# Analyse et Conception Formelles

## Lesson 7

–

## Program verification methods





## Outline





## Disclaimer

**Theorem 1 (Rice, 1953)**

*Any nontrivial property about the language recognized by a Turing machine is undecidable.*

"The more you prove the less automation you have"



## The basics

**Definition 2 (Specification)**

A complete description of the behavior of a software.

**Definition 3 (Oracle)**

An oracle is a *mechanism* determining whether a test has passed or failed, w.r.t a specification.

**Definition 4 (Domain (of Definition))**

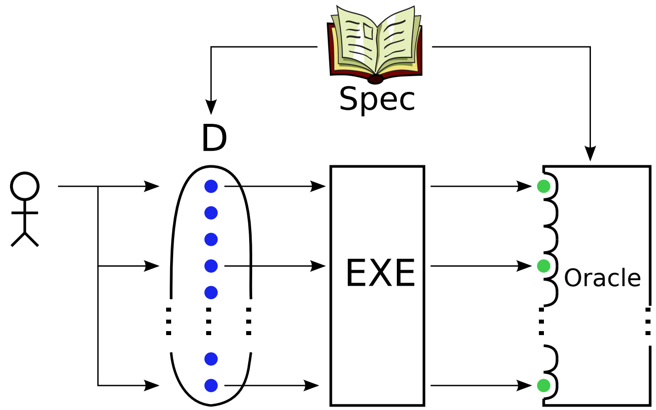
The set of all possible inputs of a program, as defined by the specification.

## Notations

| | |
|---|---|
| Spec | the specification |
| Mod | a formal model or formal prototype of the software |
| Source | the source code of the software |
| EXE | the binary executable code of the software |
| D | the domain of definition of the software |
| Oracle | an oracle |
| $D^{\#}$ | an abstract definition domain |
| $Source^{\#}$ | an abstract source code |
| $Oracle^{\#}$ | an abstract oracle |

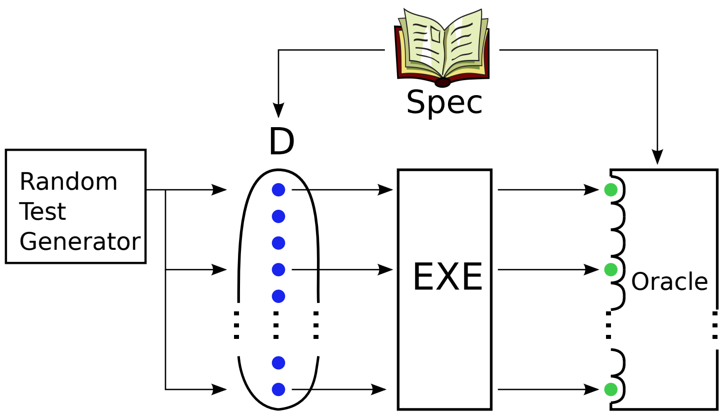


## Testing principles





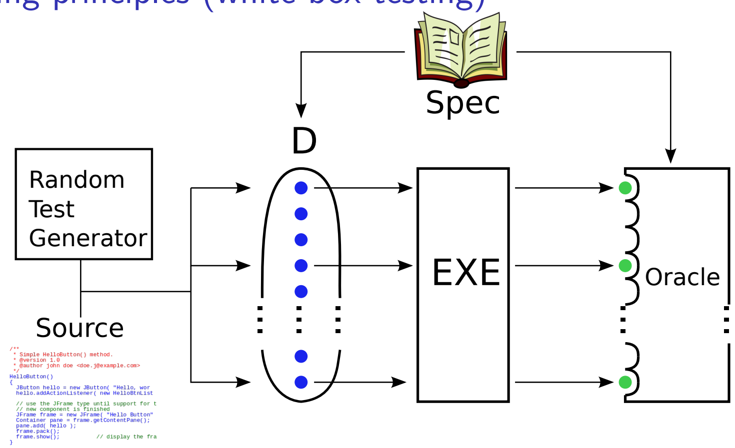

## Testing principles (random generators)




This is what Isabelle/HOL `quickcheck` does (and TP4Bis)



## Testing principles (white box testing)




**Definition 5 (Code coverage)**

The degree to which the source code of a program has been tested, *e.g.* a *statement coverage* of 70% means that 70% of all the statements of the software have been tested at least once.

## Demo of white box testing in Evosuite

Objective: cover 100% of code (and raised exceptions)

**Example 6 (Program to test with Evosuite)**

```
public static int Puzzle(int[] v, int i){
  if (v[i]>1) {
    if (v[i+2]==v[i]+v[i+1]) {
      if (v[i+3]==v[i]+18)
         throw new Error("hidden bug!");
      else return 1;}
    else return 2;}
 else return 3;
}
```



## Demo of white box testing in Evosuite

Generates tests for all branches (1, 2, 3, null array, hidden bug, etc)

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One of the **generated** JUnit test cases:

```
@Test(timeout = 4000)
public void test5()  throws Throwable  {
    int[] intArray0 = new int[18];
    intArray0[1] = 3;
    intArray0[3] = 3;
    intArray0[4] = 21;      // an array raising hidden bug!

    try {
      Main.Puzzle(intArray0, 1);
      fail("Expecting exception: Error");
    } catch(Error e) {
       verifyException("temp.Main", e);
    }
}
```



## Testing, to sum up

**Strong and weak points**

- \+ Done on the code ⟶ Finds real bugs!
- \+ Simple tests are easy to guess
- − Good tests are not so easy to guess! (Recall TP0?)
- \+ Random and white box testing automate this task. May need an oracle: a formula or a reference implementation.
- − Finds bugs but cannot prove a property
- \+ Test coverage provides (at least) a metric on software quality

**Some tool names**

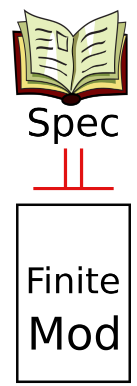
Klee, SAGE (Microsoft), PathCrawler (CEA), Evosuite, many others ...

**One killer result**

SAGE (running on 200 PCs/year) found 1/3 of security bugs in Windows 7
https://www.microsoft.com/en-us/security-risk-detection/

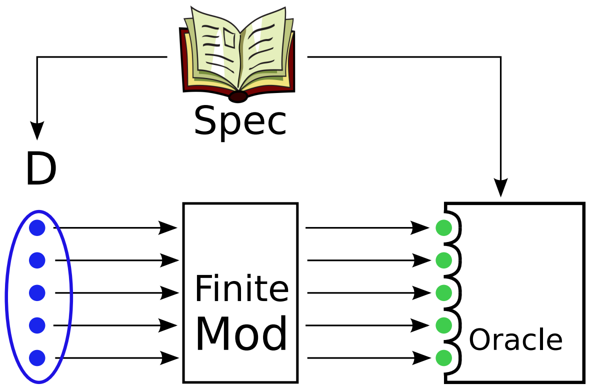


## Model-checking principles




Where $\models$ is the usual logical consequence. This property is not shown by doing a logical proof but by checking (by computation) that ...

## Model-checking principles (II)




Where D, Mod and Oracle are finite.



## Model-checking principle explained in Isabelle/HOL

Automaton `digiCode.as` and Isabelle file `cm7.thy`

**Exercise 1**

*Define the lemma stating that whatever the initial state, typing A,B,C leads execution to Final state.*

**Exercise 2**

*Define the lemma stating that the only possibility for arriving in the Final state by typing three letters is to have typed A,B,C.*



## Model-checking, to sum-up

**Strong and weak points**

+ Automatic and efficient
+ Can find bugs and prove the property
− For finite models only (*e.g* not on source code!)
+ Can deal with huge finite models ($10^{120}$ states)
  More than the number of atoms in the universe!
+ Can deal with finite abstractions of infinite models *e.g.* source code
− Incomplete on abstractions (but can find real bugs!)

**Some tool names**

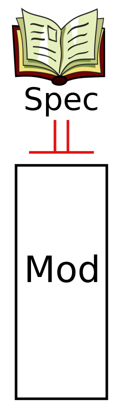
SPIN, SMV, (bug finders) CBMC, SLAM, ESC-Java, Java path finder, ...

**One killer result**

INTEL processors are commonly model-checked

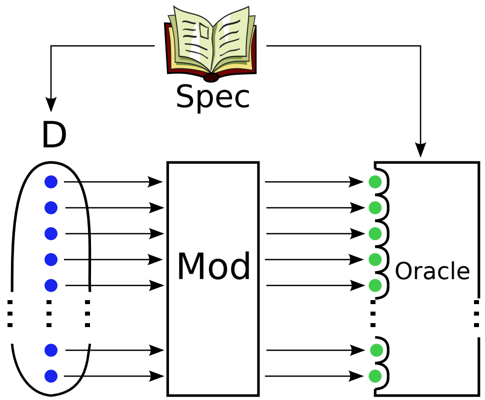


## Assisted proof principles




Where $\models$ is the usual logic consequence. This is proven directly on formulas Mod and Spec. This proof guarantees that...

## Assisted proof principles (II)




Where D, Mod, Oracle can be infinite.



## Assisted proof, to sum-up

**Strong and weak points**

- \+ Can do the proof or find bugs (with counterexample finders)
- \+ Proofs can be certified
- – Needs assistance
- – For models/prototypes only (not on source nor on EXE)
- \+ Proof holds on the source code if it is generated from the prototype

**Some tool names**

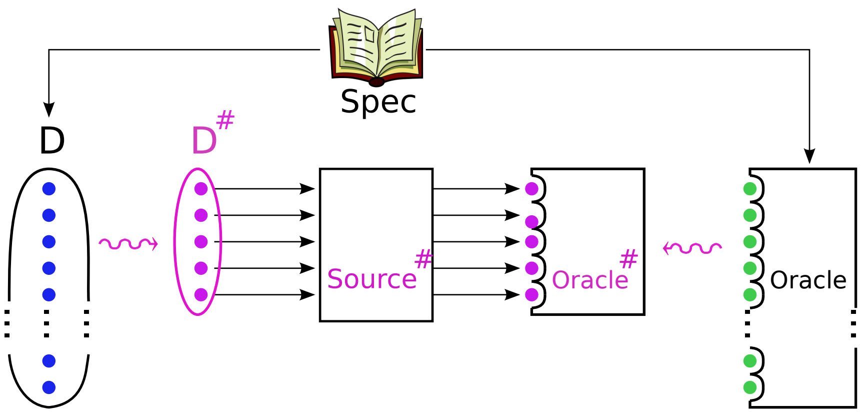
B, Coq, Isabelle/HOL, ACL2, PVS, . . . Why, Frama-C, . . .

**One killer result**

CompCert certified C compiler



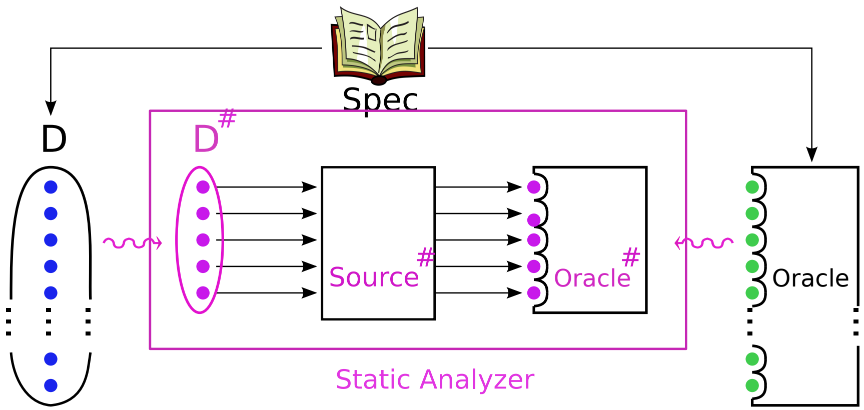
## Static Analysis principles




Where abstraction ⇝ is a correct abstraction



## Static Analysis principles (II)




Where abstraction ⇝ is a correct abstraction

## Static Analysis principles – Abstract Interpretation (III)

The abstraction '$\rightsquigarrow$' is based on the abstraction function abs:: D $\Rightarrow$ $D^{\#}$

Depending on the verification objective, precision of abs can be adapted

**Example 7 (Some abstractions of program variables for D=int)**

(1) abs:: int $\Rightarrow \{\bot, \top\}$ where $\bot \equiv$ "undefined" and $\top \equiv$ "any int"

(2) abs:: int $\Rightarrow \{\bot, \mathtt{Neg}, \mathtt{Pos}, \mathtt{Zero}, \mathtt{NegOrZero}, \mathtt{PosOrZero}, \top\}$

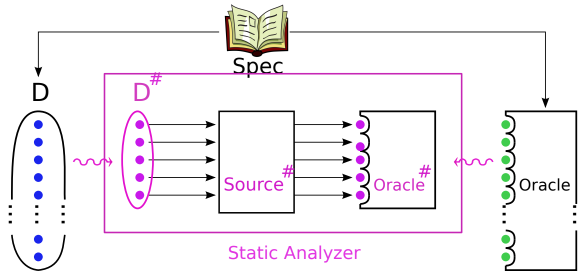
(3) abs:: int $\Rightarrow \{\bot\} \cup$ Intervals on $\mathbb{Z}$

**Example 8 (Program abstraction with abs (1), (2) and (3))**

| | (1) | (2) | (3) |
|---|---|---|---|
| x:= y+1; | x=⊥ | x=⊥ | x=⊥ |
| read(x); | x=⊤ | x=⊤ | x=]-∞;+∞[ |
| y:= x+10 | y=⊤ | y=⊤ | y=]-∞;+∞[ |
| u:= 15; | u=⊤ | u=Pos | u=[15;15] |
| x:= \|x\| | x=⊤ | x=PosOrZero | x=[0;+∞[ |
| u:= x+u; | u=⊤ | u=Pos | u=[15;+∞[ |



## Static Analysis: proving the correctness of the analyzer



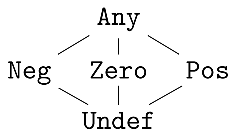

- Formalize semantics of Source language, *i.e.* formalize an `eval`
- Formalize the oracle: `BAD` predicate on program states
- Formalize the abstract domain $D^{\#}$
- Formalize the static analyser `SAn`:: program $\Rightarrow$ bool
- Prove correctness of `SAn`: $\forall$ **P**. `SAn`(**P**) $\longrightarrow$ ($\neg$ `BAD`(`eval`(**P**)))
- ... Relies on the proof that $\rightsquigarrow$ is a correct abstraction



## Static Analysis principle explained in Isabelle/HOL

To abstract `int`, we define `absInt` as the abstract domain ($D^{\#}$):

```
datatype absInt= Neg|Zero|Pos|Undef|Any
```




**Remark 1**

*Have a look at the concretization function (called* `concrete`*) defining sets of integers represented by abstract elements* `Neg`, `Zero`, *etc.*

**Exercise 3**

*Define the function* `absPlus:: absInt ⇒ absInt ⇒ absInt` *(noted* $+^{\#}$*)*

**Exercise 4 (Prove that $+^{\#}$ is a correct abstraction of $+$)**

$x \in \mathtt{concrete}(x^a) \wedge y \in \mathtt{concrete}(y^a) \longrightarrow (x+y) \in \mathtt{concrete}(x^a +^{\#} y^a)$



## Static Analysis, to sum-up

**Strong and weak points**

\+ Can prove the property
\+ Automatic
\+ On the source code
– Not designed to find bugs

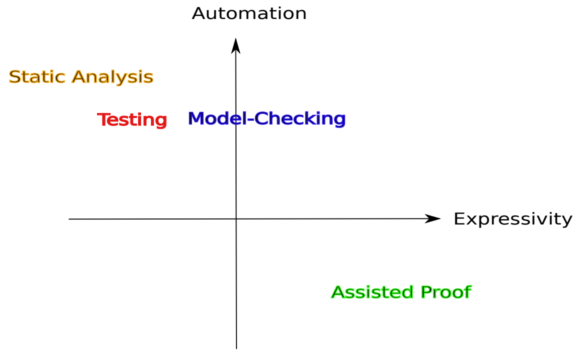
**Some tool names**

Astree (Airbus), Polyspace, Infer (Meta, though unsound and incomplete)

**Two killer results**

- Astree is used to successfully analyze $10^6$ lines of code of the Airbus A380 flight control system
- Millions of lines of Meta's production code are journally reviewed by the infer static analyzer

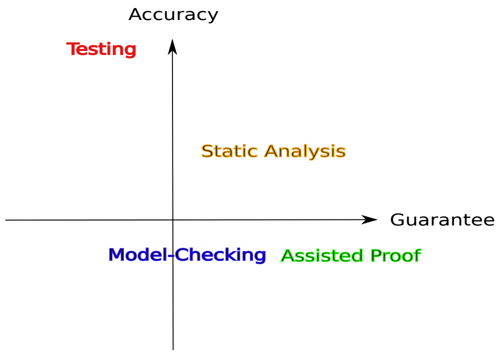
## To sum-up on all presented techniques




- Some properties are too complex to be verified using a static analyzer
- Testing can only be used to check finite properties
- Model-checking deals only with finite models (to be built by hand)
- Static analysis is always fully automatic



## To sum-up on all presented techniques

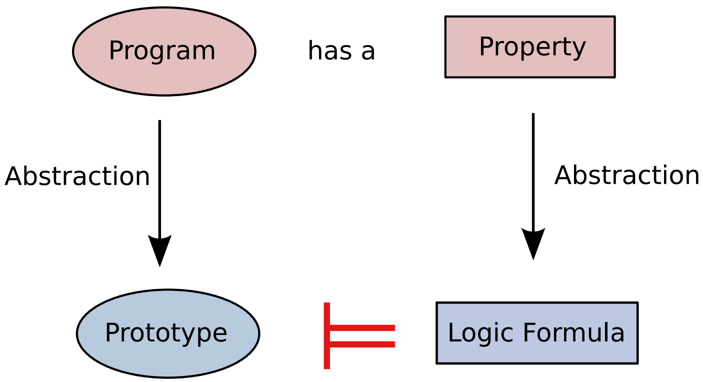



- Testing works on EXE, Static analysis on source code, others on models/prototypes
- Model-checking, assisted proof and static analysis have a similar guarantee level except that assisted proofs can be certified



## A word about models/prototypes

Program verification using "formal methods" relies on:




This is the case for model-checking and assisted proof.

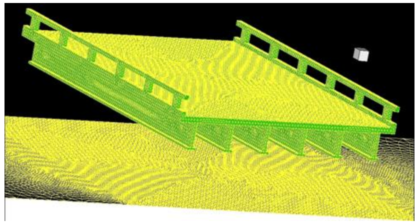


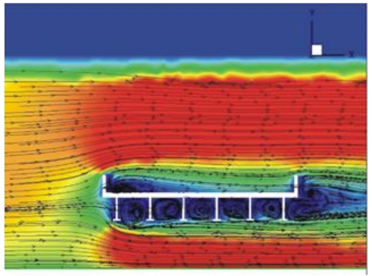
## Testing prototypes is a common practice in engineering

It is crucial for early detection of problems! Do you know Tacoma bridge?

## Testing prototypes is an engineering common practice (II)

More and more, prototypes are mathematical/numerical models





If the prototype is accurate: any detected problem is a real problem!

Problem on the prototype $\longrightarrow$ Problem on the real system

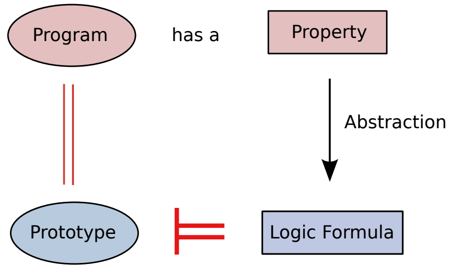
But in general, we do not have the opposite:

No problem on the prototype $\not\longrightarrow$ No problem on the real system



## Why code exportation is a great plus?

Code exportation produces the program from the model itself!




Thus, we here have a great bonus: [TP5, TP67, TP89, CompCert]

No problem on the prototype $\longrightarrow$ No problem on the real system

If the exported program is not efficient enough it can, at least, be used as a reference implementation (an oracle) for testing the optimized one.



## About "Property $\xrightarrow{\text{Abstraction}}$ Logic formula"

This is the only remaining difficulty, and this step is necessary!

**Back to TP0, it is very difficult for two reasons:**

1. The "what to do" is not as simple as it seems
   - Many tests to write and what exactly to test?
   - How to be sure that no test was missing?
   - Lack of a concise and precise way to state the property
     Defining the property with a french text is too ambigous!
2. The "how to do" was not that easy

Logic Formula = factorization of tests

- guessing 1 formula is harder than guessing 1 test
- guessing 1 formula is harder than guessing 10 tests
- guessing 1 formula is not harder than guessing 100 tests
- guessing 1 formula is faster than writing 100 tests (TP0 in Isabelle)
- proving 1 formula is stronger than writing infinitely many tests



## About formal methods and security

You have to use formal methods to secure your software
... because hackers will use them to find new attacks!

Be serious, do hackers read scientific papers?

or use academic stuff?

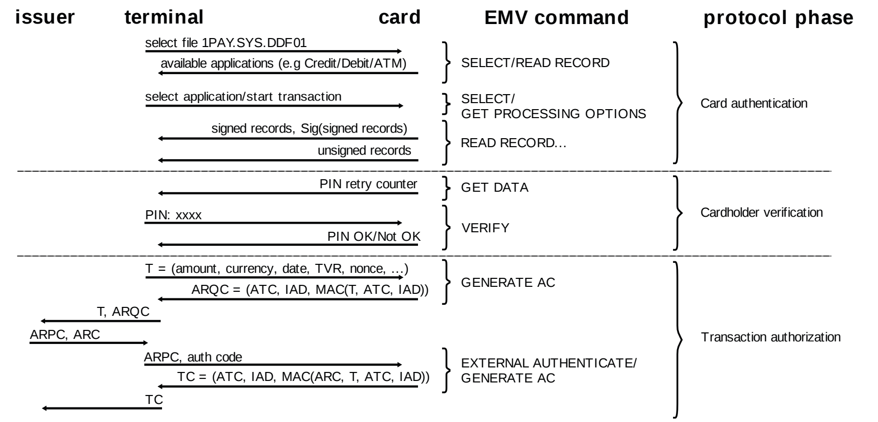
Yes, they do!

## Hackers do read scientific papers!

**Chip and PIN is Broken**

Steven J. Murdoch, Saar Drimer, Ross Anderson, Mike Bond
*University of Cambridge*
*Computer Laboratory*
*Cambridge, UK*

Conference
Security and Privacy
2010
13 pages






## Hackers do read scientific papers!

**Chip and PIN is Broken**

Steven J. Murdoch, Saar Drimer, Ross Anderson, Mike Bond
*University of Cambridge*
*Computer Laboratory*
*Cambridge, UK*

Conference
Security and Privacy
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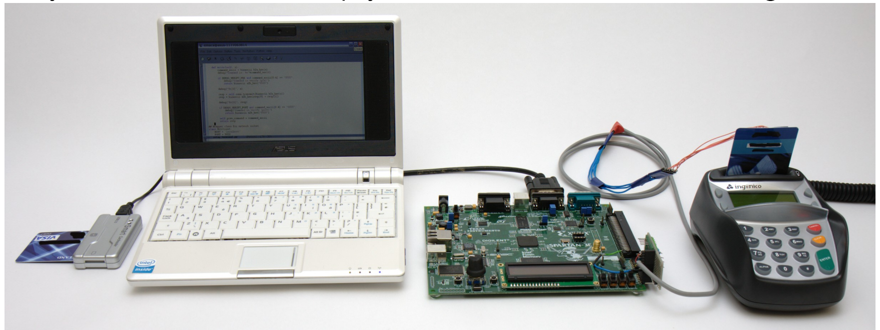
They revealed a weakness in the payment protocol of EMV

They showed how to make a payment with a card without knowing the PIN





## Hackers do read scientific papers!

**When Organized Crime Applies Academic Results**
**A Forensic Analysis of an In-Card Listening Device**

Houda Ferradi, Rémi Géraud, David Naccache, and Assia Tria

[1] École normale supérieure
Computer Science Department
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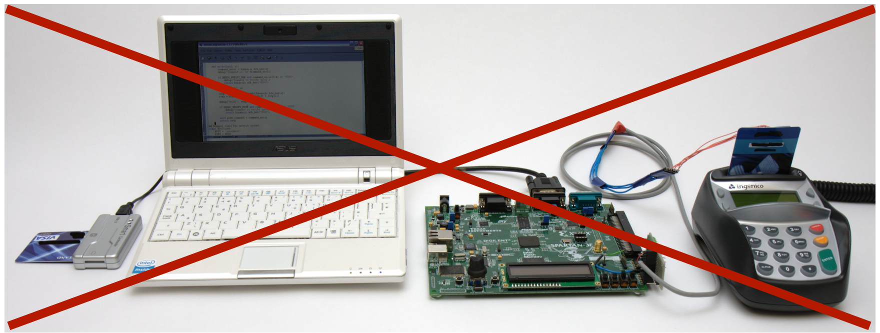
Criminals used the attack of Murdoch & al. but not:



## About formal methods and security

You have to use formal methods to secure your software

... because hackers will use them to find new attacks!

**(1 formula) + (counter-example generator) $\longrightarrow$ attack!**

- Fuzzing of implementations using model-checking
- Finding bugs (to exploit) using white-box testing
- Finding errors in protocols using counter-example gen. (e.g. TP89)

$\Longrightarrow$ You will have to formally prove security of your software!