Distributed local strategies in broadcast networks

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Motivation

Verification of networks of processes of unbounded size

Why considering such networks?

- Distributed algorithms (*mutual exclusion, leader election,...*)
- Telecommunication protocols (*routing,...*)
- Algorithms for ad-hoc networks
- Biological systems
- ...

Crowd networks

All the processes have the same behavior

They form a **crowd** [Esparza, STACS'14]

More precisely:

- Every process follows a same given protocol
- Processes can communicate, by either
  - Message passing
  - Shared variables
  - Rendez-vous communications
  - **Broadcast communications**
  - **Multi-diffusion (selective broadcasts)**

Question:
Is a goal reachable in some network with N processes ?

This talk

Decidability and complexity of reachability problems in parameterized networks

Features:

- Simple protocols with broadcast communication
- Simple reachability questions
- **Taking into account some *locality* assumptions**

Outline

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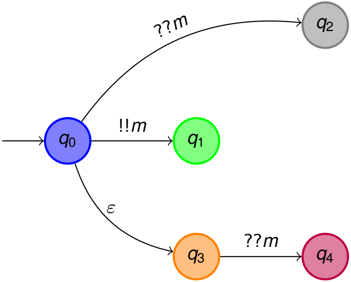
1. Networks of reconfigurable broadcast protocols
2. Restricting to local strategies
3. Conclusion

A model for reconfigurable broadcast networks

Main characteristics [Delzanno *et al.* CONCUR'10]

- No creation/deletion of nodes
- Each node executes the same finite state process
- Broadcast of the messages to the neighbors
- Communication topology evolves non-deterministically

Reconfigurable Broadcast Networks: syntax



A protocol

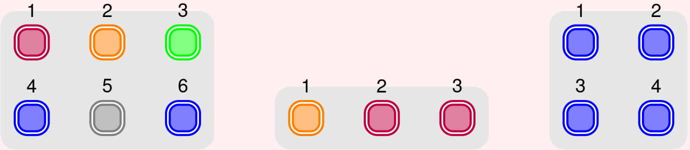
Finite state automaton. Transitions labelled with:

1. broadcast of messages - $!!m$
2. reception of messages - $??m$
3. internal actions - ε

A protocol defines a reconfigurable broadcast network

Configurations

A configuration is a vector of arbitrary size



- **Initial configurations**: **all** vertices labelled with the initial state
- *Notation :* lab(γ) *for all the labels present in* γ

Remarks:

- Size of configurations is not bounded

$\Rightarrow$ **Broadcast networks are infinite state systems**

Reconfigurable Broadcast Networks: semantics

Transition system associated with *P*

- $\mathcal{C}$: set of configurations
- $\mathcal{C}_0$: set of initial configurations
- $\rightarrow: \mathcal{C} \times \mathcal{C}$: transition relation
  - Choice a of process
  - Choice of a neighbor set
  - Execution of an action

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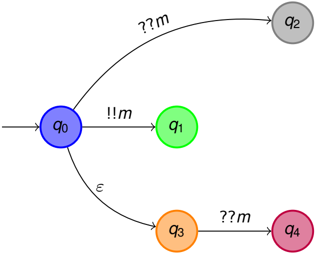
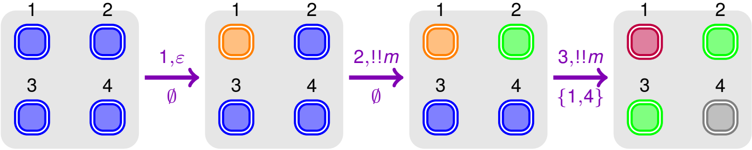
Characteristics

- The number of vertices does not change
- Two kinds of transitions
  1. **local actions** - one process performs an internal action ε
  2. **broadcast** - one process emits a message with !!*m*, all its neighbors that can receive it with ??*m* must receive it

Reconfigurable Broadcast Networks: an example





Parameterized reachability

Parameter: Number of processes

Control State Reachability (REACH)

Input: A protocol and a control state q;

Output: Does there exist $\gamma \in \mathcal{C}_0$ and $\gamma' \in \mathcal{C}$ s.t. $\gamma \rightarrow^* \gamma'$ and $q \in \mathsf{lab}(\gamma')$?

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Remarks:

- Infinite number of possible initial configurations
- Reachability of a given configuration γ' is easy, since **the number of processes is fixed**

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Theorem [Delzanno et al. FSTTCS'12]

REACH is PTIME-complete for reconfigurable broadcast networks

Outline

Local strategies

Processes do not really behave the same!

- They all follow the same protocol P, yet...
- ... if the protocol is non-deterministic, each process can take a different choice!

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Local strategies dictate processes what to do given their (local) past

Two processes with same past behave similarly

Local strategy $\sigma = (\sigma_a, \sigma_r)$

- σ_a : Path(P) $\mapsto (Q \times (\{!!m\} \cup \{\varepsilon\}) \times Q)$ active actions
- σ_r : Path(P) $\times \Sigma \mapsto (Q \times \{??m\} \times Q)$ receptions

Reachability question under local strategies

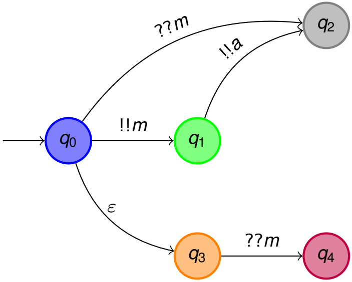
An execution respects a local strategy iff each process chooses transitions as given by the strategy

Control State Reachability (REACH[L])

Input: A protocol and a control state $q \in Q$;

Output: Does there exist $\gamma \in \mathcal{C}_0$ and $\gamma' \in \mathcal{C}$ and a local strategy σ s.t. $\gamma \to^* \gamma'$ respects σ and $q \in \mathsf{lab}(\gamma')$?

Local strategies on an example



- There is no local strategy to reach q_4
  - from q_0, either all processes move to q_1, or they all move to q_3

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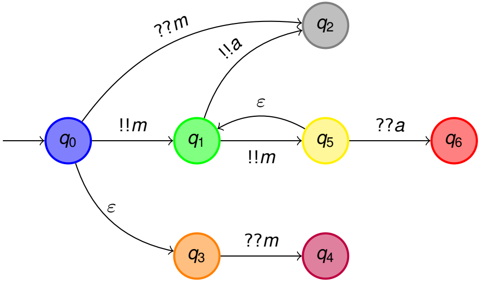

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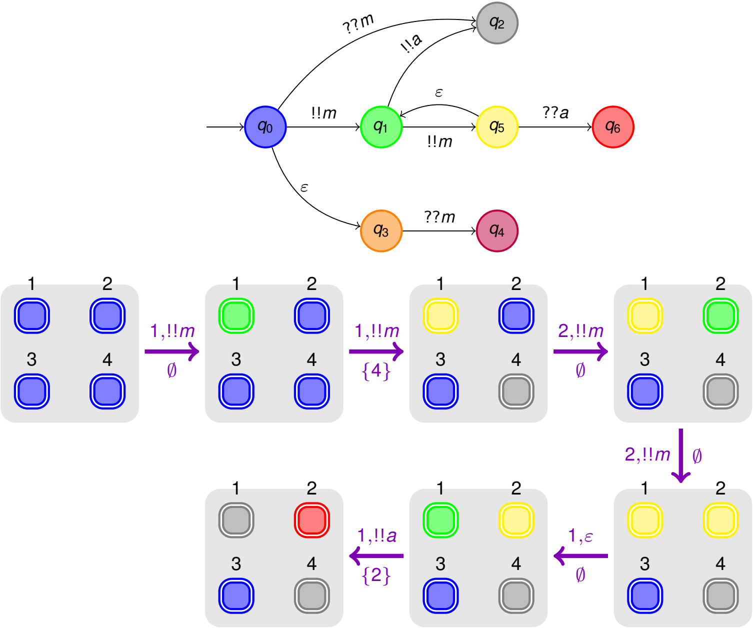
- There exists a local strategy to reach q_6

Local executions on an example



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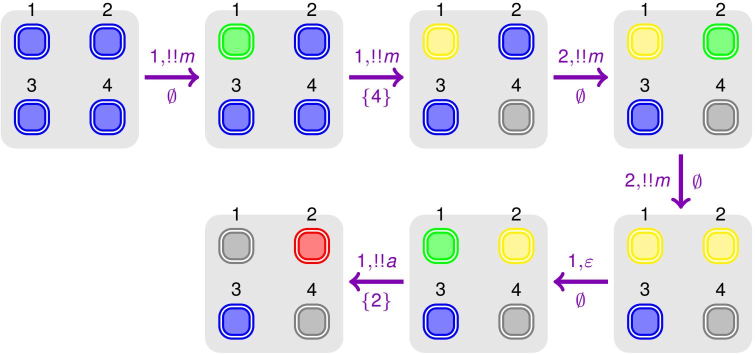
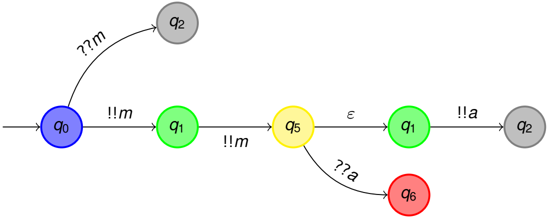


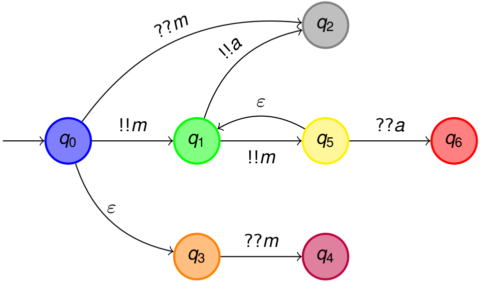


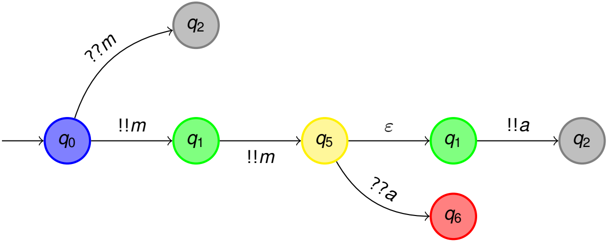
Strategy patterns for reconfigurable networks

Local strategies can be represented by trees

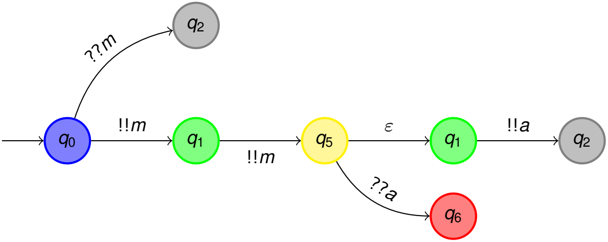
- Each path in the tree is a path in the unfolded protocol
- From each node in the tree:
  - **At most one outgoing edge is labelled by an active action (broadcast or internal action)**
  - **At most one outgoing edge labelled $??m$ per message m**
- Strategy patterns are underspecified local strategies: they may represent several local strategies

Example of strategy patterns





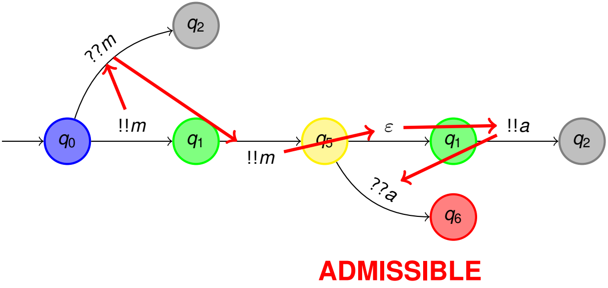
Admissible strategy patterns



An admissible strategy pattern

- A strategy pattern

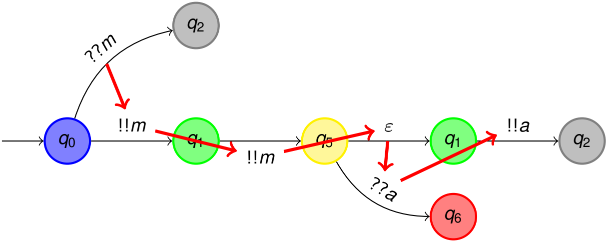
Admissible strategy patterns



An admissible strategy pattern

- A strategy pattern **+ a total order on the edges s.t.**
  - The order in the tree is satisfied
  - Each ??m is preceded by some !!m

Admissible strategy patterns



NOT ADMISSIBLE

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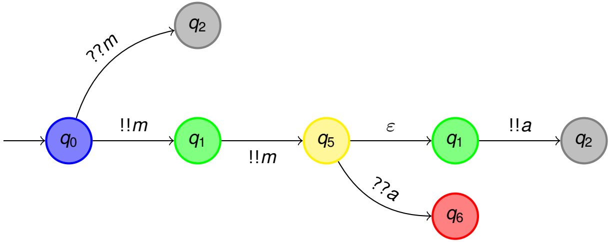

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Checking whether there exists an order such that the strategy pattern is admissible can be done in polynomial time

Solving REACH[L]

How to use strategy patterns ?

Soundness and correctness

A state is reachable in Reconfigurable Networks under a local strategy iff there is an admissible strategy pattern containing it.

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Minimization

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Theorem

REACH[L] is in NP for Reconfigurable Broadcast Networks.

NP-hardness

- Reduction from 3SAT
- 3SAT formula $\bigwedge_{i \in [1..k]} \ell_1^i \vee \ell_2^i \vee \ell_3^i$ over the variables $\{x_1, \ldots, x_r\}$

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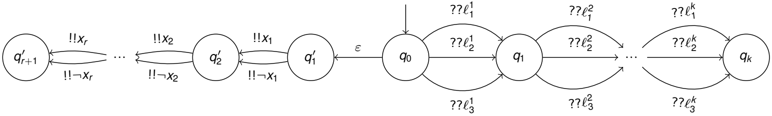

- Locality $\implies$ Uniform choice for x_i or $\neg x_i$
- Local strategy $\equiv$ Valuation
- q_k reachable iff formula satisfiable

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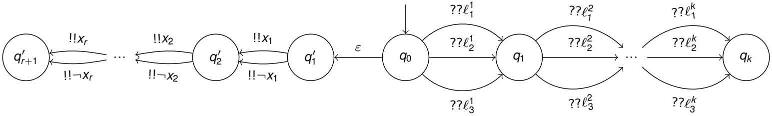

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Theorem

REACH[L] is NP-complete in Reconfigurable Broadcast Networks.

Convergence Problem

Target State Convergence (TARGET)

Input: A protocol and a set of control states T;

Output: Does there exist $\gamma \in \mathcal{C}_0$ and $\gamma' \in \mathcal{C}$ s.t. $\gamma \rightarrow^* \gamma'$ and lab(γ') $\subseteq T$?

Theorem

TARGET[L] is NP-complete in Reconfigurable Broadcast Networks.

Idea of the proof:

- Again based on strategy patterns
- Refine the notion of admissibility: the order must ensure one can 'empty' nodes towards the target set
- Still polynomial size admissible trees

Fully connected networks

Clique topology

Every broadcast necessarily reaches all participants.

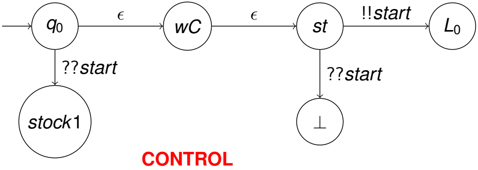
Theorem

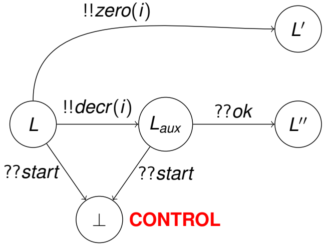
REACH[L] and TARGET[L] are undecidable in Clique Networks.

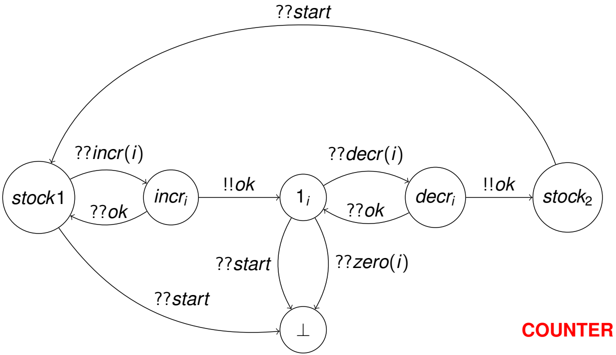
Idea of the proof:

- Encode the behavior of a Minsky machine
- For TARGET[L], as for TARGET in Clique Networks
- For REACH[L]:
  - Simulate the same run twice
  - Locality forces to do the same simulation
  - The second run can use at most as many processes for the counters as in the first run
  - Clique topology guarantees increment/decrement by 1 only

Undecidability of REACH[L] in Clique Networks







How to regain decidability?

A complete protocol

- From every state, at least one edge is labelled with an active action (internal or broadcast)
- From every state, for each message *m*, some edge is labelled with ??*m*

Property of complete protocols in clique networks: at each broadcast, all processes change their past

Theorem

REACH[L] in Clique Networks of complete protocols is decidable.

Idea of the proof:

- Abstraction: Represent processes with the same past by a single process
- Well-structured transition system

Outline

Conclusion

Local strategies in reconfigurable broadcast networks

Adding locality assumption

- all processes behave the same
- reachability and convergence are NP-complete
- technical tool: strategy patterns
- polynomial cutoff on the number of processes needed
- undecidability for clique topology
  (unless restricting to complete protocols)