Synthesis of Robust Optimal Real-Time Systems

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MFCS 2024

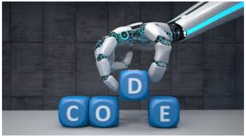
Motivation: game theory for synthesis

Classical approach

Check the correctness of a system

Game theory

Interaction between two antagonistic agents: environment and controller

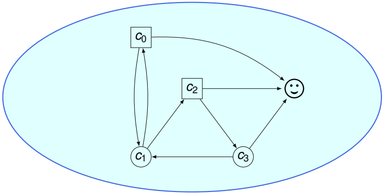


Code synthesis

Correct by construction: synthesis of controller

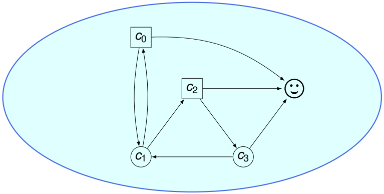
Different classes of games

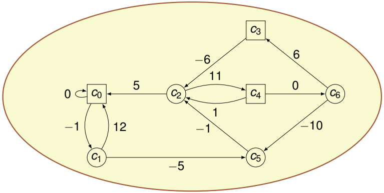
Qualitative games



Different classes of games

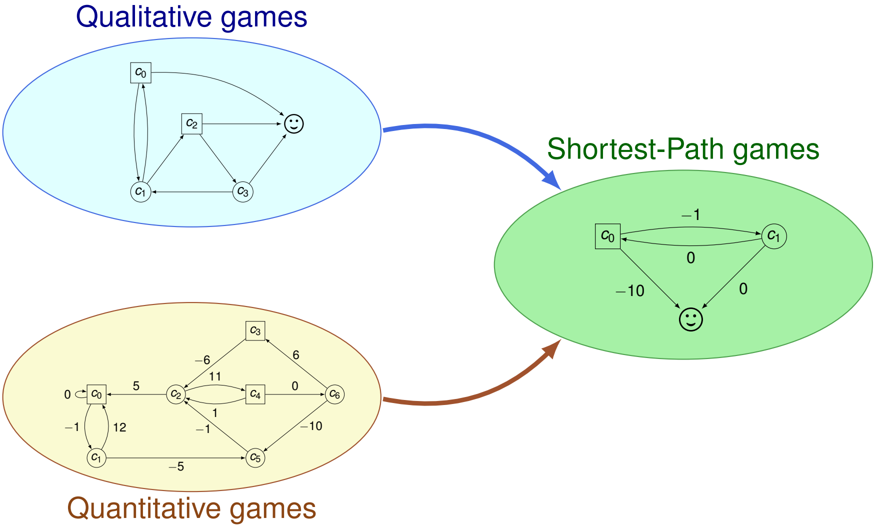
Qualitative games





Quantitative games

Different classes of games



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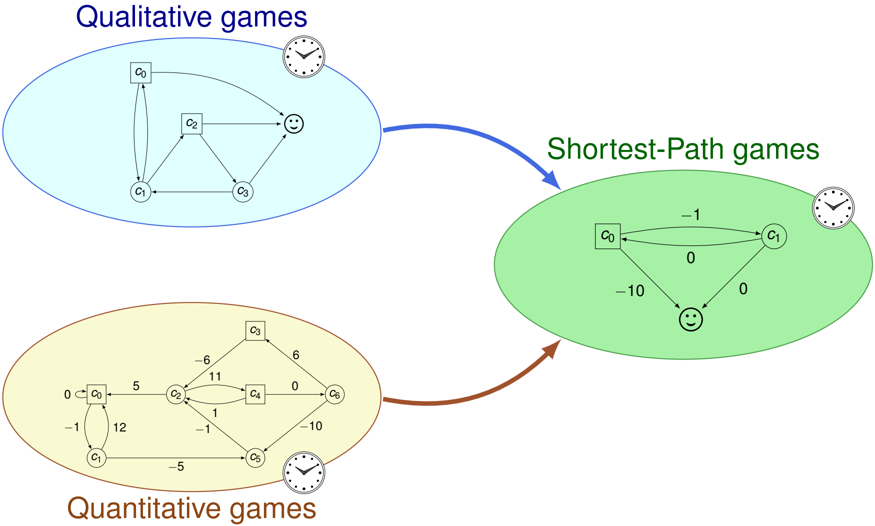
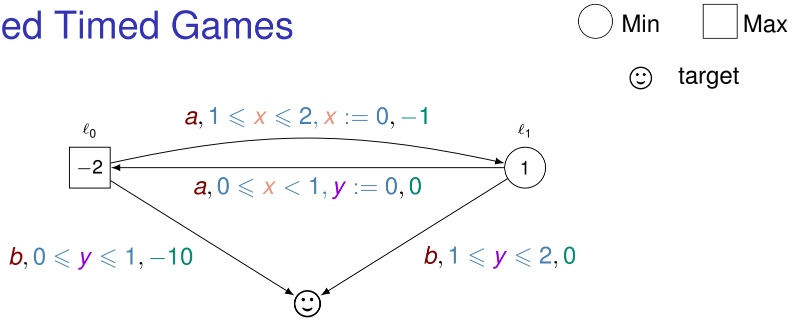


Weighted Timed Games



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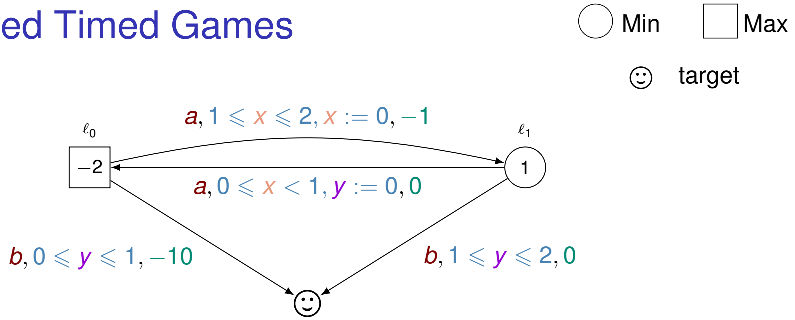
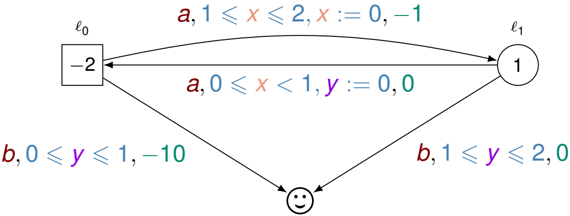


Play ρ $\left(\ell_1, \begin{bmatrix} x \mapsto 0 \\ y \mapsto 0 \end{bmatrix}\right)$

Weighted Timed Games

○ Min ☐ Max

☺ target



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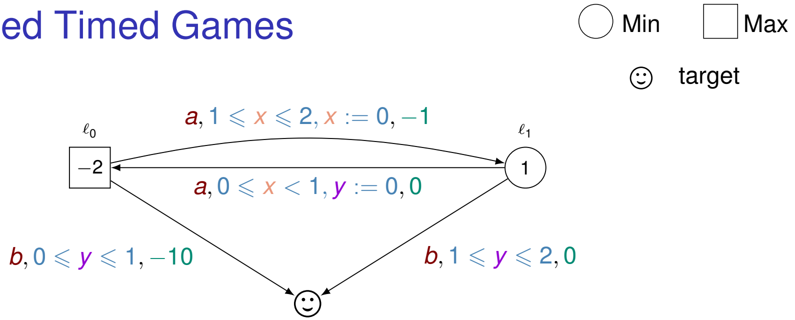
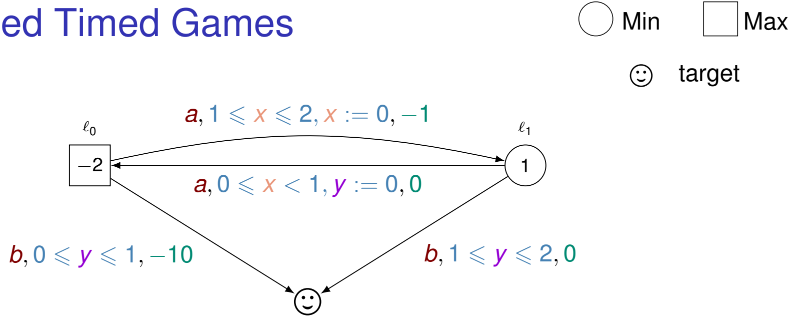
Weighted Timed Games



Play ρ $\quad (\ell_1, \begin{bmatrix} 0 \\ 0 \end{bmatrix}) \xrightarrow{0.5,\ a}$

Weighted Timed Games

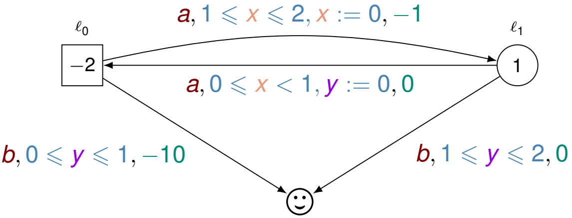


Play ρ $\quad (\ell_1, \begin{bmatrix} 0 \\ 0 \end{bmatrix}) \xrightarrow{0.5,\ a} (\ell_0, \begin{bmatrix} 0.5 \\ 0 \end{bmatrix})$

Weighted Timed Games

◯ Min ☐ Max

☺ target



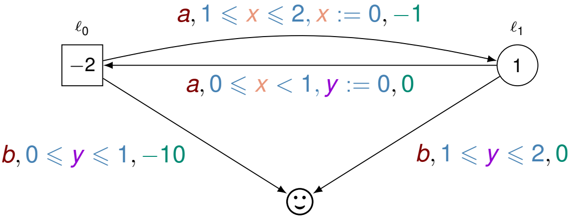
Play ρ

$$(\ell_1, \begin{bmatrix}0\\0\end{bmatrix}) \xrightarrow{0.5,\ a} (\ell_0, \begin{bmatrix}0.5\\0\end{bmatrix}) \xrightarrow{1.25,\ a} (\ell_1, \begin{bmatrix}0\\1.25\end{bmatrix}) \xrightarrow{1/3,\ b} (☺, \begin{bmatrix}1/3\\19/12\end{bmatrix})$$

Weighted Timed Games

○ Min □ Max

☺ target



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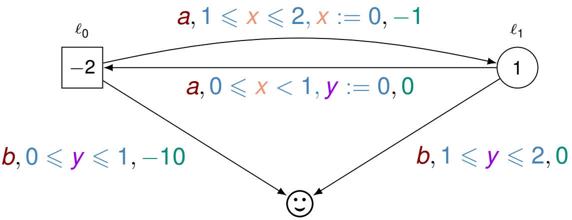
$+ \qquad +$

Weighted Timed Games

○ Min □ Max

☺ target



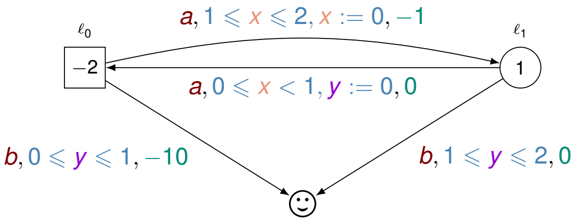
Play ρ $\quad (\ell_1, \begin{bmatrix} 0 \\ 0 \end{bmatrix}) \xrightarrow{0.5,\ a} (\ell_0, \begin{bmatrix} 0.5 \\ 0 \end{bmatrix}) \xrightarrow{1.25,\ a} (\ell_1, \begin{bmatrix} 0 \\ 1.25 \end{bmatrix}) \xrightarrow{1/3,\ b} (☺, \begin{bmatrix} 1/3 \\ 19/12 \end{bmatrix})$

$0 \quad + \quad +$

Weighted Timed Games

Min Max

target



Play ρ

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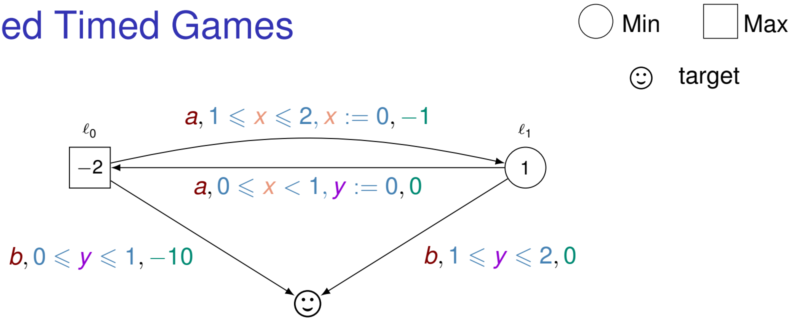
$$1 \times 0.5 + 0 \quad + \quad +$$

Weighted Timed Games

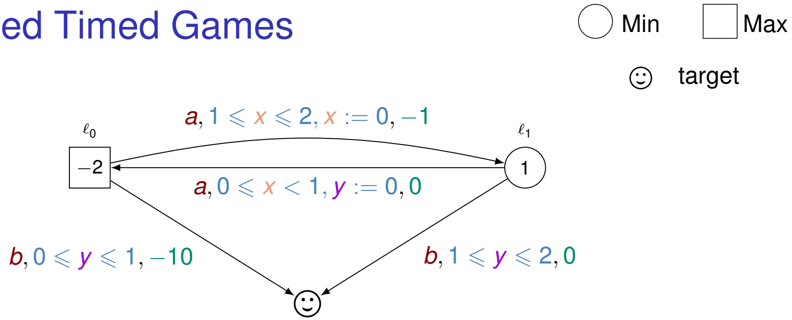


Play ρ

$$(\ell_1, \begin{bmatrix} 0 \\ 0 \end{bmatrix}) \xrightarrow{0.5,\ a} (\ell_0, \begin{bmatrix} 0.5 \\ 0 \end{bmatrix}) \xrightarrow{1.25,\ a} (\ell_1, \begin{bmatrix} 0 \\ 1.25 \end{bmatrix}) \xrightarrow{1/3,\ b} (☺, \begin{bmatrix} 1/3 \\ 19/12 \end{bmatrix}) \rightsquigarrow -\frac{8}{3}$$

$$1 \times 0.5 + 0 \quad + \quad -2 \times 1.25 - 1 \quad + \quad 1 \times \tfrac{1}{3} + 0$$

Weighted Timed Games



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Deterministic strategy

Choose an edge and a delay

Weighted Timed Games

○ Min □ Max

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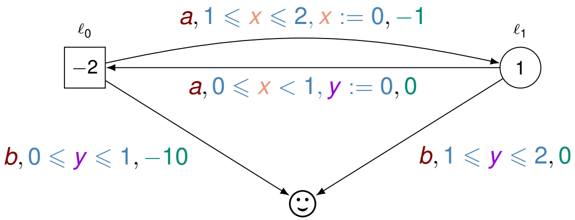

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Deterministic strategy

Choose an edge and a delay

From $(\ell_1, \begin{bmatrix} 0 \\ 0 \end{bmatrix})$

Choose a with $t = \frac{1}{3}$

How Min wins in a weighted timed game?

σ Min

τ Max

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σ Min

τ Max

Value

$$\mathsf{Val}(c) = \inf_{\sigma} \sup_{\tau} \mathbf{cost}(\mathsf{Play}(c, \sigma, \tau))$$

Pseudopolynomial iterative algorithm to solve total-payoff games and min-cost reachability games, T. Brihaye, G. Geeraerts, A. Haddad and B. Monmege, 2016, Acta Informatica

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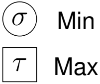

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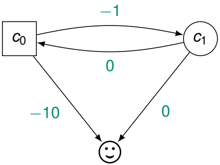

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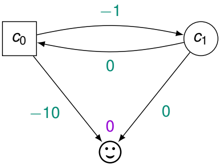

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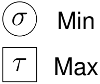

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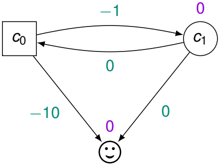

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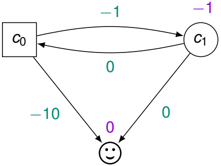

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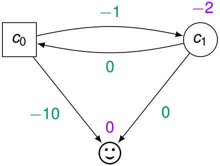

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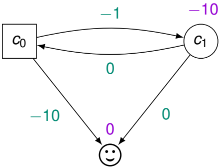

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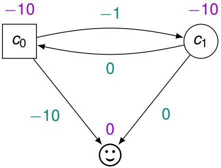

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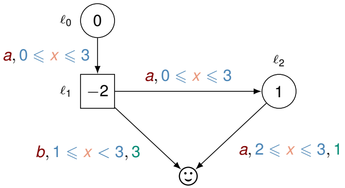

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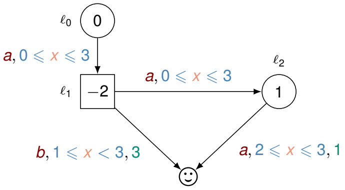

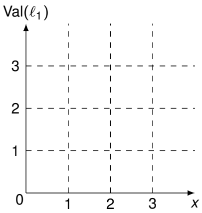

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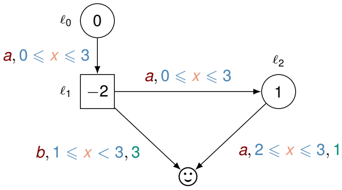

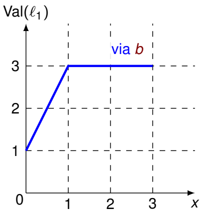

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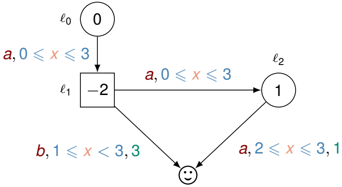

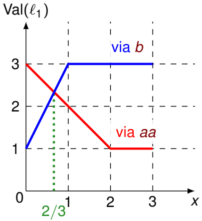

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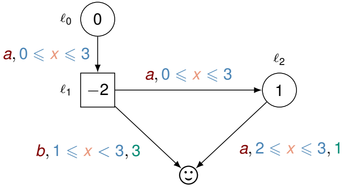

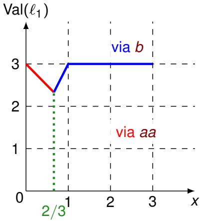

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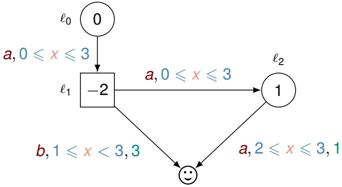
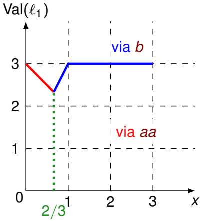
$$\mathsf{Val}(c) = \inf_\sigma \underbrace{\sup_\tau \mathbf{cost}(\mathsf{Play}(c,\sigma,\tau))}_{\mathsf{Val}^\sigma(c)}$$

Optimal strategy for Min

$$\mathsf{Val}^\sigma(c) \leqslant \mathsf{Val}(c)$$





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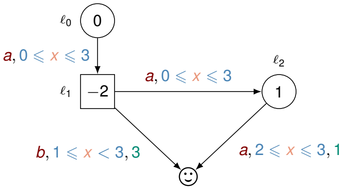

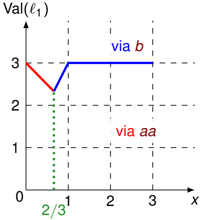

Infinite precision

From ℓ_0, Min wants to reach the valuation $2/3$

How Min wins in a weighted timed game?

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τ Max

Value

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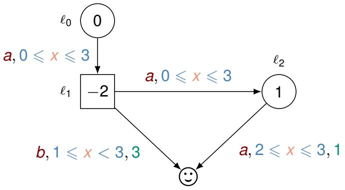
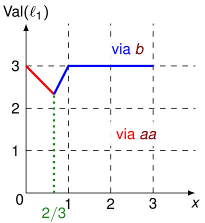




Infinite precision

From ℓ_0, Min wants to reach the valuation $2/3$

- if $x \leqslant 2/3$: Min plays $2/3 - x$

How Min wins in a weighted timed game?

σ Min

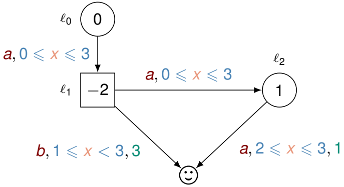
τ Max

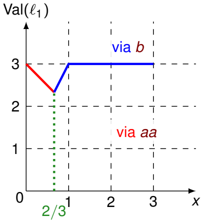
Value

$$\mathsf{Val}(c) = \inf_\sigma \underbrace{\sup_\tau \mathbf{cost}(\mathsf{Play}(c,\sigma,\tau))}_{\mathsf{Val}^\sigma(c)}$$

Optimal strategy for Min

$$\mathsf{Val}^\sigma(c) \leqslant \mathsf{Val}(c)$$





Infinite precision

From ℓ_0, Min wants to reach the valuation $2/3$

- if $x \leqslant 2/3$: Min plays $2/3 - x$
- otherwise, Min plays 0

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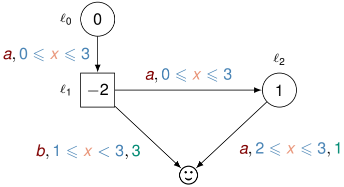
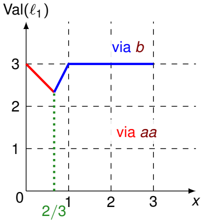


Optimal strategy for Min

$$\mathrm{Val}^{\sigma}(c) \leqslant \mathrm{Val}(c)$$



Infinite precision

From ℓ_0, Min wants to reach the valuation $2/3$

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Can Min play without the infinite precision on its strategies?

Robustness in weighted timed games

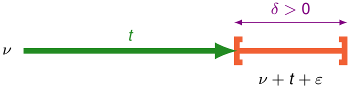
$$\nu \xrightarrow{t} \nu + t$$

Robustness in weighted timed games

Give to Max the power to perturb the delay chosen by Min

Robustness in weighted timed games

Give to Max the power to perturb the delay chosen by Min



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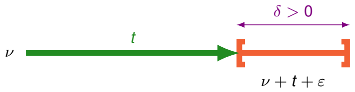

Fixed-δ semantics

Check the guard after the perturbation:

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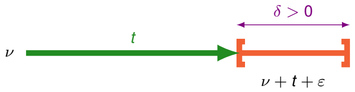

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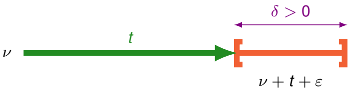

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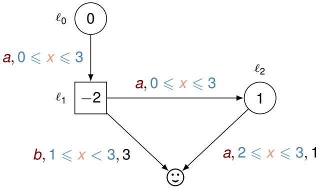

Fixed-δ play when $\delta = \frac{1}{3}$

$(\ell_0, 0)$

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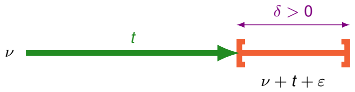

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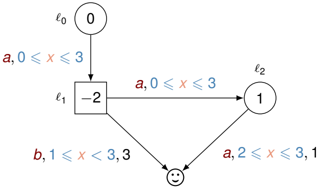

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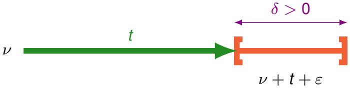

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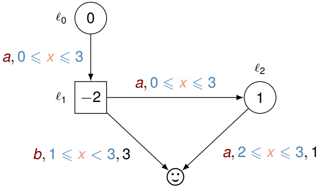

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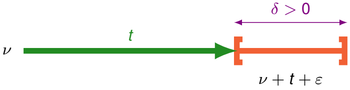

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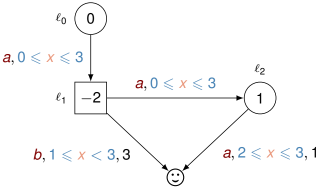

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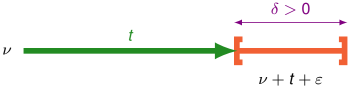

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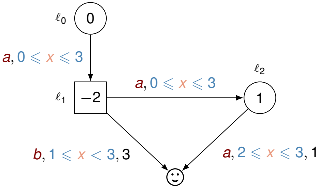

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for a weight of 3

Robust values problem

Value problem: deciding if a value is at most equal to λ?

	WTG			
Val	undecidable			

On Optimal Timed Strategies, T. Brihaye, V. Bruyère and J.-F. Raskin, 2005, FORMATS

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	WTG	acyclic	divergent	1-clock
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Divergence

All SCCs contain only cycles with a weight $\leqslant -1$ or $\geqslant 1$.

A Theory of Timed Automata, R. Alur and D. Dill, 1994, Theoretical Computer Science

Optimal Strategies in Priced Timed Game Automata, P. Bouyer, F. Cassez, E. Fleury, and K. Larsen, 2004, TCS

Optimal Reachability in Divergent Weighted Timed Games, D. Busatto-Gaston, B. Monmege, and P.-A. Reynier, 2017, FoSSaCS

Decidability of One-Clock WTG with Arbitrary Weights, B. Monmege, J. Parreaux, and P.-A. Reynier, 2022, CONCUR

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Robust Weighted Timed Automata and Games, P. Bouyer, N. Markey, and O. Sankur, 2013, FORMATS

Revisiting Robustness in Priced Timed Game, S. Guha, S. Krishna, L. Manasa, and A. Trivedi, 2015, FSTTCS

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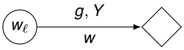

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The robust value when δ tends to 0

Fixpoint symbolic computation

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Fixpoint symbolic computation

A combination of two existing methods

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Optimal reachability for weighted timed games, R. Alur, M. Bernadsky and P. Madhusuda, 2004, ICALP

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Affine equations:
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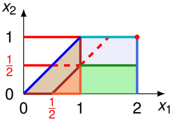

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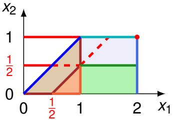

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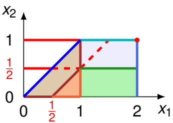

Shrunk DBM

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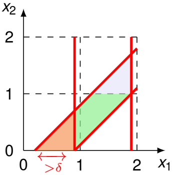

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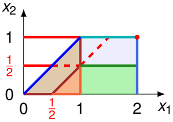

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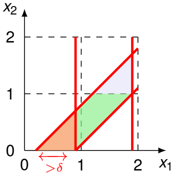

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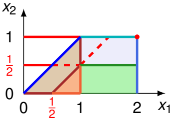

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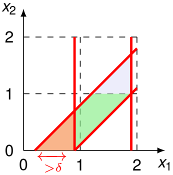

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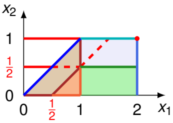

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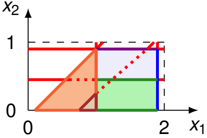

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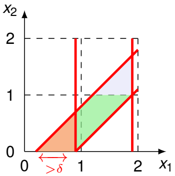

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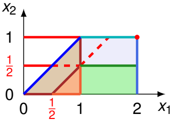

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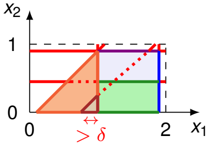

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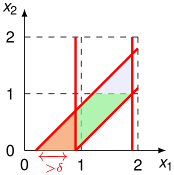

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Thank you! Questions?