Bi-harmonic Decomposition-based Maximum Loglikelihood Estimator for Carrier Phase Estimation of Coherent Optical *M*-QAM

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Abstract: A simple feedforward *M*–QAM carrier phase estimation method is presented and compared to VVMPE–ML and BPS algorithms with 64–QAM and 128–QAM. The laser-linewidth tolerance of the proposed improved method outperforms that of VVMPE–ML and BPS.

OCIS codes: (060.1660) Coherent communications; (060.4080) Modulation; (060.2360) Fiber optics links and subsystems.

1. Introduction

Coherent optical transmission systems using advanced modulation formats (*M*–ary quadrature amplitude modulation (*M*–QAM)) and free-running lasers have become a reality thanks to advances in high-speed digital signal processing (DSP) [1], in which carrier phase estimation (CPE) is an indispensable function to compensate for the random phase shifts induced by laser phase noise both at transmitter and receiver sides. In real implementations, blind feedforward CPE techniques are attractive solutions since they do not require a feedback loop [1]. Several feedforward CPE algorithms have been reported in the literature. Those include the minimum distance blind-phase-search (BPS) method [1], which is however relatively complex, some methods based on QPSK partitioning [2], which become increasingly complex with large *M*–QAM constellations, as well as the re-weighted symbol amplitudes method based on the Viterbi-Viterbi monomial-based and maximum likelihood (ML) estimator (VVMPE–ML) [3], with significant complexity reduction in comparison to the BPS method. However, these methods have been studied only with square *M*–QAM (i.e. 64–QAM). Recently, a CPE method based on crossed constellation transformation has been proposed [4], which makes it possible to operate on cross *M*–QAM (i.e. 128–QAM), at the cost of many processing stages and the use of a complex transform. A re-weighted symbol amplitude method using the fourth harmonic of the circular harmonic expansion (CHE) of a loglikelihood function (LLF), called CHE4, has been recently shown to be effective with both square and cross *M*–QAM [5], with potential reduced complexity.

In this paper, we investigate an improved CPE method that utilizes both the fourth and eighth harmonics of the CHE of LLFs, referred to as CHE48 [6], and combine it with an ML estimator in order to refine the constellation as in [3]-[5]. Instead of using power elevation for re-weighting the symbols as in VVMPE-ML, the proposed CPE uses look-up tables (LUTs) based on optimum weighting functions, resulting in not only a better phase noise estimation but also a reduced computational effort. Although the CHE4 method has been studied in the context of optical coherent communication systems for *M*–QAM in [5], CHE48 is investigated here for the first time, to the best of our knowledge, and is shown to demonstrate better performance compared to CHE4 only, especially with large *M*–QAM constellations at low signal-to-noise ratios (SNRs) (e.g. for 128–QAM at a bit-error-ratio of 1.9×10^{-2}). Moreover, sliding-average CPE [7] is used here to achieve a better tolerance towards laser linewidth compared to block-average CPE as in [5]. The proposed improved method is numerically investigated for 64–QAM and 128–QAM, showing tolerated linewidths up to 2.6 MHz and 560 kHz, respectively, for 40 Gbaud signals.

2. Harmonic and bi-harmonic methods based on circular harmonic expansion

It is assumed that the received samples at the symbol rate, $x(k) = a(k)\cdot\exp(j\cdot\phi(k))+n(k)$, are affected by complex discrete additive white Gaussian noise (AWGN), $n(k)$, with variance σ^2 for both real and imaginary parts. In this work, only the impact of unknown phase shifts due to laser phase noise is studied. The LLF for such an unknown phase shift over N_1 symbols, assuming that the sequence $\{x(k)\}$ is received, is defined as [6]

$$LLF\left(\phi(k)\,|\,\{x(k)\}\right)=\sum_{k=1}^{N_1}\log\left[\frac{1}{\left(2\pi\sigma^2 M\right)}\sum_{m=1}^{M}\exp\left(-\left|x(k)e^{-j\phi(k)}-C_m\right|^2\Big/\left(2\sigma^2\right)\right)\right] \tag{1}$$

where M and C_m (m = 1, ..., M) are the constellation size and the ideal symbol values on the constellation, respectively. Expressing the received samples in polar coordinates as $x(k)=r(k)\cdot e^{j\cdot\varphi(k)}$, and expanding (1) in Fourier series along the phase φ, the LLF approximation retaining only the first nonzero harmonic components (orders 4 and 8) is as follows

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$$LLF\left(\phi(k)\,|\,\{x(k)\}\right) \approx \Re\left(F_4\left(\{x(k)\}\right)e^{-j4\phi(k)} + F_8\left(\{x(k)\}\right)e^{-j8\phi(k)}\right) \quad (2)$$

in which $\Re$ is the real part operator, and the weighting function is defined as $F_{4n}\left(\{x(k)\}\right) = \sum_{k=1}^{N_1} A_{4n}\left(r(k)\right)e^{j4n\varphi(k)}$, where A_{4n} is the magnitude of the $4n$-th harmonic in the Fourier series [6]. The nonlinearly transformed symbols amplitudes are pre-calculated and stored into LUTs. For simplicity, we note $F4 = F_4(\{x(k)\})$ and $F8 = F_8(\{x(k)\})$.

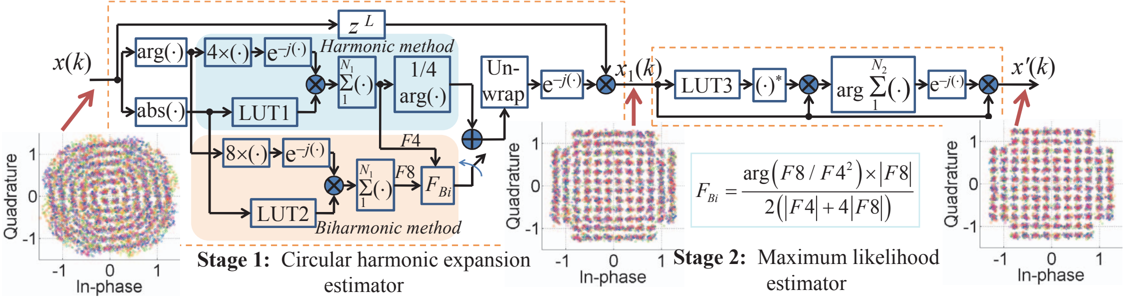

Fig. 1. CHE-ML based CPE method. The insets show 128-QAM constellations at each stage with SNR = 26.6 dB and $\Delta\nu\cdot T_S = 10^{-5}$.

Fig. 1 schematically shows the 2-stage block diagram of the proposed CPE, in which the first estimator stage, operating over N_1 symbols, can use either only the first CHE harmonic, based on the rule $\hat{\phi} = \frac{1}{4}\arg F4$, or combined first and second CHE harmonics, based on the rule $\hat{\phi} = \frac{1}{4}\arg F4 + F_{Bi}$, where $F_{Bi} = (\arg(F8/F4^2)\cdot F8/2/(F4+4F8))$, to form two CPE algorithms [6]. Sliding block averaging, based on $\hat{\phi}_{k,sliding} = \frac{1}{4}\sum_{n=-L}^{N_1-L} x\left(k-n\right)$, is used for all CPE algorithms to further improve the performance [7], where the filter half-width, L, is respectively set to $N_1/2$ or $(N_1-1)/2$, if N_1 is an even or odd number, respectively. The output symbols $x_1(k)$ are fed to the second stage carrying out standard ML operation over N_2 symbols to refine the phase estimation. The ML phase is calculated according to $\widehat{\phi_{ML}} = \arg\left[\sum_{k=1}^{N_2} DD\left(x_1(k)\right)^* x_1(k)\right]$, where DD is the direct-detection operation based on LUT3. The insets of Fig. 1 present an example of application of the CHE48-ML method for the CPE of a 128–QAM signal with a normalized linewidth (product of laser linewidth, $\Delta\nu$, and symbol duration, T_S) of $\Delta\nu\cdot T_S = 10^{-5}$.

3. Results and discussion

The performance of the proposed CPE method has been numerically studied with 64–QAM and 128–QAM constellations under the impact of laser phase noise. The M–QAM signal is generated by differentially encoding and mapping a pseudo-random binary sequence (PRBS) with length of $2^{13}-1$ onto the constellation to form ~130000 symbols. The phase noise caused by the laser linewidth is modeled as a discrete time random walk $\phi_n = \phi_{n-1} + \Delta_n$, in which Δ_n is a Gaussian random variable with zero mean and variance $2\pi\cdot\Delta\nu\cdot T_S$. The signals are transmitted and corrupted by AWGN, which is specified by the SNR. To focus on the CPE alone, only one polarization is studied and other impairments (i.e. carrier frequency offset, chromatic dispersion) are assumed to be completely compensated. In the following study, 64– and 128–QAM signals at the respective SNRs of 24.5/21.6 dB and 26.6/23.1 dB, corresponding to 1 dB penalty at the hard/soft forward-error correction (FEC) bit-error ratio (BER) limit of $10^{-3}/1.9\times10^{-2}$ are considered [8]. The numbers of phase tests used in the BPS method implemented for comparison are 64 and 128 for 64– and 128–QAM, respectively [1].

The minimum block length, defined as the shortest block length enabling reaching the target BER, is first studied. For a fair comparison, only the block lengths of the first CPE stage of CHE4-ML and CHE48-ML (N_1) are compared to those of the VVMPE–ML and BPS methods. For 64–QAM signals, the resulting BERs obtained with a normalized linewidth of 10^{-5} are shown in Fig. 2(a) for 1 dB penalty at the FEC limits of 10^{-3} (upper row) and of 1.9×10^{-2} (lower row). It can be observed that the VVMPE-ML method requires more symbols compared to other methods to reach the target BER. The required symbol number for CPE is reduced for CHE4-ML, while the CHE48-ML and BPS algorithms need fewer symbols than the others. More specifically, the minimum block lengths are 70/22/46/28 (at the FEC limit of 10^{-3}) and 64/34/46/34 (at the FEC limit of 1.9×10^{-2}) for VVMPE–ML/BPS/CHE4-ML/CHE48-ML algorithms, respectively. Fig. 2(b) presents the laser phase noise as estimated by the different algorithms compared to the actual one (normalized linewidth of 10^{-5}) for 128–QAM signals. The variances of the differences between the actual and estimated phases are $3.7\times10^{-4}/6.0\times10^{-6}/2.9\times10^{-5}/4.2\times10^{-6}$ for the VVMPE–ML/BPS/ CHE4-ML/CHE48-ML algorithms, respectively, in which the smallest variation is provided by the CHE48-ML method showing the most accurate estimation. These

CPEs are carried out using the nonlinear transformed amplitude functions, as plotted in Fig. 2(c). The block length impact for 128–QAM is evaluated in the same way without plotting the results due to space constraints, resulting in minimum block length values of –/140/220/160 (at the FEC limit of 10^{-3}) and –/160/280/160 (at the FEC limit of 1.9×10^{-2}). Note that, the VVMPE–ML method is not suitable for 128–QAM, as pointed out in [5].

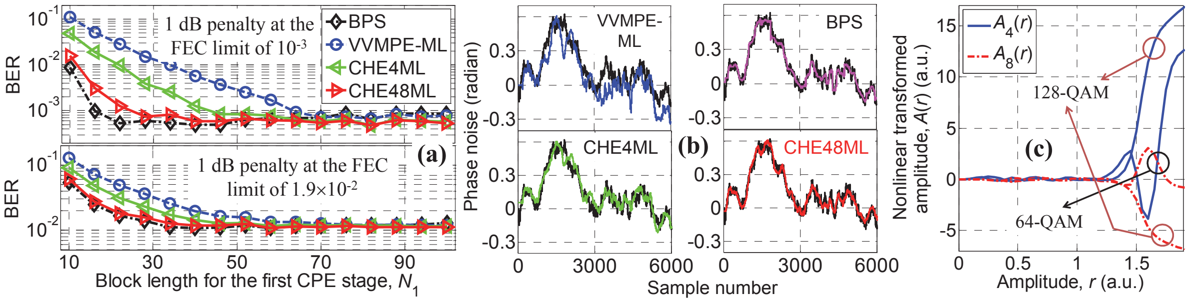

Fig. 2. (a) BER versus block length for the first CPE stage, N_1; (b) Examples of the phase noise estimated by the algorithms compared to the actual phase noise; (c) Nonlinear transformed amplitude function for 64– and 128–QAM signals.

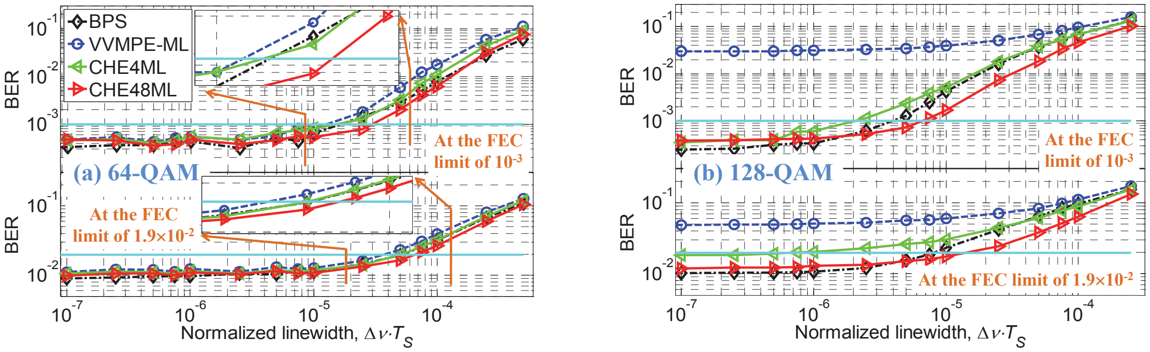

Fig. 3. BER versus normalized linewidth at the two FEC limits of 10^{-3} and 1.9×10^{-2} for (a) 64- and (b) 128-QAM.

In the next step, the laser linewidth tolerance of the CHE4-ML and CHE48-ML methods is compared to that of the other methods. The minimum block lengths for each method, as obtained from the previous investigation, are used in this study. Fig. 3 presents the results of BER calculations as a function of normalized laser linewidth for 64–and 128–QAM at the two FEC limits. As expected, the BER is deteriorated when the linewidth increases. It is seen that the proposed methods outperform the VVMPE-ML method, especially with cross *M*–QAM signals. Compared to BPS, CHE4-ML consistently shows similar performance for 64–QAM and worse performance for 128–QAM, whereas CHE48-ML provides a better performance in all cases. The detailed linewidth tolerance for each method is summarized in Table 1, in which the proposed CHE48-ML method can tolerate normalized linewidths equal to 6.6×10^{-5} and 1.4×10^{-6} for 64– and 128–QAM, respectively, with 1 dB penalty at a BER of 1.9×10^{-2}. At 40 Gbaud, the maximum linewidth requirement is therefore 2.6 MHz/560 kHz for 64–/128–QAM, showing the effectiveness of the proposed method.

Table 1. Normalized laser linewidth tolerance of the different CPE algorithms

	1 dB penalty at BER = 10^{-3}				1 dB penalty at BER = 1.9×10^{-2}			
	VVMPE-ML	BPS	CHE4-ML	CHE48-ML	VVMPE-ML	BPS	CHE4-ML	CHE48-ML
64-QAM	1.3×10^{-5}	1.6×10^{-5}	1.6×10^{-5}	3.1×10^{-5}	3.7×10^{-5}	5.0×10^{-5}	5.0×10^{-5}	6.6×10^{-5}
128-QAM	–	3.7×10^{-6}	2.1×10^{-6}	6.6×10^{-6}	–	7.5×10^{-6}	1.0×10^{-6}	1.4×10^{-5}

4. Conclusion

A simple CPE algorithm based on bi-harmonic expansion of a loglikelihood function is proposed and numerically validated with 64– and 128–QAM, showing its compatibility with commercial lasers, with linewidths of 2.6 MHz and 560 kHz for 40 Gbaud signals, respectively. The proposed method performance is better than that of BPS and of VVMPE–ML in term of laser linewidth tolerance.

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