Weighted Timed Games: Decidability, Randomisation and Robustness

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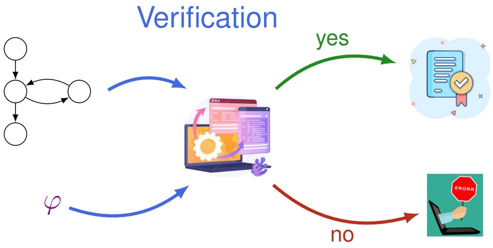
University of Warsaw

Séminaire M2F

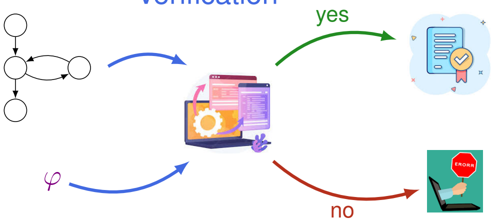
Joint work with Benjamin Monmege and Pierre–Alain Reynier

Correctness and performance of real-time systems



Correctness and performance of real-time systems

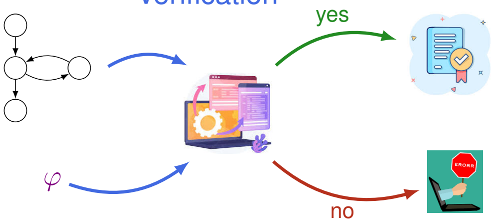
Verification



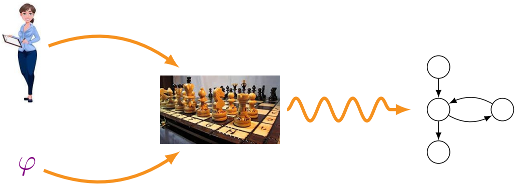
Synthesis

Correctness and performance of real-time systems

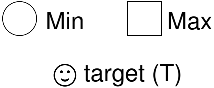
Verification

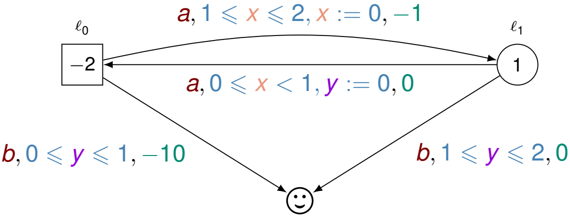


Synthesis

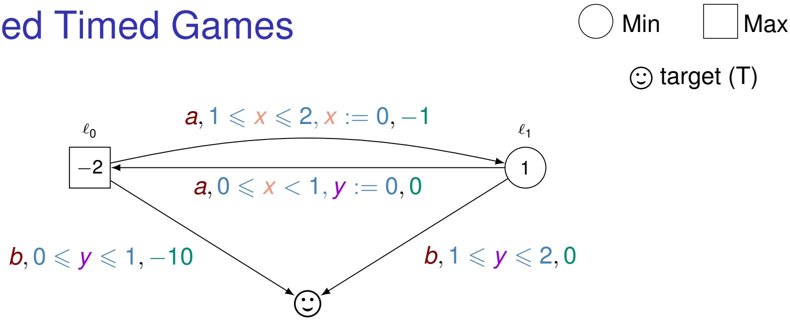


Weighted Timed Games



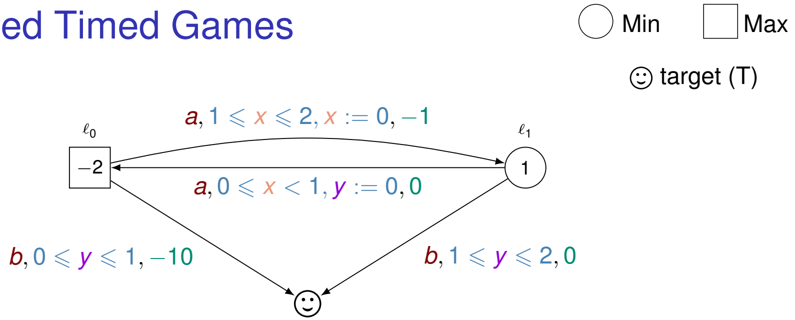


Weighted Timed Games



Play ρ $\left(\ell_1, \begin{bmatrix} x \mapsto 0 \\ y \mapsto 0 \end{bmatrix}\right)$

Weighted Timed Games

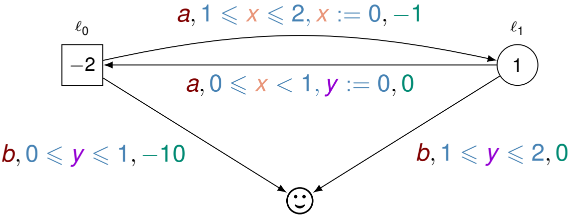


Play ρ $\quad (\ell_1, \begin{bmatrix} 0 \\ 0 \end{bmatrix})$

Weighted Timed Games

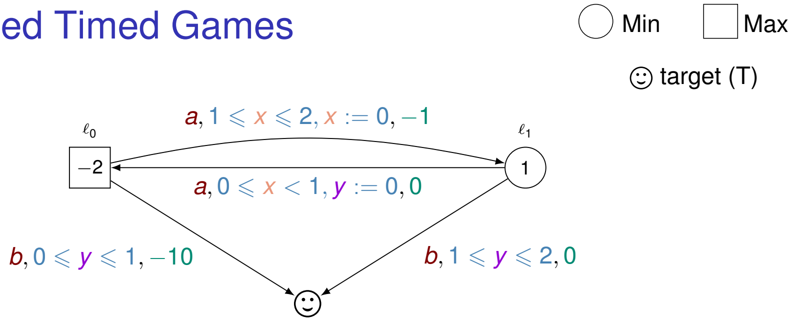
○ Min □ Max

☺ target (T)



Play ρ $\quad (\ell_1, \begin{bmatrix} 0 \\ 0 \end{bmatrix}) \xrightarrow{0.5,\ a}$

Weighted Timed Games

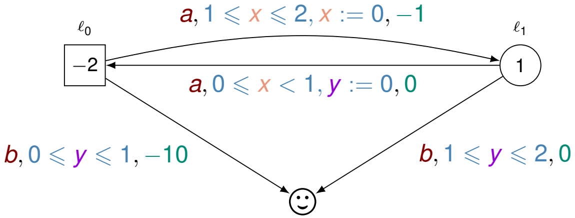


Play ρ $\quad (\ell_1, \begin{bmatrix} 0 \\ 0 \end{bmatrix}) \xrightarrow{0.5,\ a} (\ell_0, \begin{bmatrix} 0.5 \\ 0 \end{bmatrix})$

Weighted Timed Games

○ Min ☐ Max

☺ target (T)

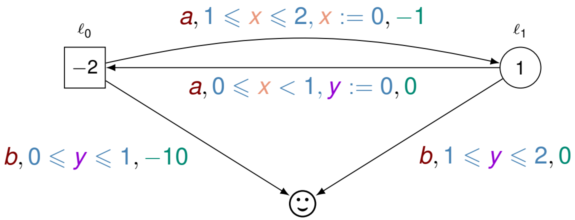


Play ρ $\quad (\ell_1, \begin{bmatrix} 0 \\ 0 \end{bmatrix}) \xrightarrow{0.5,\ a} (\ell_0, \begin{bmatrix} 0.5 \\ 0 \end{bmatrix}) \xrightarrow{1.25,\ a} (\ell_1, \begin{bmatrix} 0 \\ 1.25 \end{bmatrix}) \xrightarrow{1/3,\ b} (☺, \begin{bmatrix} 1/3 \\ 19/12 \end{bmatrix})$

Weighted Timed Games

○ Min □ Max

☺ target (T)



Play ρ

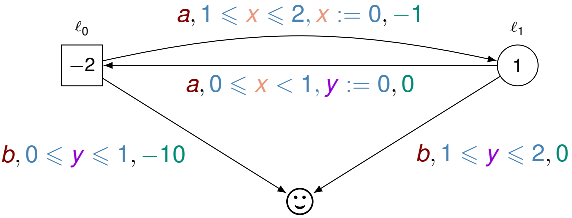
$$(\ell_1, \begin{bmatrix} 0 \\ 0 \end{bmatrix}) \xrightarrow{0.5,\ a} (\ell_0, \begin{bmatrix} 0.5 \\ 0 \end{bmatrix}) \xrightarrow{1.25,\ a} (\ell_1, \begin{bmatrix} 0 \\ 1.25 \end{bmatrix}) \xrightarrow{1/3,\ b} (☺, \begin{bmatrix} 1/3 \\ 19/12 \end{bmatrix})$$

$$+ \qquad\qquad\qquad +$$

Weighted Timed Games

○ Min □ Max

☺ target (T)



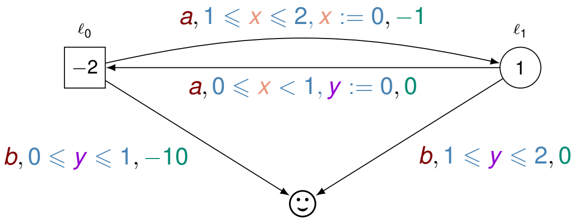
Play ρ $\quad (\ell_1, \begin{bmatrix} 0 \\ 0 \end{bmatrix}) \xrightarrow{0.5,\ a} (\ell_0, \begin{bmatrix} 0.5 \\ 0 \end{bmatrix}) \xrightarrow{1.25,\ a} (\ell_1, \begin{bmatrix} 0 \\ 1.25 \end{bmatrix}) \xrightarrow{1/3,\ b} (☺, \begin{bmatrix} 1/3 \\ 19/12 \end{bmatrix})$

$0 \quad + \quad\quad\quad +$

Weighted Timed Games

○ Min □ Max

☺ target (T)



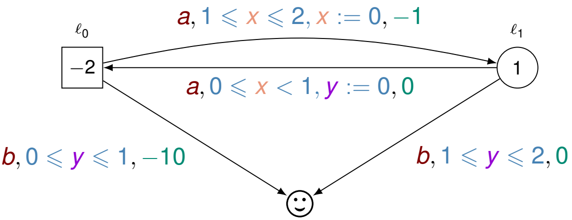
Play ρ $\quad (\ell_1, \begin{bmatrix} 0 \\ 0 \end{bmatrix}) \xrightarrow{0.5,\ a} (\ell_0, \begin{bmatrix} 0.5 \\ 0 \end{bmatrix}) \xrightarrow{1.25,\ a} (\ell_1, \begin{bmatrix} 0 \\ 1.25 \end{bmatrix}) \xrightarrow{1/3,\ b} (☺, \begin{bmatrix} 1/3 \\ 19/12 \end{bmatrix})$

$1 \times 0.5 + 0 \quad + \quad\quad +$

Weighted Timed Games

○ Min □ Max

☺ target (T)



Play ρ

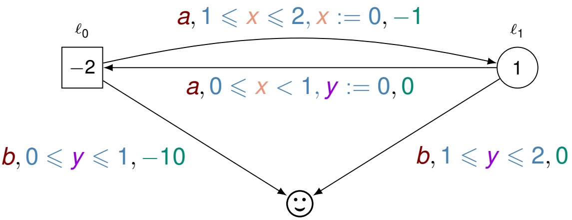
$$(\ell_1, \begin{bmatrix} 0 \\ 0 \end{bmatrix}) \xrightarrow{0.5,\ a} (\ell_0, \begin{bmatrix} 0.5 \\ 0 \end{bmatrix}) \xrightarrow{1.25,\ a} (\ell_1, \begin{bmatrix} 0 \\ 1.25 \end{bmatrix}) \xrightarrow{1/3,\ b} (☺, \begin{bmatrix} 1/3 \\ 19/12 \end{bmatrix}) \rightsquigarrow -\frac{8}{3}$$

$$1 \times 0.5 + 0 \quad + \quad -2 \times 1.25 - 1 \quad + \quad 1 \times \tfrac{1}{3} + 0$$

Weighted Timed Games

○ Min ☐ Max

☺ target (T)



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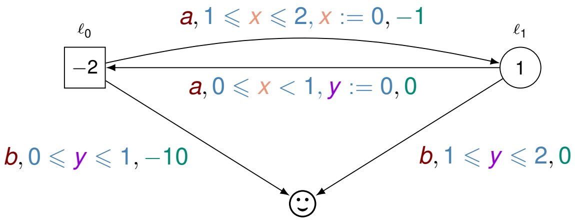
Deterministic strategy

Choose an edge and a delay

Weighted Timed Games

○ Min □ Max

☺ target (T)



Play ρ $\quad (\ell_1, \begin{bmatrix} 0 \\ 0 \end{bmatrix}) \xrightarrow{0.5,\ a} (\ell_0, \begin{bmatrix} 0.5 \\ 0 \end{bmatrix}) \xrightarrow{1.25,\ a} (\ell_1, \begin{bmatrix} 0 \\ 1.25 \end{bmatrix}) \xrightarrow{1/3,\ b} (☺, \begin{bmatrix} 1/3 \\ 19/12 \end{bmatrix})$

Deterministic strategy

Choose an edge and a delay

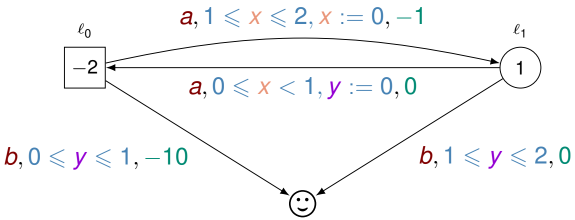
From $(\ell_1, \begin{bmatrix} 0 \\ 0 \end{bmatrix})$

Choose a with $t = \frac{1}{3}$

Weighted Timed Games

○ Min □ Max

☺ target (T)



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Deterministic strategy

Choose an edge and a delay

From $(\ell_1, \begin{bmatrix} 0 \\ 0 \end{bmatrix})$

Choose a with $t = \frac{1}{3}$

What features on strategies are needed for Min?

Features on strategies needed for Min

σ Min

τ Max

Features on strategies needed for Min

σ Min

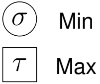
τ Max

Deterministic value

$\mathsf{dVal}(c) = \inf_{\sigma} \sup_{\tau} \mathbf{cost}(\mathsf{Play}(c, \sigma, \tau))$

Pseudopolynomial iterative algorithm to solve total-payoff games and min-cost reachability games, T. Brihaye, G. Geeraerts, A. Haddad and B. Monmege, 2016, Acta Informatica

Features on strategies needed for Min



Deterministic value

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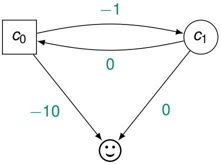

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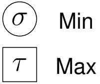
Features on strategies needed for Min



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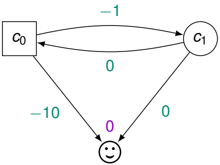

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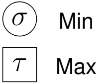

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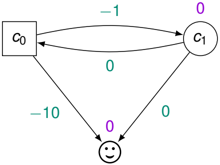

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Features on strategies needed for Min

σ Min

τ Max

Deterministic value

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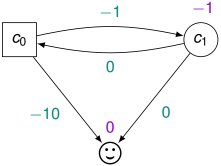

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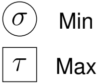

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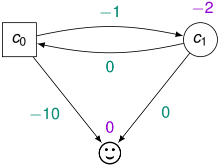

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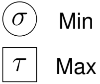

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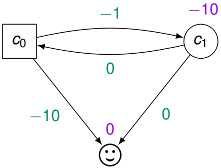

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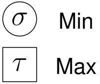

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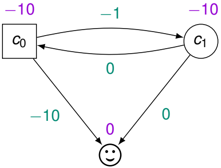

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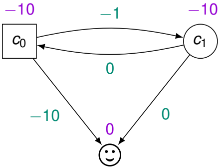
Features on strategies needed for Min

σ Min

τ Max

Deterministic value

$$\mathrm{dVal}(c) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbf{cost}(\mathrm{Play}(c, \sigma, \tau))}_{\mathrm{dVal}^{\sigma}(c)}$$



Optimal strategy for Min

$$\mathrm{dVal}^{\sigma}(c) \leqslant \mathrm{dVal}(c)$$

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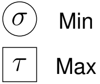
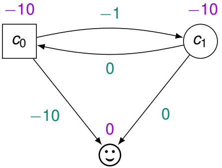


Deterministic value

$$\mathrm{dVal}(c) = \inf_\sigma \underbrace{\sup_\tau \mathbf{cost}(\mathrm{Play}(c, \sigma, \tau))}_{\mathrm{dVal}^\sigma(c)}$$



Optimal strategy for Min

$$\mathrm{dVal}^\sigma(c) \leqslant \mathrm{dVal}(c)$$

Finite memory

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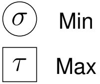
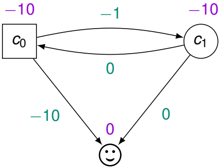
Features on strategies needed for Min



Deterministic value

$$\mathsf{dVal}(c) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbf{cost}(\mathsf{Play}(c, \sigma, \tau))}_{\mathsf{dVal}^{\sigma}(c)}$$



Optimal strategy for Min

$$\mathsf{dVal}^{\sigma}(c) \leqslant \mathsf{dVal}(c)$$

Finite memory

Switching strategy:

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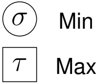
Features on strategies needed for Min



Deterministic value

$$\mathrm{dVal}(c) = \inf_\sigma \underbrace{\sup_\tau \mathbf{cost}(\mathrm{Play}(c,\sigma,\tau))}_{\mathrm{dVal}^\sigma(c)}$$

Optimal strategy for Min

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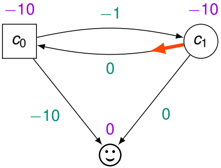

Finite memory

Switching strategy:

- σ_1: reach cycle with a weight $\leqslant -1$

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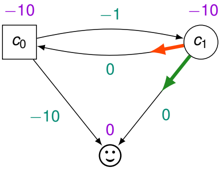
Features on strategies needed for Min

σ Min

τ Max

Deterministic value

$$\mathsf{dVal}(c) = \inf_\sigma \underbrace{\sup_\tau \mathbf{cost}(\mathsf{Play}(c,\sigma,\tau))}_{\mathsf{dVal}^\sigma(c)}$$



Optimal strategy for Min

$$\mathsf{dVal}^\sigma(c) \leqslant \mathsf{dVal}(c)$$

Finite memory

Switching strategy:

- σ_1: reach cycle with a weight $\leqslant -1$
- σ_2: reach ☺

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Features on strategies needed for Min

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Deterministic value

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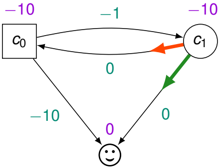

Optimal strategy for Min

$$\text{dVal}^{\sigma}(c) \leqslant \text{dVal}(c)$$

Finite memory

Switching strategy:

- σ_1: reach cycle with a weight $\leqslant -1$
- σ_2: reach ☺
- K: number of turns before switch

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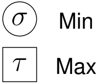

Deterministic value

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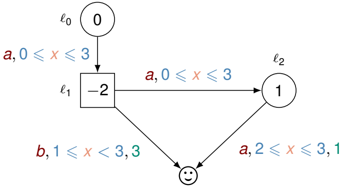

Optimal strategy for Min

$\mathsf{dVal}^\sigma(c) \leqslant \mathsf{dVal}(c)$

Finite memory

Switching strategy:

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- σ_2: reach ☺
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Features on strategies needed for Min

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Deterministic value

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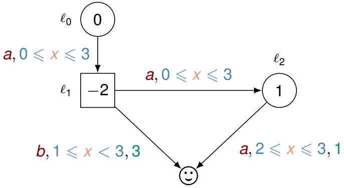
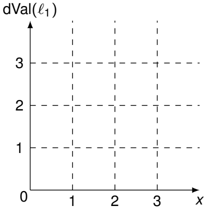


Optimal strategy for Min

$$\mathrm{dVal}^{\sigma}(c) \leqslant \mathrm{dVal}(c)$$



Finite memory

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Pseudopolynomial iterative algorithm to solve total-payoff games and min-cost reachability games, T. Brihaye, G. Geeraerts, A. Haddad and B. Monmege, 2016, Acta Informatica

Features on strategies needed for Min

σ Min

τ Max

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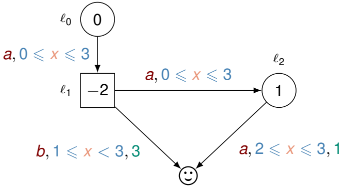

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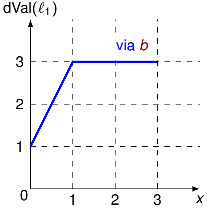

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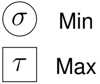

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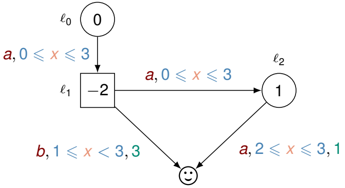

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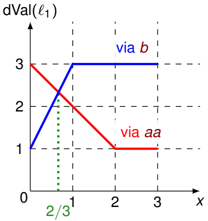

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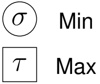

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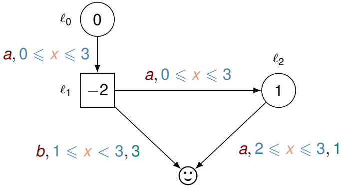

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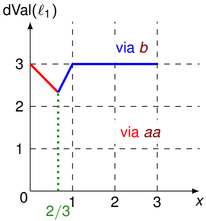

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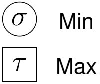

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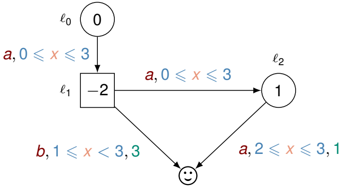

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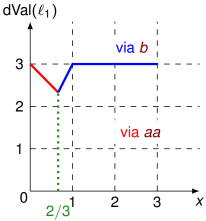

Finite memory

Switching strategy:

- σ_1: reach cycle with a weight $\leqslant -1$
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- K: number of turns before switch

Infinite precision

From ℓ_0, Min wants to reach the valuation $2/3$

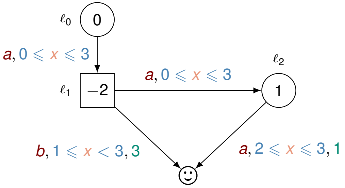
Features on strategies needed for Min

σ Min

τ Max

Deterministic value

$$\mathsf{dVal}(c) = \inf_\sigma \underbrace{\sup_\tau \mathbf{cost}(\mathsf{Play}(c,\sigma,\tau))}_{\mathsf{dVal}^\sigma(c)}$$



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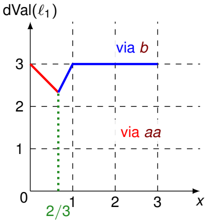

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From ℓ_0, Min wants to reach the valuation $2/3$

- if $x \leqslant 2/3$: Min plays $2/3 - x$

Features on strategies needed for Min

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Deterministic value

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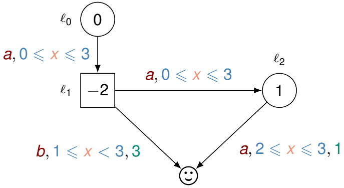

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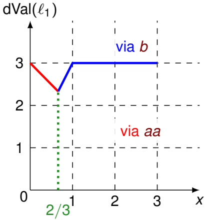

Finite memory

Switching strategy:

- σ_1: reach cycle with a weight $\leqslant -1$
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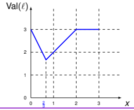
Infinite precision

From ℓ_0, Min wants to reach the valuation $2/3$

- if $x \leqslant 2/3$: Min plays $2/3 - x$
- otherwise, Min plays 0

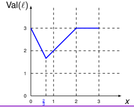
Problems on weighted timed games

Deterministic value problem



Problems on weighted timed games

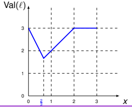
Deterministic value problem



Trading memory with probabilities

Problems on weighted timed games

Deterministic value problem

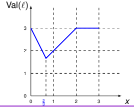


Trading memory with probabilities

Robust optimal strategies

Problems on weighted timed games

Deterministic value problem



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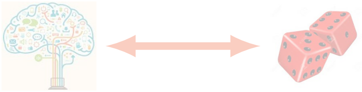

Robust optimal strategies

Deterministic value problem

Deciding if $\mathrm{dVal}(c) \leqslant \lambda$?

Deterministic value problem

Deciding if dVal(c) $\leqslant \lambda$?

	WTG			
$\mathbb{N}$	undecidable			
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On Optimal Timed Strategies, T. Brihaye, V. Bruyère and J.-F. Raskin, 2005, FORMATS

Adding Negative Prices to Priced Timed Games, T. Brihaye, G. Geeraerts, S. Krishna, L. Manasa, B. Monmege, and A. Trivedi, 2014, CONCUR

Deterministic value problem

Deciding if $\mathrm{dVal}(c) \leqslant \lambda$?

	WTG	0-clock		
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Deterministic value problem

Deciding if dVal(c) $\leqslant \lambda$?

	WTG	0-clock		
$\mathbb{N}$	undecidable	PTIME		
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On Short Paths Interdiction Problems: Total and Node-Wise Limited Interdiction, L. Khachiyan, E. Boros, K. Borys, K. Elbassioni, V. Gurvich, G. Rudolf, and J. Zhao, 2008, Theory of Computing Systems

Pseudopolynomial iterative algorithm to solve total-payoff games and min-cost reachability games., T. Brihaye, G. Geeraerts, A. Haddad, and B. Monmege, 2017, Acta Informatica

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	WTG	0-clock	divergent	
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	WTG	0-clock	divergent	
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Property of divergence

All SCCs of the WTG contain only cycles with a weight $\leqslant -1$ or $\geqslant 1$

Optimal Reachability for Weighted Timed Game., R. Alur, M. Bernadsky, and P. Madhusudan, 2004, ICALP

Optimal Strategies in Priced Timed Game Automata, P. Bouyer, F. Cassez, E.l Fleury, and K. Larsen, 2004, FSTTCS

Optimal Reachability in Divergent Weighted Timed Games., D. Busatto-Gaston, B. Monmege, and P.-A. Reynier, 2017, FOSSACS

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Almost optimal strategies in one clock priced timed games, P. Bouyer, K. Larsen, N. Markey, and J. Rasmussen, 2006, FSTTCS

Two-Player Reachability-Price Games on Single Clock Timed Automata., M. Rutkowski, 2011, QAPL

A Faster Algorithm for Solving One-Clock Priced Timed Games, T. Dueholm Hansen, R. Ibsen-Jensen, and P. Bro Miltersen, 2013, CONCUR

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PSPACE lower bound

The deterministic value problem is PSPACE-hard for 1-clock WTG

One-Clock Priced Timed Games are PSPACE-hard., J. Fearnley, R. Ibsen-Jensen, and R. Savani, 2020, LICS

Deterministic value problem

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- Back-time algorithm: compute $c \mapsto \mathrm{Val}(c)$ from $x = 1$ to 0

Deterministic value problem

Deciding if $\mathrm{dVal}(c) \leqslant \lambda$?

	WTG	0-clock	divergent	1-clock
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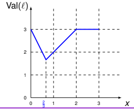
$c \mapsto \mathrm{Val}(c)$ is computable in exponential time

- Back-time algorithm: compute $c \mapsto \mathrm{Val}(c)$ from $x = 1$ to 0
- Value iteration algorithm: deterministic value is a fixed point of a given operator

Problems on weighted timed games

Deterministic value problem



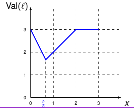
Trading memory with probabilities

Robust optimal strategies

Problems on weighted timed games

Decidability for 1-clock WTG

Deterministic value problem



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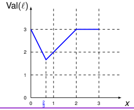
Problems on weighted timed games

Decidability for 1-clock WTG

Software prototype for 1-clock WTG

Deterministic value problem



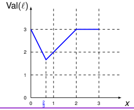
Trading memory with probabilities

Robust optimal strategies

Problems on weighted timed games

Fixpoint characterisation

Deterministic value problem



Decidability for 1-clock WTG

Software prototype for 1-clock WTG

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Robust optimal strategies

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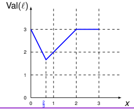
Fixpoint characterisation

Switching strategies in divergent WTG

Deterministic value problem



Decidability for 1-clock WTG

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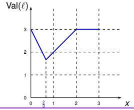


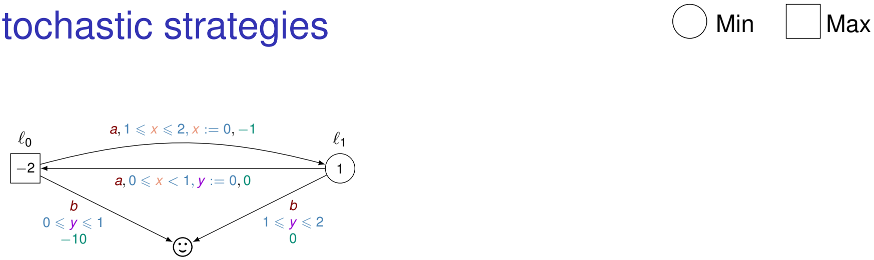
Decidability for 1-clock WTG

Software prototype for 1-clock WTG

Trading memory with probabilities

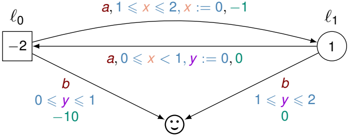
Robust optimal strategies

Stochastic strategies



Stochastic Timed Automata, N. Bertrand, P. Bouyer, T. Brihaye, Q. Menet, C. Baier, M. Grosser, and M. Jurdzinzki, 2014, Logical Methods in Computer Science

Stochastic strategies



Stochastic strategy

Distribution over possible choices

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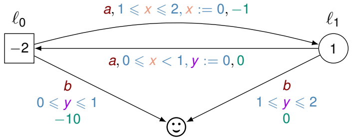
Stochastic strategies

Min Max



Stochastic strategy

Distribution over possible choices

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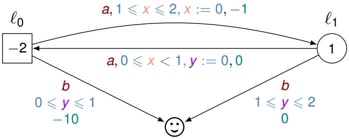
Stochastic strategies

Min Max



Stochastic strategy

Distribution over possible choices

1. Edge *a*: finite distribution
2. Delay for *a*: infinite distribution

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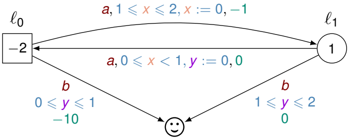
Stochastic strategies

○ Min □ Max



From $(\ell_1, \begin{bmatrix} 0 \\ 0 \end{bmatrix})$

Choose between *a* or *b* with $\mathcal{B}(\frac{1}{2})$

Stochastic strategy

Distribution over possible choices

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2. Delay for *a*: infinite distribution

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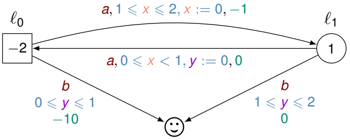
Stochastic strategies

◯ Min ☐ Max



From $(\ell_1, \begin{bmatrix} 0 \\ 0 \end{bmatrix})$

Choose between a or b with $\mathcal{B}(\frac{1}{2})$

- a: choose t with $\mathcal{U}([0, 1))$

Stochastic strategy

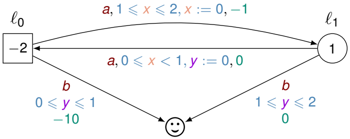
Distribution over possible choices

1. Edge a: finite distribution
2. Delay for a: infinite distribution

Stochastic Timed Automata, N. Bertrand, P. Bouyer, T. Brihaye, Q. Menet, C. Baier, M. Grosser, and M. Jurdzinzki, 2014, Logical Methods in Computer Science

Stochastic strategies

○ Min ☐ Max



From $(\ell_1, \begin{bmatrix} 0 \\ 0 \end{bmatrix})$

Choose between *a* or *b* with $\mathcal{B}(\frac{1}{2})$

- *a*: choose *t* with $\mathcal{U}([0, 1))$
- *b*: choose *t* with $\delta_{1.5}$

Stochastic strategy

Distribution over possible choices

1. Edge *a*: finite distribution
2. Delay for *a*: infinite distribution

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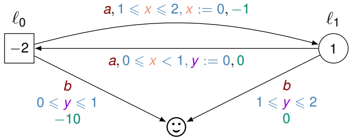
η Min θ Max



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When we fix two strategies

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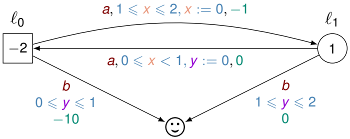

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When we fix two strategies

- Infinite Markov Chain

Stochastic strategies

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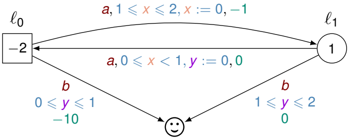

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Stochastic strategies

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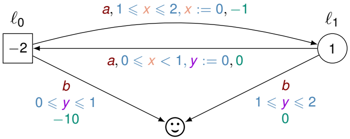

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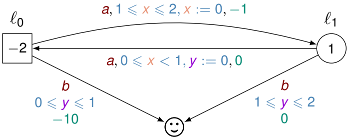
Measurability conditions on η and θ

Stochastic strategies

η Min θ Max



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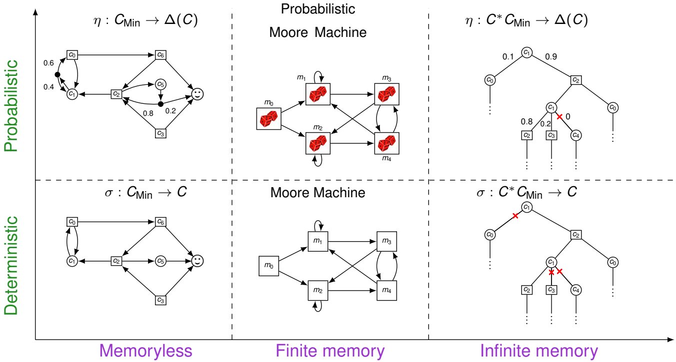

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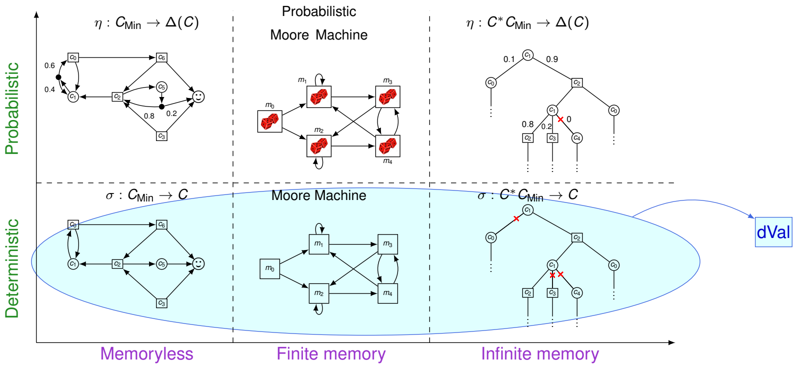

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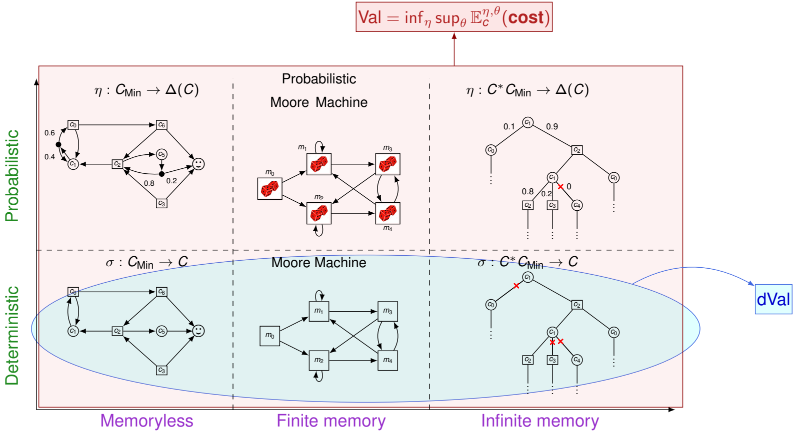

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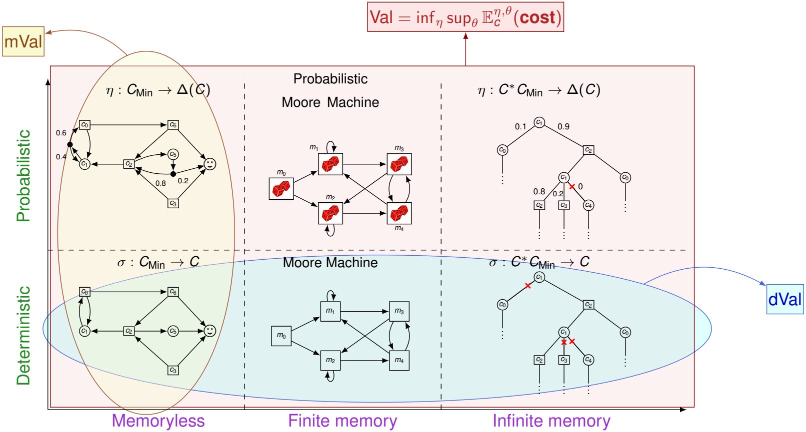

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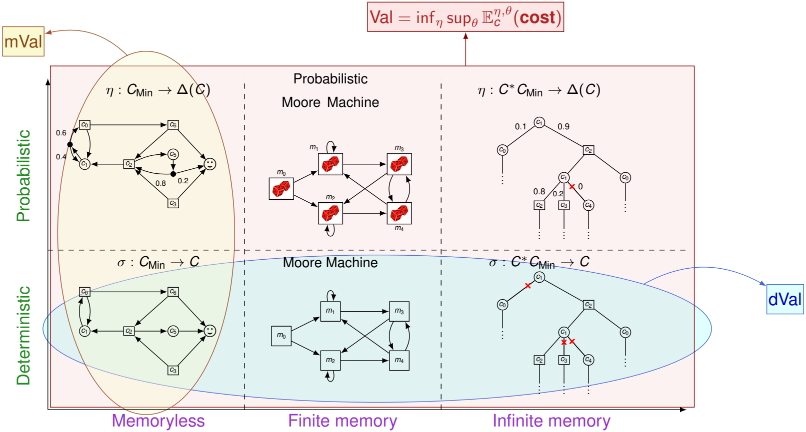
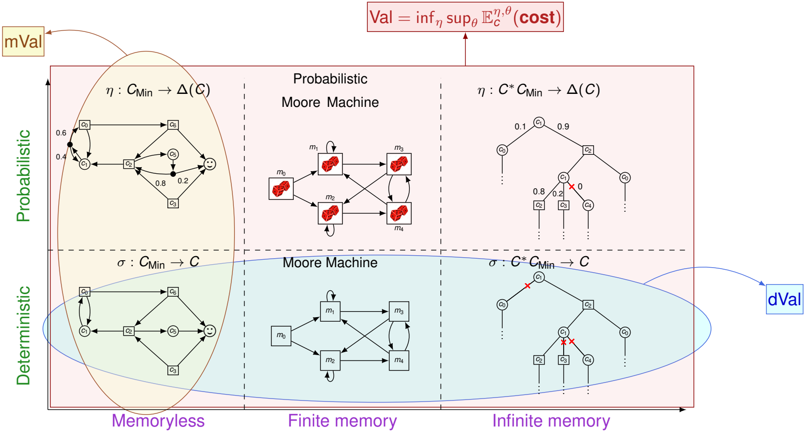


Theorem (CONCUR'20, ICALP'21): Trading memory with probabilities

Stochastic values



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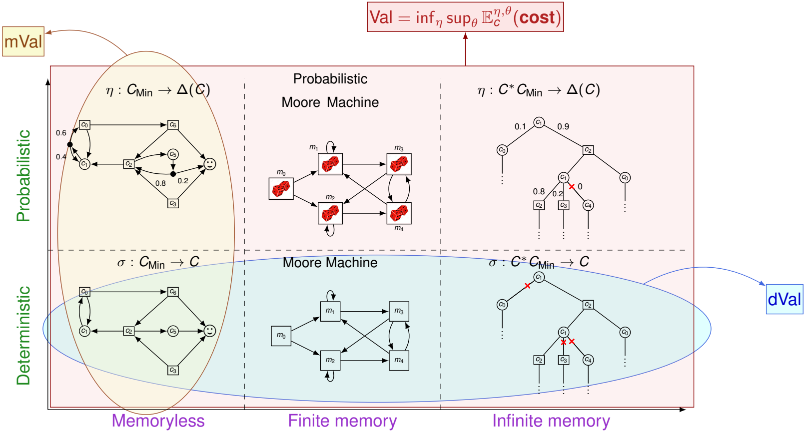
Stochastic values



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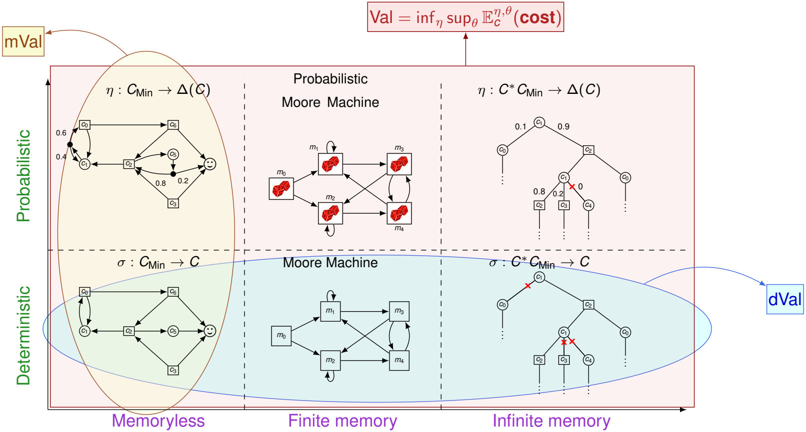

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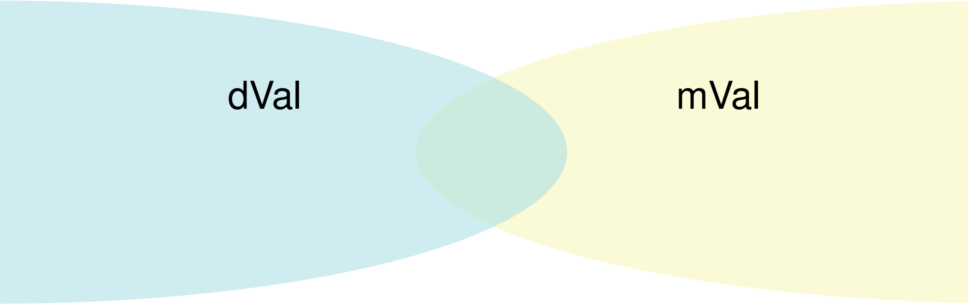
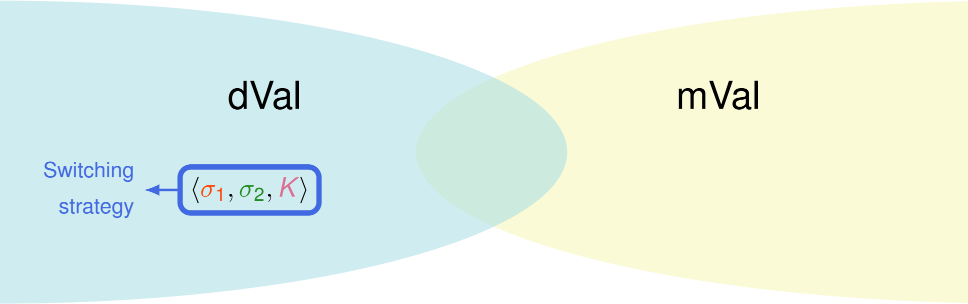
- 0-clock weighted timed games
- divergent weighted timed games

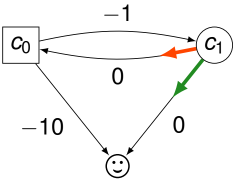
Trading memory with probabilities



Trading memory with probabilities

○ Min □ Max





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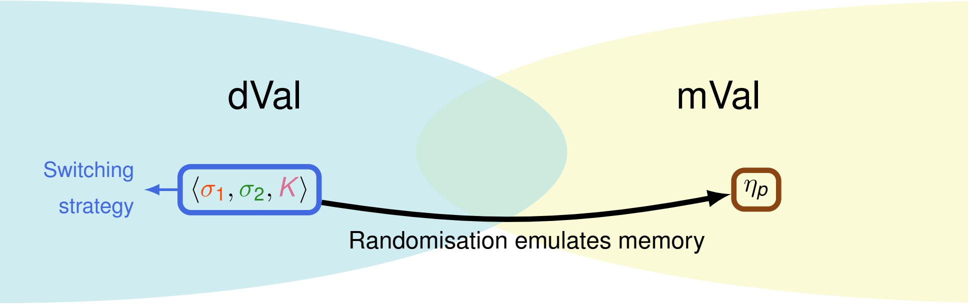

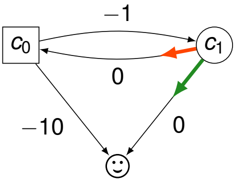

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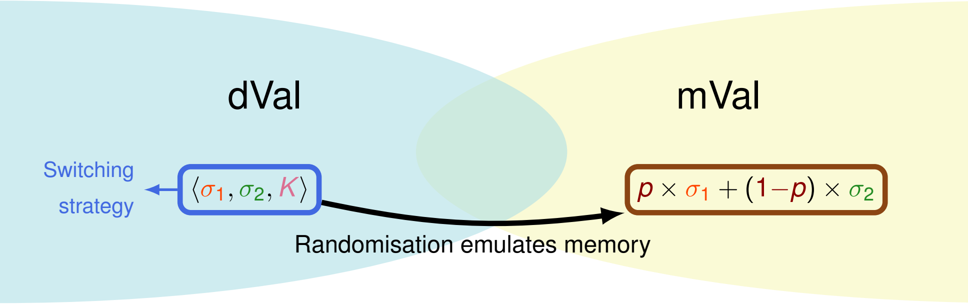

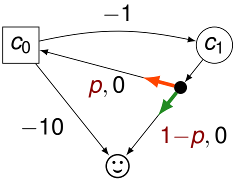

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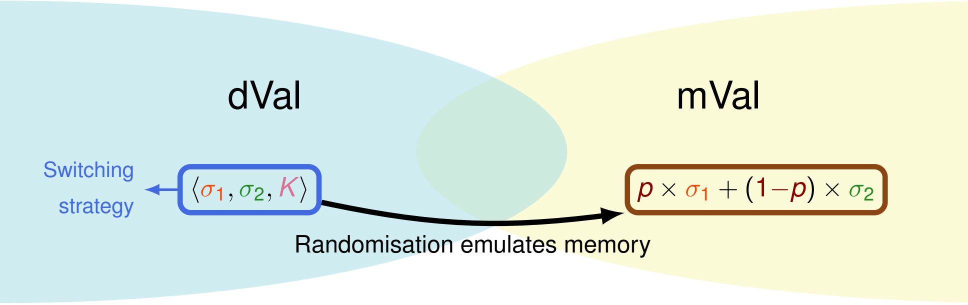

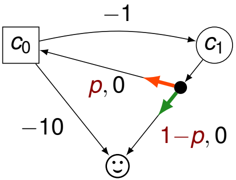

- Max has a best response deterministic memoryless strategy: τ

Trading memory with probabilities

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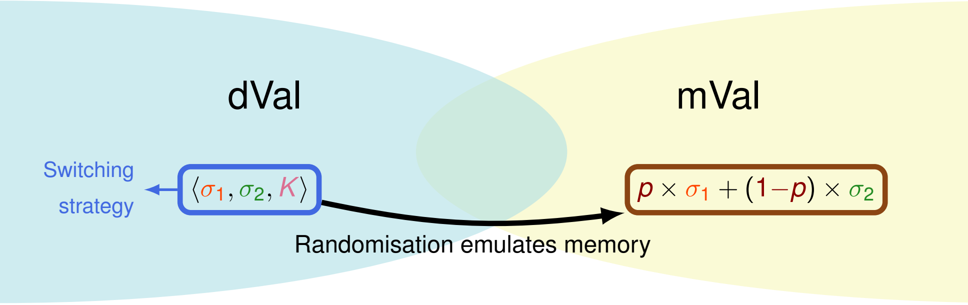

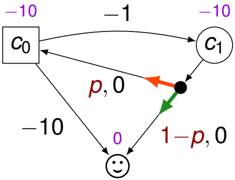

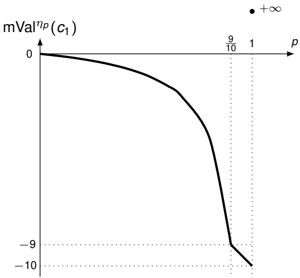

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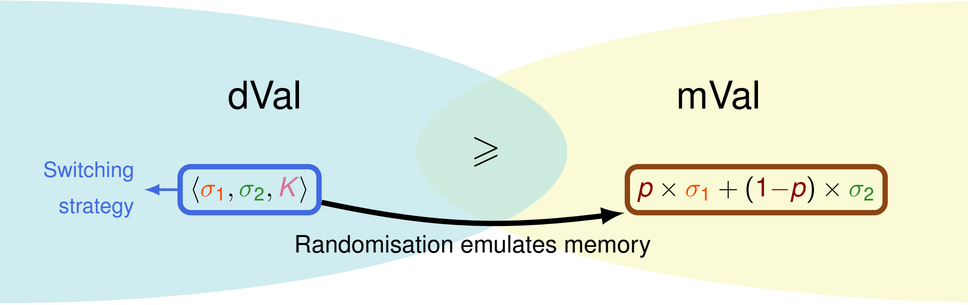

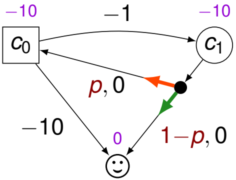

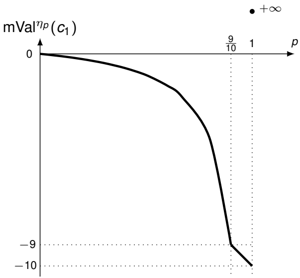

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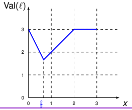
Problems on weighted timed games

Fixpoint characterisation

Switching strategies in divergent WTG

Deterministic value problem



Decidability for 1-clock WTG

Software prototype for 1-clock WTG

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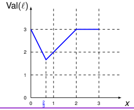

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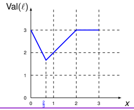

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Memory is useless in divergent WTG and 0-clock WTG

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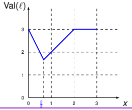

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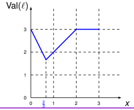

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Robustness in weighted timed games

$$\nu \xrightarrow{t} \nu + t$$

Robustness in weighted timed games

Give to Max the power to perturb the delay chosen by Min

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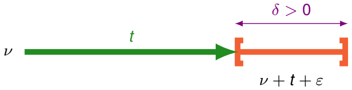

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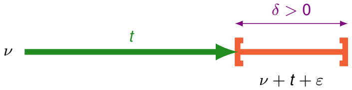

Robust semantics

Check the guard after the perturbation:

Robustness in weighted timed games

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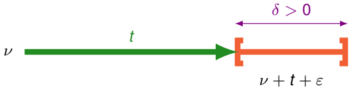

Robust semantics

Check the guard after the perturbation: $\forall \varepsilon \in [0, \delta], \nu + t + \varepsilon$ satisfies the guard

Robustness in weighted timed games

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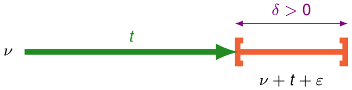

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Two problems induced by our knowledge on δ

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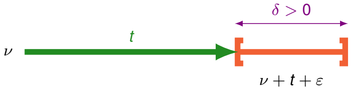

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χ Min

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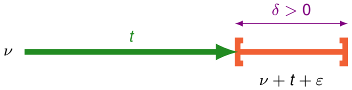

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$$\mathrm{rVal}^{\delta}(c) = \inf_{\substack{\chi \\ \delta\text{-robust}}} \sup_{\substack{\zeta \\ \delta\text{-robust}}} \mathbf{cost}(\mathrm{Play}(c, \chi, \zeta))$$

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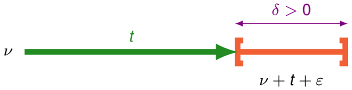

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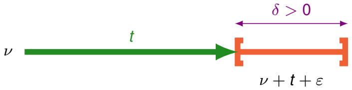

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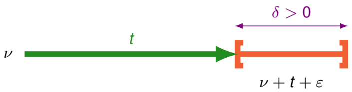

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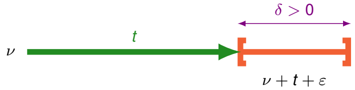

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Encoding fixed-δ semantics into exact one

Need a new clock

- δ tends to 0

$$\mathsf{rVal}(c) = \lim_{\substack{\delta \to 0 \\ \delta > 0}} \mathsf{rVal}^\delta(c)$$

rVal^δ is monotonic in δ

Robust value problems

Deciding if $\mathrm{rVal}^{\delta}(c)$ (resp. $\mathrm{rVal}(c)$) is at most equal to λ?

Robust value problems

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	WTG			
rVal^{δ}	undecidable			
rVal	undecidable			

Robust Weighted Timed Automata and Games, P. Bouyer, N. Markey, and O. Sankur, 2013, FORMATS

Robust value problems

Deciding if $\mathrm{rVal}^{\delta}(c)$ (resp. $\mathrm{rVal}(c)$) is at most equal to λ?

	WTG	acyclic	divergent	1-clock
rVal^{δ}	undecidable			
rVal	undecidable			

Robust value problems

Deciding if $\mathrm{rVal}^{\delta}(c)$ (resp. $\mathrm{rVal}(c)$) is at most equal to λ?

	WTG	acyclic	divergent	1-clock
rVal^{δ}	undecidable			decidable (in $\mathbb{N}$)
rVal	undecidable			

Revisiting Robustness in Priced Timed Game, S. Guha, S. Krishna, L. Manasa, and A. Trivedi, 2015, FSTTCS

Robust value problems

Deciding if $\mathrm{rVal}^{\delta}(c)$ (resp. $\mathrm{rVal}(c)$) is at most equal to λ?

	WTG	acyclic	divergent	1-clock
rVal^{δ}	undecidable	decidable	decidable	decidable (in $\mathbb{N}$)
rVal	undecidable			

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Optimal reachability for weighted timed games, R. Alur, M. Bernadsky and P. Madhusudan, 2004, ICALP

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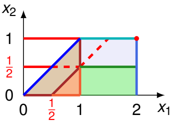
$y = \sum_i a_i\, x_i + b$



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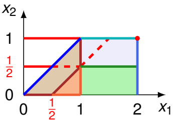
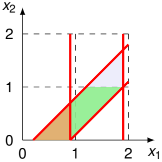


Shrunk DBM



Shrinking timed automata, O. Sankur, P. Bouyer, and N. Markey, 2011, FSTTCS

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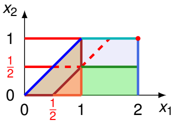

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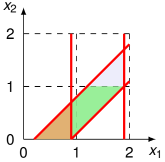

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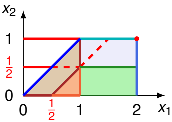

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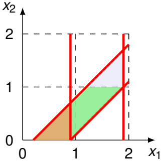

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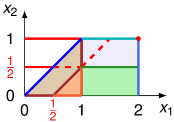

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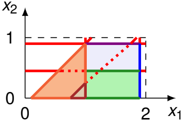

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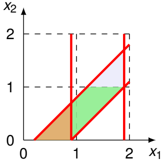

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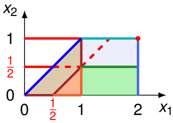

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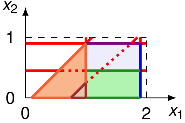

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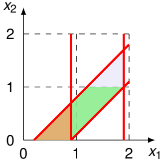
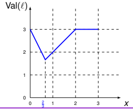


Problems on weighted timed games

Fixpoint characterisation

Switching strategies in divergent WTG

Deterministic value problem



Decidability for 1-clock WTG

Software prototype for 1-clock WTG

Definition of stochastic values

Memory is useless in divergent WTG and 0-clock WTG

Trading memory with probabilities

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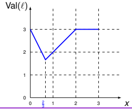

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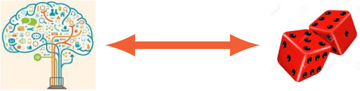

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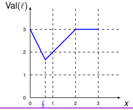

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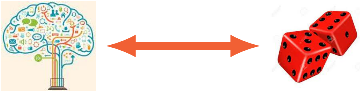

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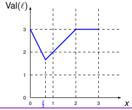

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Computing robust values in divergent (and acyclic) WTG

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Decidability of rVal(c) $< +\infty$ in all WTGs

Counterfactual causality

Joint work with Christel Baier and Jakob Piribauer at Dresden (Germany)

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Why?

Counterfactual causality

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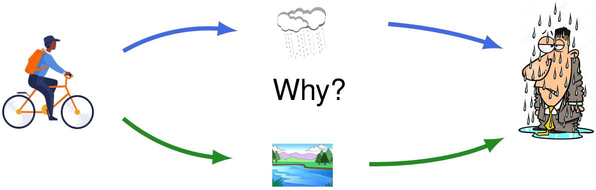

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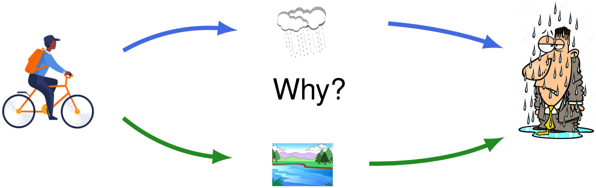
Why the specification does not hold in the counterexample?



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¬Cause (in the system) implies ¬Effect (in *closed* execution)

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Definition (GandALF'23): Definition of counterfactual causes in transitions systems and games

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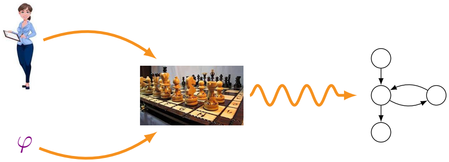
Definition (GandALF'23): Definition of counterfactual causes in transitions systems and games

Using distance over executions (strategies) to define *close*

Timed Church synthesis (work on progress)

Joint work with Sławomir Lasota at Warsaw (Poland)



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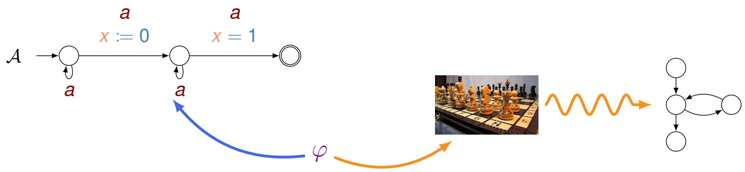

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Timed Church synthesis

$A \times \mathbb{Q}_{\geqslant 0}$

Timed Church synthesis (work on progress)

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Timed Church synthesis

$A \times \mathbb{Q}_{\geqslant 0}$

B

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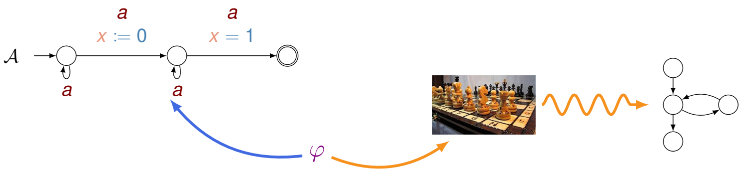

Timed Church synthesis

(a_1, t_1)

$A \times \mathbb{Q}_{\geqslant 0}$

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Timed Church synthesis

Produce $w \in (A \times B \times \mathbb{Q}_{\geqslant 0})^\omega$

$A \times \mathbb{Q}_{\geqslant 0}$ (a_1, t_1) (a_2, t_2) $\cdots$

B b_1 b_2 $\cdots$

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Existence winning strategy		

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Existence winning strategy	 wins $\Leftrightarrow$ $w \in \mathcal{L}(\mathcal{A})$	

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$(a_1, t_1) \quad (a_2, t_2) \quad \cdots$

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	Undecidable	

Timed Games and Deterministic Separability, L. Clemente, S. Lasota, and R. Piórkowski, 2020, ICALP

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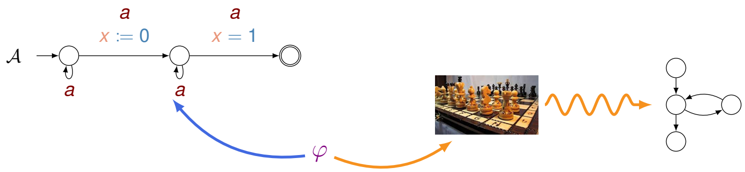

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Existence winning strategy	 wins $\Leftrightarrow$ $w \in \mathcal{L}(\mathcal{A})$	 wins $\Leftrightarrow$ $w \in \mathcal{L}(\mathcal{A})$
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- Reduce power of one player

Determinacy

Deterministic separability for timed automata

Emulate the memory

Memory versus probabilities

- Characterisation of winning strategy without memory
- New algorithms to solve games

Memory versus clocks

Memory versus control

Robustness

Synthesis of robust systems

- Parametric timed automata
- Stochastic robustness

Quantify the robustness of a system

How to synthesis a real-time system usable in the real word?

Memory from the specification

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Thank you. Questions?