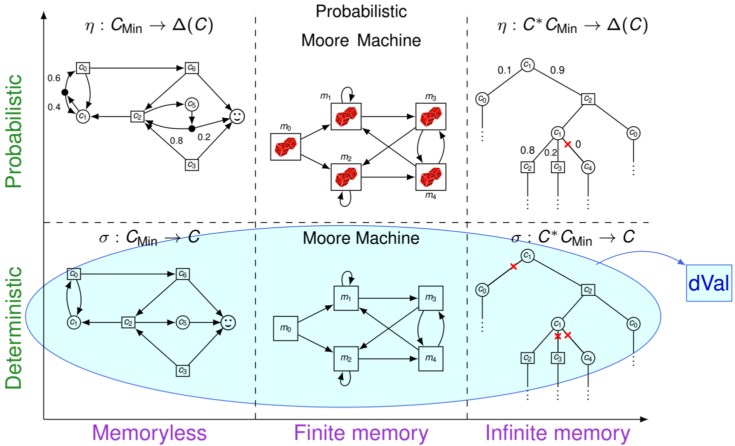


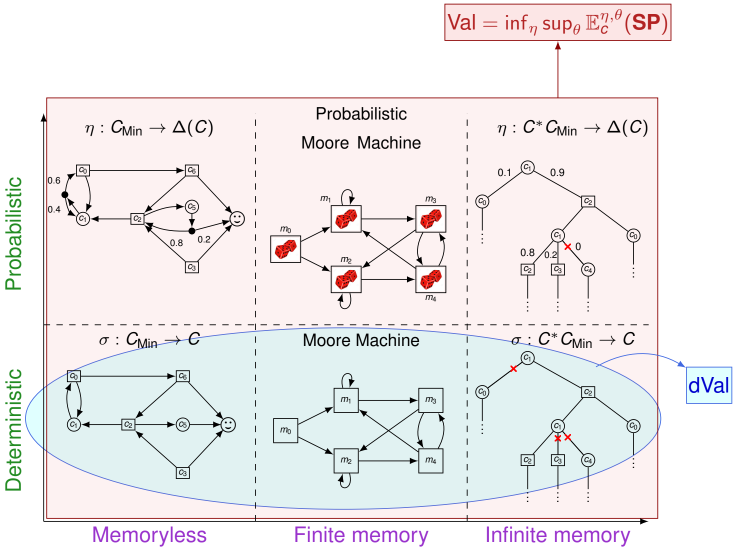
Stochastic values



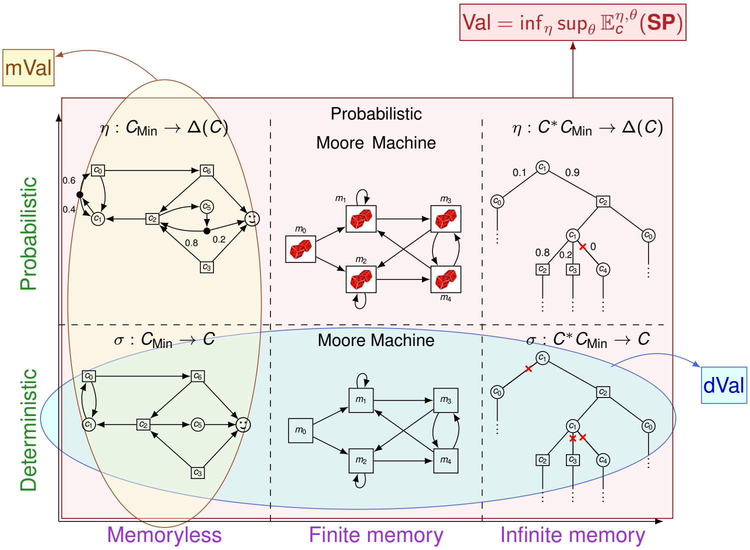
Stochastic values



Stochastic values



Stochastic values



Contribution

$$\mathrm{dVal} \quad = \quad \mathrm{Val} \quad = \quad \mathrm{mVal}$$

Contribution

Trade-off between memory and randomness

$$\text{dVal} \quad = \quad \text{Val} \quad = \quad \text{mVal}$$

Contribution

Trade-off between memory and randomness

- Stochastic games with qualitative objectives

$$\mathrm{dVal} \quad = \quad \mathrm{Val} \quad = \quad \mathrm{mVal}$$

Trading Memory for Randomness, K. Chatterjee, L. Alfaro and T. Henzinger, 2004, QEST

Contribution

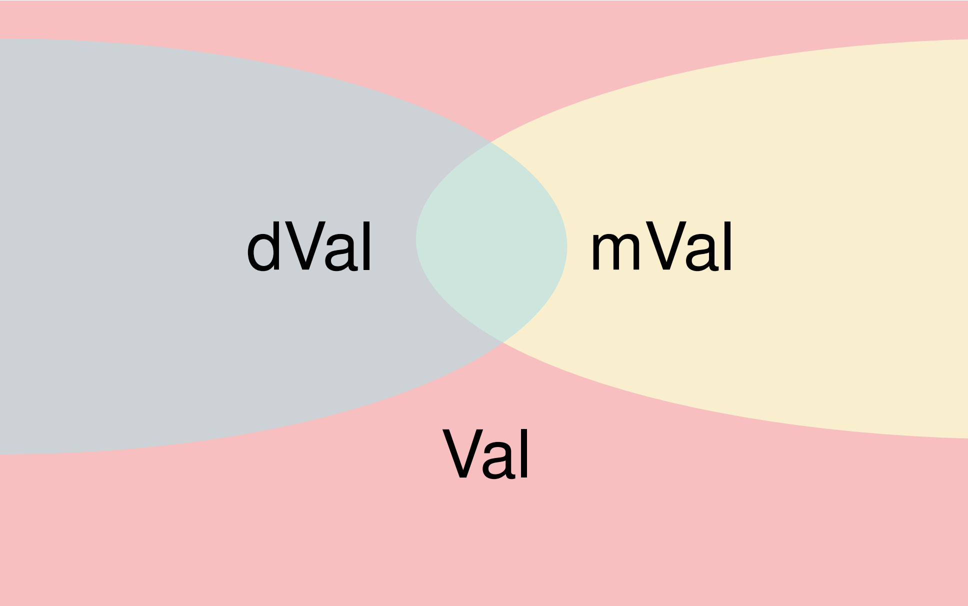
Trade-off between memory and randomness

- Stochastic games with qualitative objectives
- Reachability Timed Games

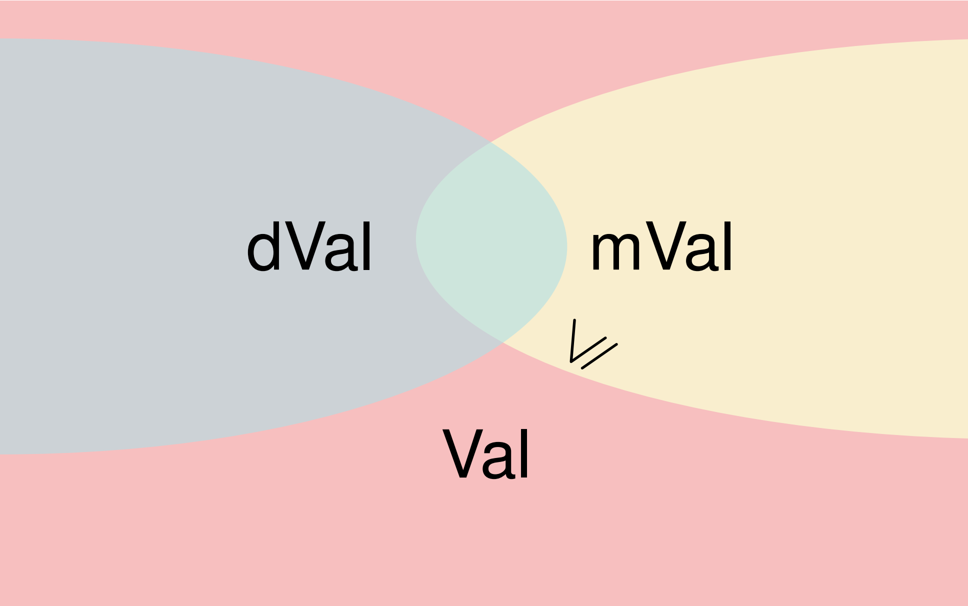
$$\text{dVal} \quad = \quad \text{Val} \quad = \quad \text{mVal}$$

Trading Infinite Memory for Uniform Randomness in Timed Games, K. Chatterjee, T. Henzinger and S. Vinayak, 2008, HSCC

Contribution



Contribution



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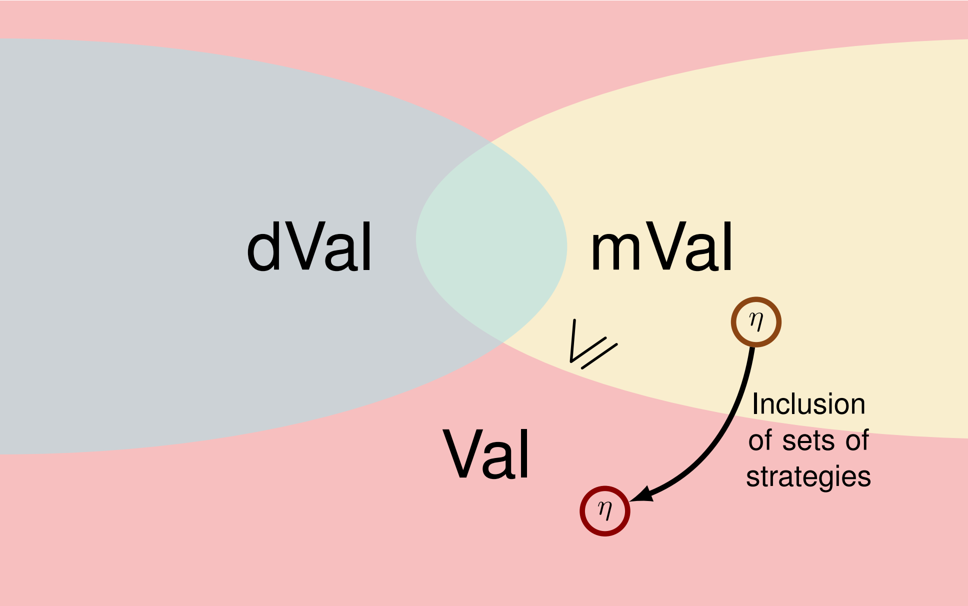

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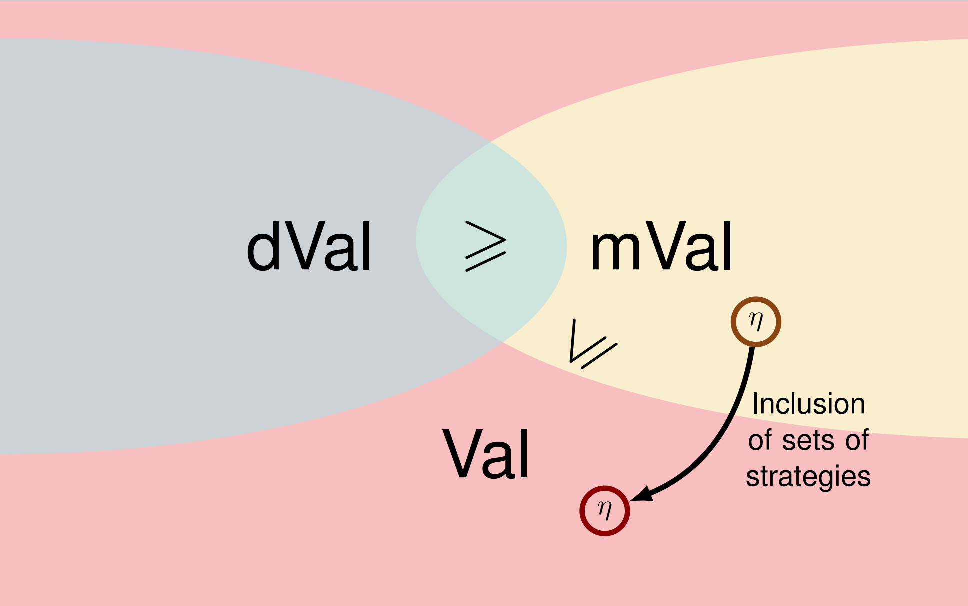

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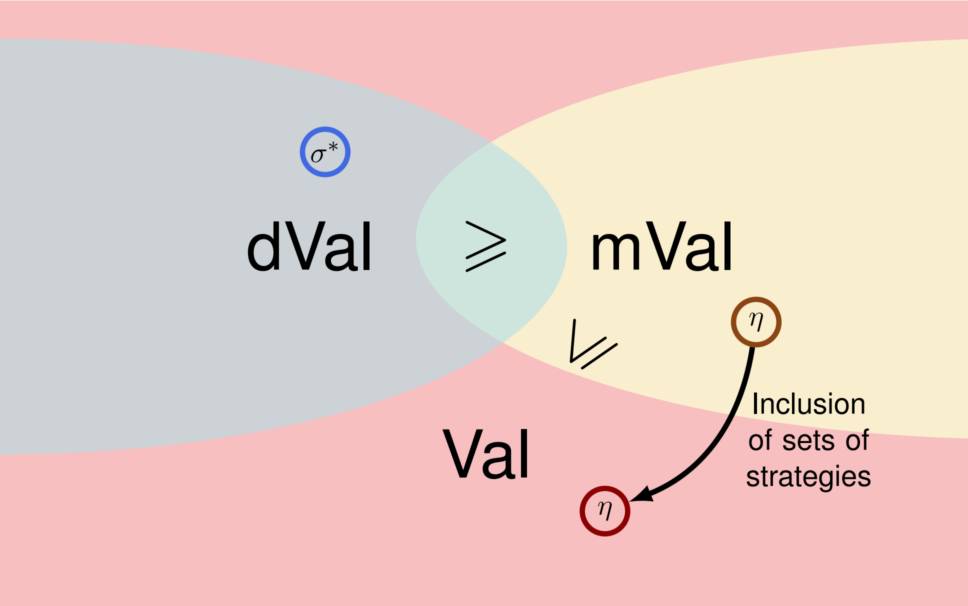

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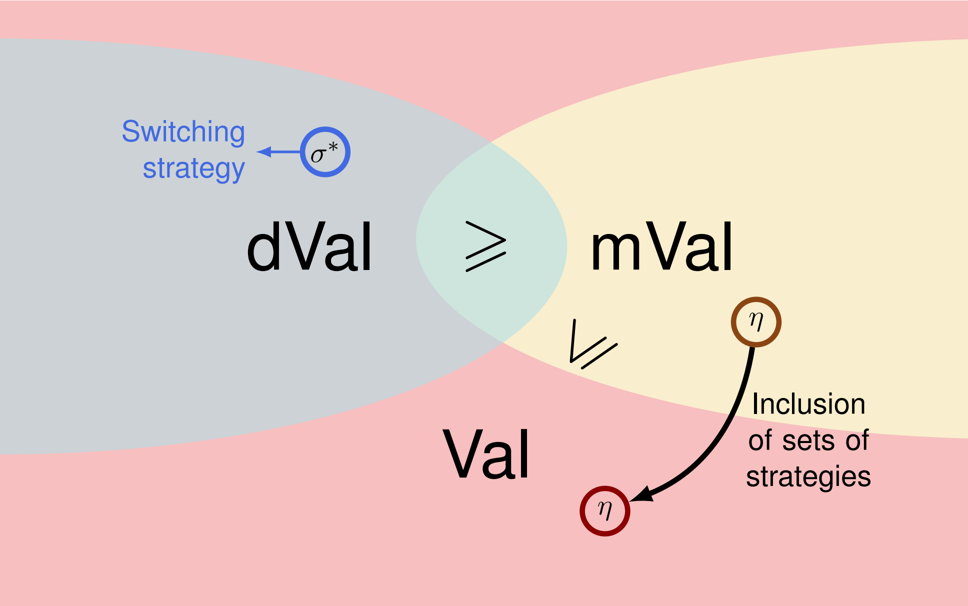

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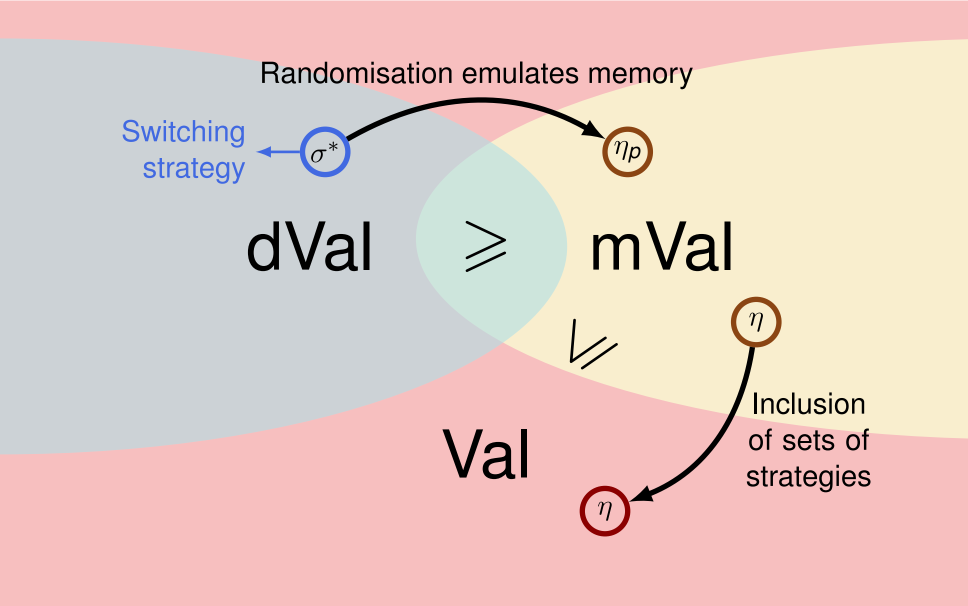

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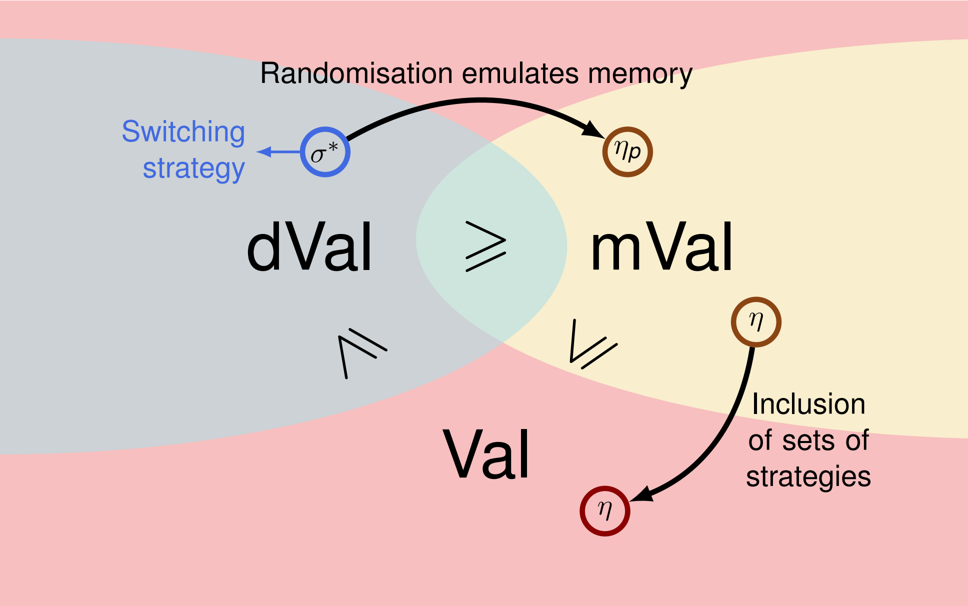

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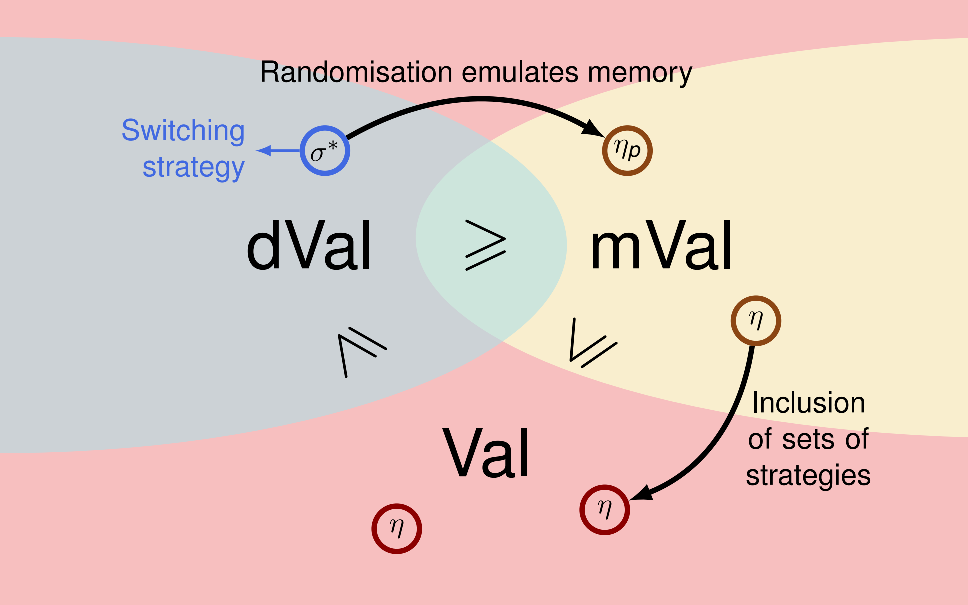

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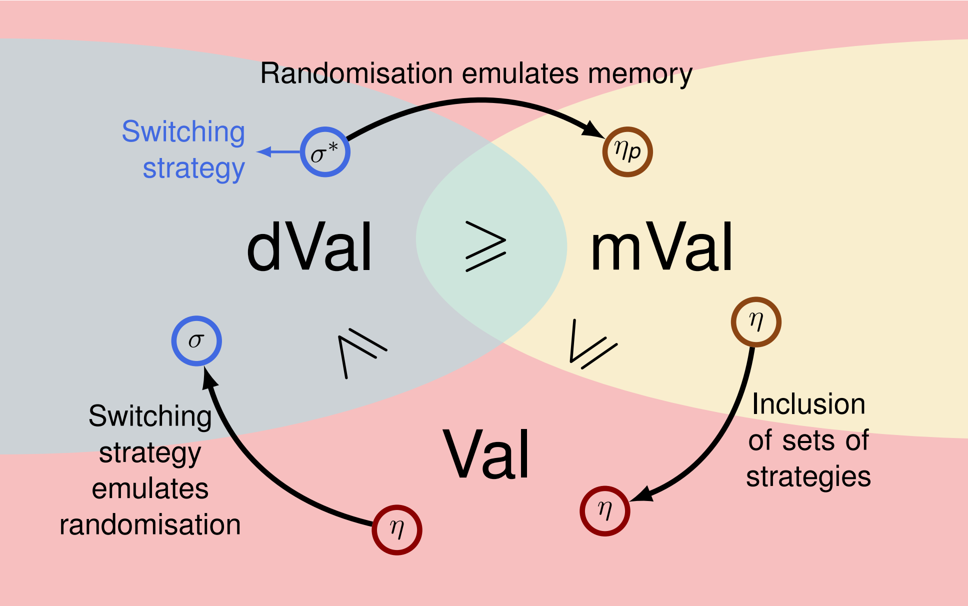

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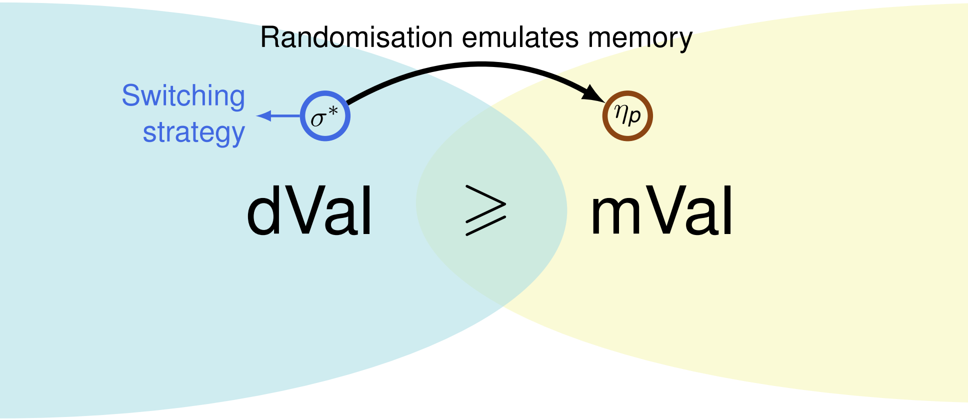
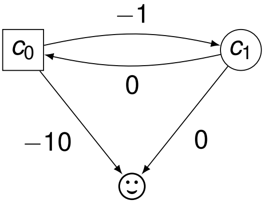


Randomisation emulates memory

Min Max



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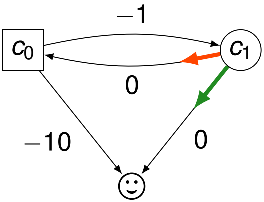
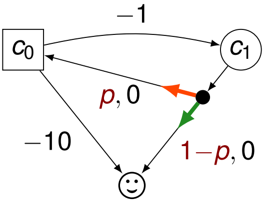


Strategy η_p

Let $\langle \sigma_1, \sigma_2, K \rangle$ be an optimal switching strategy,

Randomisation emulates memory



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Let $\langle \sigma_1, \sigma_2, K \rangle$ be an optimal switching strategy, $\forall p \in (0, 1)$,

$$\eta_p = p \times \sigma_1 + (1-p) \times \sigma_2$$

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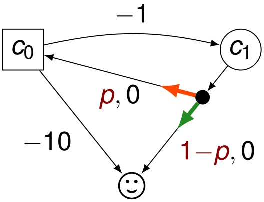

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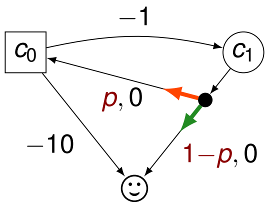
Properties of η_p

- For all θ, $\mathbb{P}_c^{\eta_p,\theta}(\diamond ☺) = 1$

Randomisation emulates memory

Min Max



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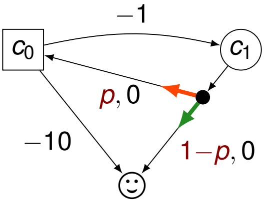
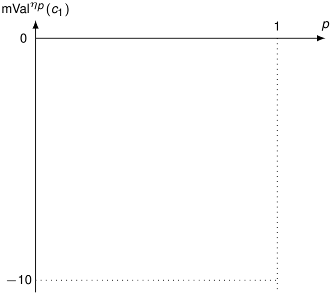
- Max has a best response deterministic memoryless strategy: τ

Randomisation emulates memory



Computation of $\mathrm{mVal}^{\eta_p}(c_1)$



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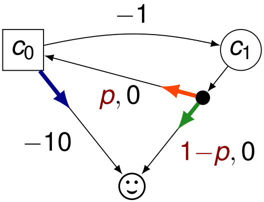

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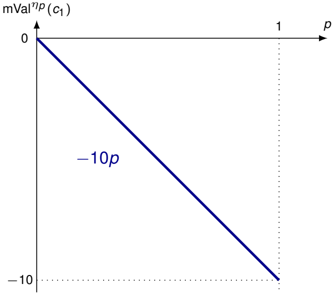

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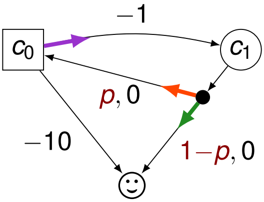

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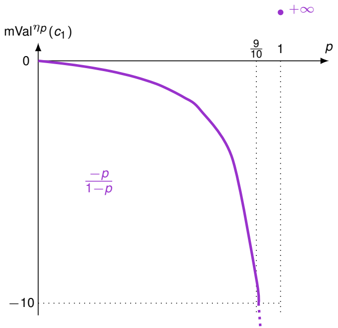

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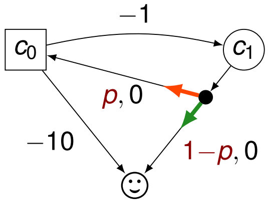

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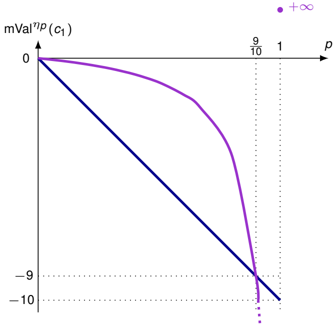

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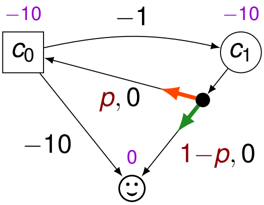

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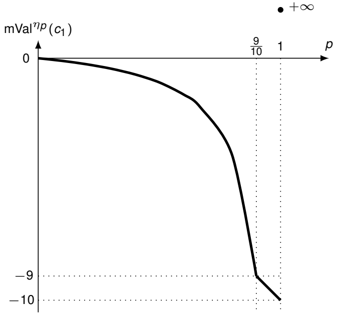

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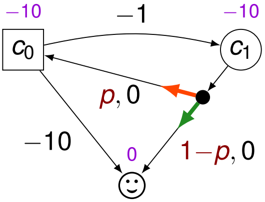

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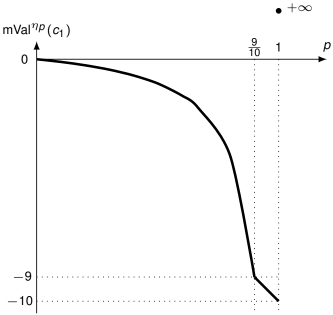

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Claim

For all c,

$$\lim_{\substack{p \to 1 \\ p < 1}} \mathbb{E}_c^{\eta_p, \tau}(\mathbf{SP}) \leqslant \mathrm{dVal}^{\langle \sigma_1, \sigma_2, K \rangle}(c)$$

Properties of η_p

- For all θ, $\mathbb{P}_c^{\eta_p, \theta}(\diamond ☺) = 1$
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Computation of the expectation $\mathbb{E}_c^{\eta_p,\tau}(\mathbf{SP})$

$$\mathbb{E}_c^{\eta_p,\tau}(\mathbf{SP}) = \sum_{\substack{\rho \\ \rho \models \diamond \smiley}} \mathbf{SP}(\rho)\,\mathbb{P}(\rho)$$

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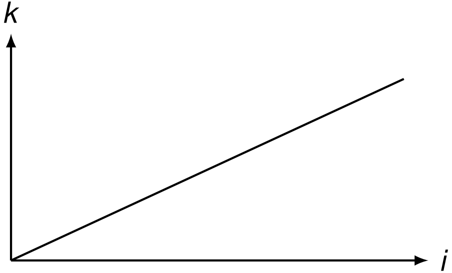
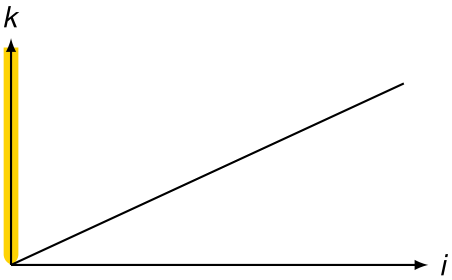


k size of play reaching the target

i number of choices given by σ_2

Computation of the expectation $\mathbb{E}_c^{\eta_p,\tau}(\mathbf{SP})$

$$\mathbb{E}_c^{\eta_p,\tau}(\mathbf{SP}) = \sum_{\substack{\rho \\ \rho \models \diamond ☺}} \mathbf{SP}(\rho)\,\mathbb{P}(\rho) = \mathbb{E} + \quad +$$



Yellow zone

All plays conforming to σ_1

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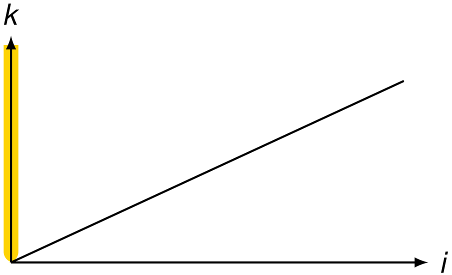

Yellow zone

All plays conforming to σ_1

$\mathbf{SP}(\rho) \leqslant \mathrm{dVal}^{\langle \sigma_1, \sigma_2, K \rangle}(c)$

k size of play reaching the target

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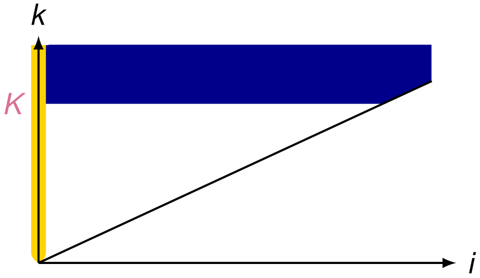
$$\mathbb{E}_c^{\eta_p,\tau}(\mathbf{SP}) = \sum_{\substack{\rho \\ \rho \models \diamond \smiley}} \mathbf{SP}(\rho)\,\mathbb{P}(\rho) = \mathbb{E} + \mathbb{E} +$$



Yellow zone

All plays conforming to σ_1

$\mathbf{SP}(\rho) \leqslant \mathrm{dVal}^{\langle \sigma_1, \sigma_2, K \rangle}(c)$

Blue zone

Plays with many negative cycles

k size of play reaching the target

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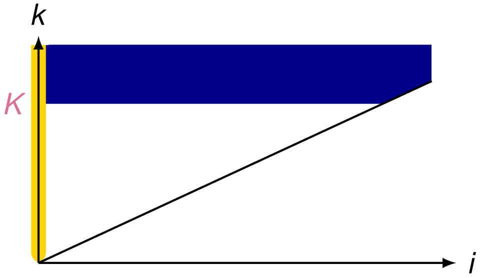

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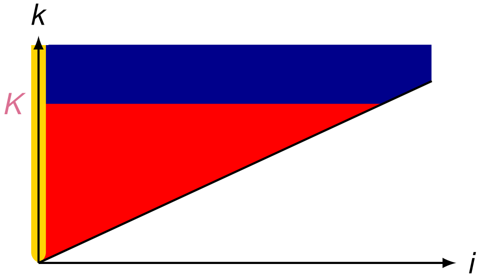

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Red zone

Rest of plays

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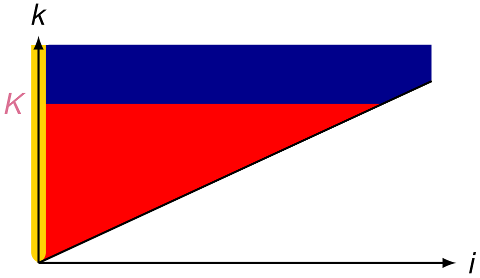

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Rest of plays $\quad \mathbb{E} \underset{\substack{p \to 1 \\ p < 1}}{\longrightarrow} 0$

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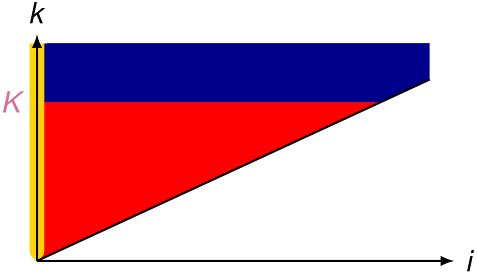

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$\mathbf{SP}(\rho) \leqslant \mathrm{dVal}^{\langle \sigma_1, \sigma_2, K \rangle}(c)$

$$\lim_{\substack{p \to 1 \\ p < 1}} \mathbb{E} + \mathbb{E} \leqslant \mathrm{dVal}^{\langle \sigma_1, \sigma_2 K \rangle}(c)$$

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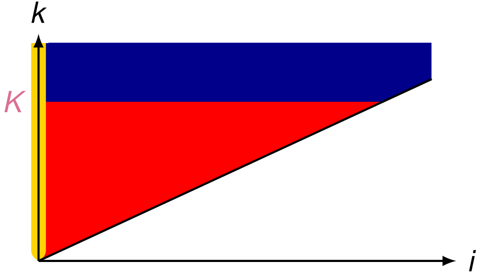

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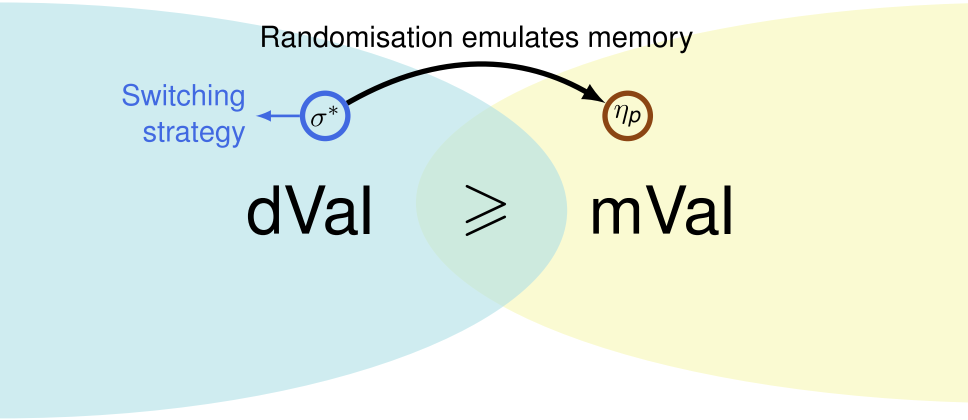
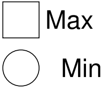
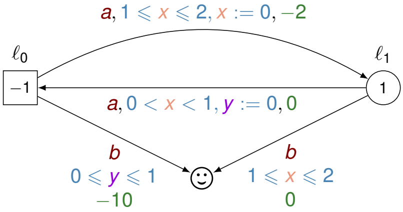
Contribution



Existence of an ε-optimal switching strategy





Switching strategy

- σ_1: reach cycle with a weight $\leqslant -1$
- σ_2: reach ☺
- K: number of turns before switch

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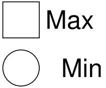

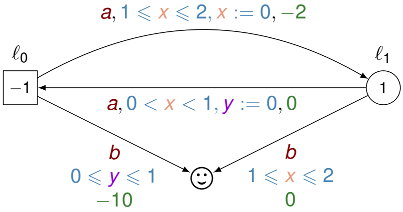

Divergent weighted timed game

All SCCs contain only cycles with a weight $\leqslant -1$ or $\geqslant 1$

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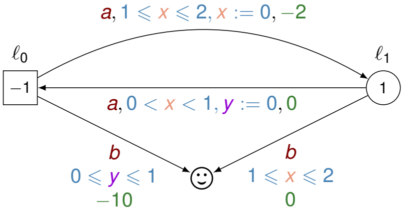
Optimal Reachability in Divergent Weighted Timed Games, D. Busatto-Gaston, B. Monmege and P.-A. Reynier, 2017, ETAPS

Existence of an ε-optimal switching strategy

□ Max

○ Min



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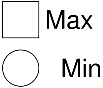
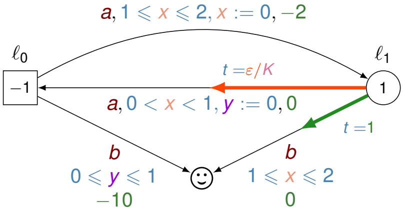
Theorem

Min has an ε-optimal switching strategy

Existence of an ε-optimal switching strategy





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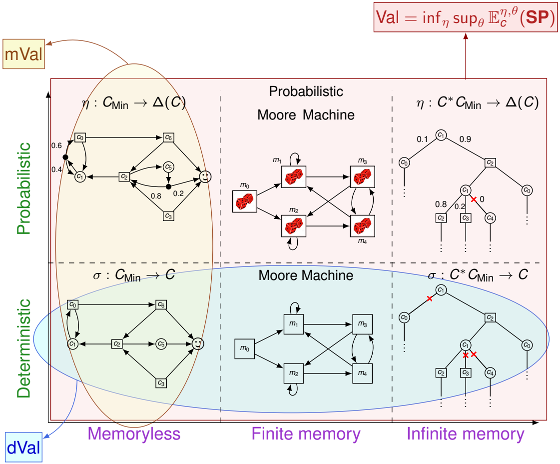
Min has an ε-optimal switching strategy : $\langle \sigma_1, \sigma_2, K \rangle$

Summary



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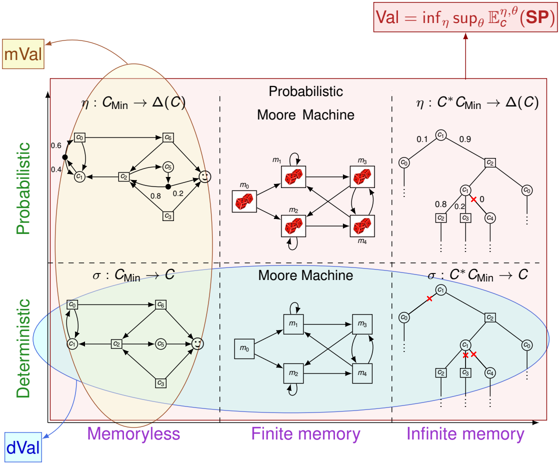
Summary



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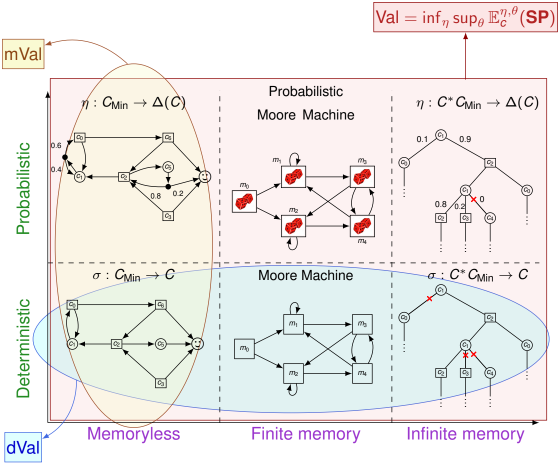
Summary



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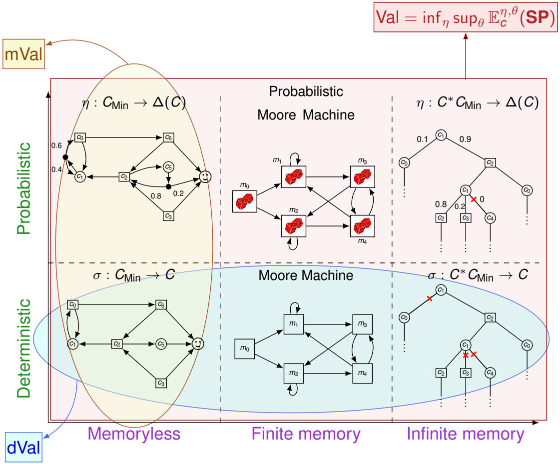
For which classes of games?

- Finite shortest path games

Theorem: $\text{Val} = \text{dVal} = \text{mVal}$

Summary



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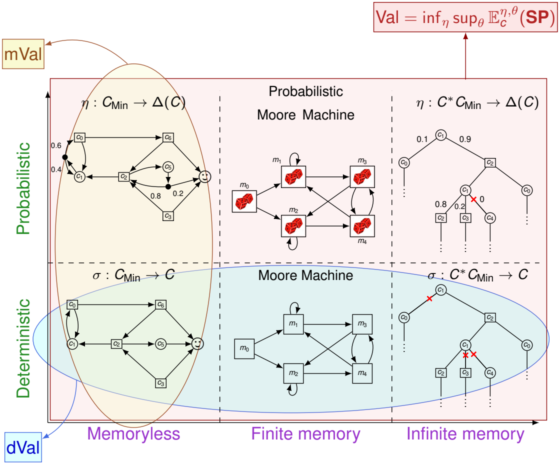

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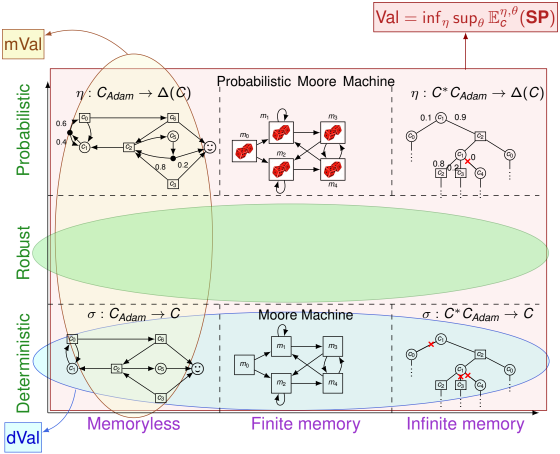

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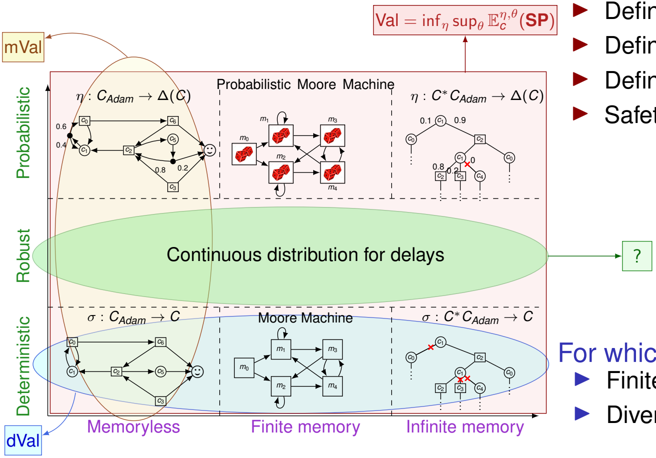
Trading Infinite Memory for Uniform Randomness in Timed Games, K. Chatterjee, T. Henzinger and S. Vinayak, 2008, HSCC

Summary: perspectives



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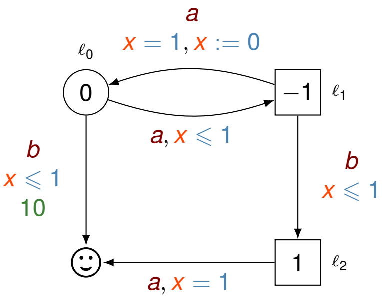
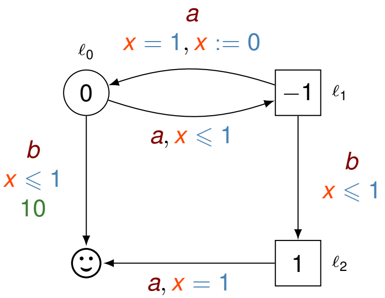
Value problem in 1-clock Weighted Timed Games



Value problem in 1-clock Weighted Timed Games

Value Problem

Decide if $\mathrm{dVal}(c) \leqslant \lambda$?



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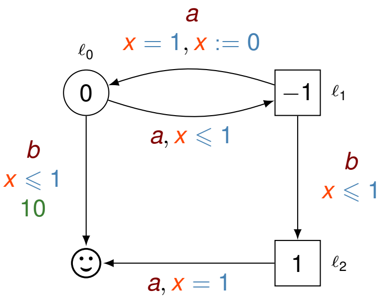
State of the art



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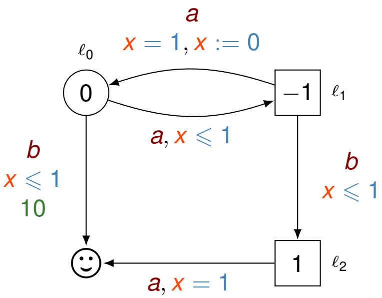

State of the art

☹ Undecidable for at least two clocks

On Optimal Timed Strategies, T. Brihaye, V. Bruyère and J.-F. Raskin, 2005, FORMATS

On the Value Problem in Weighted Timed Games, P. Bouyer, S. Jaziri, and N. Markey, 2015, CONCUR.

Value problem in 1-clock Weighted Timed Games

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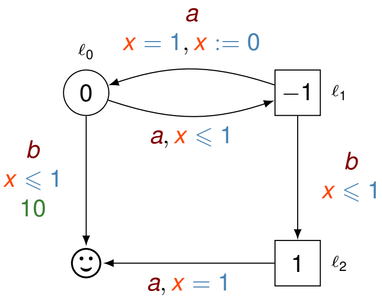

State of the art

- ☹ Undecidable for at least two clocks
- ☺ Decidable for 1-clock with non-negative weights

Almost Optimal Strategies in One Clock Priced Timed Games, P. Bouyer, K. Larsen, N. Markey, and J. Rasmussen, 2006, FSTTCS

Value problem in 1-clock Weighted Timed Games

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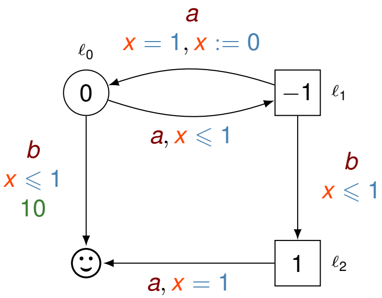

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Simple Priced Timed Games Are Not That Simple, T. Brihaye, G. Geeraerts, A. Haddad, E. Lefaucheux, and B. Monmege, 2015, FSTTCS

Value problem in 1-clock Weighted Timed Games

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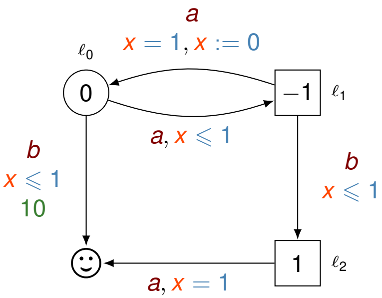

State of the art

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- ☺ A lower-bound: PSPACE-hard

One-clock priced timed games are PSPACE-hard., J. Fearnley, R. Ibsen-Jensen, and R. Savani, 2020, LICS

Value problem in 1-clock Weighted Timed Games

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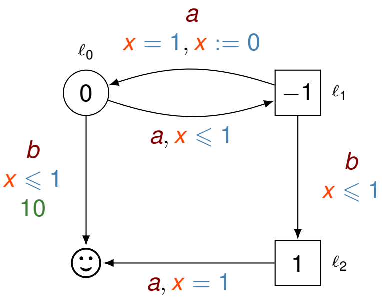

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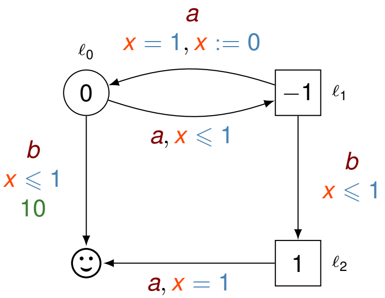

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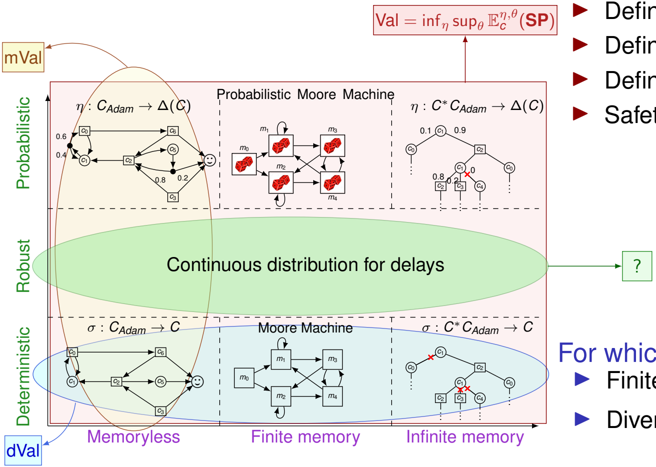
Decidable in 1-clock Weighted Timed Games

$c \mapsto \mathrm{dVal}(c)$ is computable in exponential time

Summary : perspectives



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Theorem: $\text{Val} = \text{dVal} = \text{mVal}$

Thank you! Questions?

Existence of the expectation: $\mathbb{E}_c^{\eta,\theta}(\mathbf{SP})$

η Min $\quad$ $\boxed{\theta}$ Max

Existence of the expectation: $\mathbb{E}_c^{\eta,\theta}(\mathbf{SP})$

η Min θ Max

$\eta, \theta : C^*C \to \Delta(C)$

Distribution over possible choices

1. Edge a: finite distribution $\eta_E(c)$
2. Delay for a: infinite distribution: $\eta_{\mathbb{R}^+}(c, a)$

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Probability of a path

$$\mathbb{P}_c^{\eta,\theta}(a\,\pi) = \int_{t \in I(c,a)} \eta_E(c)(a)\, \mathbb{P}_{c_1}^{\eta,\theta}(\pi)\, \mathrm{d}\eta_{\mathbb{R}^+}(c, a)(t)$$

Stochastic Timed Automata, N. Bertrand, P. Bouyer, T. Brihaye, Q. Menet, C. Baier, M. Grosser, and M. Jurdzinzki, 2014, Logical Methods in Computer Science

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⚠ Measurability conditions on η and θ

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Probability of a path

$$\mathbb{P}_c^{\eta,\theta}(a\,\pi) = \int_{t \in I(c,a)} \eta_E(c)(a)\, \mathbb{P}_{c_1}^{\eta,\theta}(\pi)\, \mathrm{d}\eta_{\mathbb{R}^+}(c,a)(t)$$

Expectation of **SP** in a path

$$\mathbb{E}_c^{\eta,\theta}(a\,\pi) = \int_{t \in I(c,a)} \eta_E(c)(a) \left[(t\,\mathrm{wt}(c) + \mathrm{wt}(a))\, \mathbb{P}_{c_1}^{\eta,\theta}(\pi) + \mathbb{E}_{c_1}^{\eta,\theta}(\pi) \right] \mathrm{d}\eta_{\mathbb{R}^+}(c,a)(t)$$

Existence of the expectation: $\mathbb{E}_c^{\eta,\theta}(\mathbf{SP})$

η Min θ Max

$\eta, \theta : C^* C \to \Delta(C)$

Distribution over possible choices

1. Edge a: finite distribution $\eta_E(c)$
2. Delay for a: infinite distribution: $\eta_{\mathbb{R}^+}(c, a)$

⚠ Measurability conditions on η and θ

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Restrictions on strategies for Min

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Restrictions on strategies for Min

- For all θ, $\mathbb{P}_c^{\eta,\theta}(\diamond☺) = 1$

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$$\mathbb{E}_c^{\eta,\theta}(\mathbf{SP}) = \sum_{\substack{\pi \\ \pi \models \diamond \smile}} \mathbb{E}_c^{\eta,\theta}(\pi)$$

Convergence ?

Restrictions on strategies for Min

- For all θ, $\mathbb{P}_c^{\eta,\theta}(\diamond \smile) = 1$

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$|\mathbb{E}_c^{\eta,\theta}(\pi)|$

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$$|\mathbb{E}_c^{\eta,\theta}(\pi)| \leqslant k|\pi| \, \alpha^{-|\pi|}$$

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$$|\mathbb{E}_c^{\eta,\theta}(\pi)| \leqslant \underbrace{k|\pi|}_{|\mathbf{SP}(\pi)|} \ \alpha^{-|\pi|}$$

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- For all θ, $\mathbb{P}_c^{\eta,\theta}(\diamond ☺) = 1$

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Restrictions on strategies for Min

- For all θ, $\mathbb{P}_c^{\eta,\theta}(\diamond ☺) = 1$
- ☺ must be reached quickly enough